Vectors and Coordinate Systems

Wind has both a speed and a direction, hence the motion of the wind is described by a vector.

Looking Ahead
The goals of Chapter 3 are to learn how vectors are represented and used. In this chapter you will learn to:

- Understand and use the basic properties of vectors.
- Decompose a vector into its components and reassemble vector components into a magnitude and direction.
- Add and subtract vectors both graphically and using components.

Looking Back
This chapter continues the development of vectors that was begun in Chapter 1. Please review:

- Section 1.3 Vector addition and subtraction.

Many of the quantities that we use to describe the physical world are simply numbers. For example, the mass of an object is 2 kg, its temperature is 21°C, and it occupies a volume of 250 cm³. A quantity that is fully described by a single number (with units) is called a scalar quantity. Mass, temperature, and volume are all scalars. Other scalar quantities include pressure, density, energy, charge, and voltage. We will often use an algebraic symbol to represent a scalar quantity. Thus $m$ will represent mass, $T$ temperature, $V$ volume, $E$ energy, and so on. Notice that scalars, in printed text, are shown in italics.

Our universe has three dimensions, so some quantities also need a direction for a full description. If you ask someone for directions to the post office, the reply “Go three blocks” will not be very helpful. A full description might be, “Go three blocks south.” A quantity having both a size and a direction is called a vector quantity.

You met examples of vector quantities in Chapter 1: position, displacement, velocity, and acceleration. You will soon make the acquaintance of others, such as force, momentum, and the electric field. Now, before we begin a study of forces, it’s worth spending a little time to look more closely at vectors.

3.1 Scalars and Vectors

Suppose you are assigned the task of measuring the temperature at various points throughout a building and then showing the information on a building floor plan. To do this, you could put little dots on the floor plan, to show the points at which you made measurements, then write the temperature at that point beside the dot.
Challenge Problems

74. 64 m = 0 m + (32 m/s)(4 s - 0 s) + \(\frac{1}{2}a_d(4 s - 0 s)^2\)

75. (10 m/s)^2 = v_y^2 - 2(9.8 m/s^2)(10 m - 0 m)

76. (0 m/s)^2 = (5 m/s)^2 - 2(9.8 m/s^2)(\sin 10^\circ)(x_i - 0 m)

77. \(v_t = 0 m/s + (20 m/s^2)(5 s - 0 s)\)
\(x_1 = 0 m + (0 m/s)(5 s - 0 s) + \frac{1}{2}(20 m/s^2)(5 s - 0 s)^2\)
\(x_2 = x_1 + v_{ts}(10 s - 5 s)\)

STOP TO THINK ANSWERS

Stop to Think 2.1: d. The particle starts with positive \(x\) and moves to negative \(x\).

Stop to Think 2.2: c. The velocity is the slope of the position graph. The slope is positive and constant until the position graph crosses the axis, then positive but decreasing, and finally zero when the position graph is horizontal.

Stop to Think 2.3: b. A constant positive \(v\) corresponds to a linearly increasing \(x\), starting from \(x_i = -10 m\). The constant negative \(v\) then corresponds to a linearly decreasing \(x\).

Stop to Think 2.4: a or b. The velocity is constant while \(a = 0\), it decreases linearly while \(a\) is negative. Graphs a, b, and c all have the same acceleration, but only graphs a and b have a positive initial velocity that represents a particle moving to the right.

Stop to Think 2.5: d. The acceleration vector \(\vec{a}\) points downhill (negative \(s\)-direction) and has the constant value \(-g \sin \theta\) throughout the motion.

Stop to Think 2.6: c. Acceleration is the slope of the graph. The slope is zero at \(B\). Although the graph is steepest at \(A\), the slope at that point is negative, and so \(a_A < a_B\). Only \(C\) has a positive slope, so \(a_C > a_B\).
In other words, as Figure 3.1a shows, you can represent the temperature at each point with a simple number (with units). Temperature is a scalar quantity.

Having done such a good job on your first assignment, you are next assigned the task of measuring the velocities of several employees as they move about in their work. Recall from Chapter 1 that velocity is a vector; it has both a size and a direction. Simply writing each employee's speed is not sufficient because speed doesn't take into account the direction in which the person moved. After some thought, you conclude that a good way to represent the velocity is by drawing an arrow whose length is proportional to the speed and that points in the direction of motion. Further, as Figure 3.1b shows, you decide to place the tail of an arrow at the point where you measured the velocity.

As this example illustrates, the geometric representation of a vector is an arrow, with the tail of the arrow (not its tip!) placed at the point where the measurement is made. The vector then seems to radiate outward from the point to which it is attached. An arrow makes a natural representation of a vector because it inherently has both a length and a direction.

The mathematical term for the length, or size, of a vector is magnitude, so we can say that a vector is a quantity having a magnitude and a direction. As an example, Figure 3.2 shows the geometric representation of a particle's velocity vector \( \vec{v} \). The particle's speed at this point is 5 m/s, and it is moving in the direction indicated by the arrow. The arrow is drawn with its tail at the point where the velocity was measured.

NOTE: Although the vector arrow is drawn across the page, from its tail to its tip, this does not indicate that the vector "stretches" across this distance. Instead, the vector arrow tells us the value of the vector quantity only at the one point where the tail of the vector is placed.

Arrows are good for pictures, but we also need an algebraic representation of vectors to use in labels and in equations. We do this by drawing a small arrow over the letter that represents the vector: \( \vec{r} \) for position, \( \vec{v} \) for velocity, \( \vec{a} \) for acceleration, and so on.

The magnitude of a vector is indicated by the letter without the arrow. For example, the magnitude of the velocity vector in Figure 3.2 is \( v = 5 \text{ m/s} \). This is the object's speed. The magnitude of the acceleration vector \( \vec{a} \) is written \( a \). The magnitude of a vector is a scalar quantity.

NOTE: The magnitude of a vector cannot be a negative number; it must be positive or zero, with appropriate units.

It is important to get in the habit of using the arrow symbol for vectors. If you omit the vector arrow from the velocity vector \( \vec{v} \) and write only \( v \), then you're referring only to the object's speed, not its velocity. The symbols \( \vec{r} \) and \( r \), or \( \vec{v} \) and \( v \), do not represent the same thing, so if you omit the vector arrow from vector symbols you will soon have confusion and mistakes.
The boat's displacement is the straight-line connection from its initial to its final position.

3.2 Properties of Vectors

Recall from Chapter 1 that the displacement is a vector drawn from an object's initial position to its position at some later time. Because displacement is an easy concept to think about, we can use it to introduce some of the properties of vectors. However, these properties apply to all vectors, not just to displacement.

Suppose that Sam starts from his front door, walks across the street, and ends up 200 ft to the northeast of where he started. Sam's displacement, which we will label \( \vec{S} \), is shown in Figure 3.3a. The displacement vector is a straight-line connection from his initial to his final position, not necessarily his actual path. The dotted line indicates a possible route Sam might have taken, but his displacement is the vector \( \vec{S} \).

To describe a vector, we must specify both its magnitude and its direction. We can write Sam's displacement as

\[
\vec{S} = (200 \text{ ft, northeast})
\]

where the first number specifies the magnitude and the second number is the direction. The magnitude of Sam's displacement is \( S = 200 \text{ ft} \), the distance between his initial and final points.

Sam's next-door neighbor Bill also walks 200 ft to the northeast, starting from his own front door. Bill's displacement \( \vec{B} = (200 \text{ ft, northeast}) \) has the same magnitude and direction as Sam's displacement \( \vec{S} \). Because vectors are defined by their magnitude and direction, two vectors are equal if they have the same magnitude and direction. This is true regardless of the starting points of the vectors. Thus the two displacements in Figure 3.3b are equal to each other, and we can write \( \vec{B} = \vec{S} \).

**NOTE** A vector is unchanged if you move it to a different point on the page as long you don't change its length or the direction it points. We used this idea in Chapter 1 when we moved velocity vectors around in order to find the average acceleration vector \( \vec{a} \).

**Vector Addition**

Figure 3.4 shows the displacement of a hiker who starts at point P and ends at point S. She first hikes 4 miles to the east, then 3 miles to the north. The first leg of the hike is described by the displacement \( \vec{A} = (4 \text{ mi, east}) \). The second leg of the hike has displacement \( \vec{B} = (3 \text{ mi, north}) \). Now, by definition, a vector from the initial position P to the final position S is also a displacement. This is vector \( \vec{C} \) on the figure. \( \vec{C} \) is the net displacement because it describes the net result of the hiker's first having displacement \( \vec{A} \), then displacement \( \vec{B} \).

If you earn $50 on Saturday and $60 on Sunday, your net income for the weekend is the sum of $50 and $60. With scalars, the word net implies addition. The
same is true with vectors. The net displacement \( \vec{C} \) is an initial displacement \( \vec{A} \) plus a second displacement \( \vec{B} \), or

\[
\vec{C} = \vec{A} + \vec{B}
\]  

(3.1)

The sum of two vectors is called the \textbf{resultant vector}. It's not hard to show that vector addition is commutative: \( \vec{A} + \vec{B} = \vec{B} + \vec{A} \). That is, you can add vectors in any order you wish.

Look back at Tactics Box 1.1 on page 10 to see the three-step procedure for adding two vectors. This tip-to-tail method for adding vectors, which is used to find \( \vec{C} = \vec{A} + \vec{B} \) in Figure 3.4, is called \textbf{graphical addition}. Any two vectors of the same type—two velocity vectors or two force vectors—can be added in exactly the same way.

The graphical method for adding vectors is straightforward, but we need to do a little geometry to come up with a complete description of the resultant vector \( \vec{C} \). Vector \( \vec{C} \) of Figure 3.4 is defined by its magnitude \( C \) and by its direction. Because the three vectors \( \vec{A}, \vec{B}, \) and \( \vec{C} \) form a right triangle, the magnitude, or length, of \( \vec{C} \) is given by the Pythagorean theorem:

\[
C = \sqrt{A^2 + B^2} = \sqrt{(4 \text{ mi})^2 + (3 \text{ mi})^2} = 5 \text{ mi}
\]  

(3.2)

Notice that Equation 3.2 uses the magnitudes \( A \) and \( B \) of the vectors \( \vec{A} \) and \( \vec{B} \). The angle \( \theta \), which is used in Figure 3.4 to describe the direction of \( \vec{C} \), is easily found for a right triangle:

\[
\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{3 \text{ mi}}{4 \text{ mi}}\right) = 37^\circ
\]  

(3.3)

 Altogether, the hiker's net displacement is

\[
\vec{C} = \vec{A} + \vec{B} = (5 \text{ mi}, 37^\circ \text{ north of east})
\]  

(3.4)

\textbf{NOTE} Vector mathematics makes extensive use of geometry and trigonometry. Appendix A, at the end of this book, contains a brief review of these topics.

\textbf{EXAMPLE 3.1 Using graphical addition to find a displacement}  

A bird flies 100 m due east from a tree, then 200 m northwest (that is, 45° north of west). What is the bird's net displacement?  

\textbf{VISUALIZE} Figure 3.5 shows the two individual displacements, which we've called \( \vec{A} \) and \( \vec{B} \). The net displacement is the vector sum \( \vec{C} = \vec{A} + \vec{B} \), which is found graphically.

\textbf{SOLVE} The two displacements are \( \vec{A} = (100 \text{ m, east}) \) and \( \vec{B} = (200 \text{ m, northwest}) \). The net displacement \( \vec{C} = \vec{A} + \vec{B} \) is found by drawing a vector from the initial to the final position. But describing \( \vec{C} \) is a bit trickier than the example of the hiker because \( \vec{A} \) and \( \vec{B} \) are not at right angles. First, we can find the magnitude of \( \vec{C} \) by using the law of cosines from trigonometry:

\[
C^2 = A^2 + B^2 - 2AB\cos(45^\circ)
\]

\[
= (100 \text{ m})^2 + (200 \text{ m})^2 - 2(100 \text{ m})(200 \text{ m})\cos(45^\circ)
\]

\[
= 21,720 \text{ m}^2
\]

Thus \( C = \sqrt{21,720 \text{ m}^2} = 147 \text{ m} \). Then a second use of the law of cosines can determine angle \( \phi \) (the Greek letter phi):

\[
\phi = \cos^{-1}\left[\frac{A^2 + C^2 - B^2}{2AC}\right] = 106^\circ
\]

It is easier to describe \( \vec{C} \) with the angle \( \theta = 180^\circ - \phi = 74^\circ \). The bird's net displacement is

\[
\vec{C} = (147 \text{ m, 74}^\circ \text{ north of west})
\]
When two vectors are to be added, it is often convenient to draw them with their tails together, as shown in Figure 3.6a. To evaluate $\vec{D} + \vec{E}$, you could move vector $\vec{E}$ over to where its tail is on the tip of $\vec{D}$, then use the tip-to-tail rule of graphical addition. The gives vector $\vec{F} = \vec{D} + \vec{E}$ in Figure 3.6b. Alternatively, Figure 3.6c shows that the vector sum $\vec{D} + \vec{E}$ can be found as the diagonal of the parallelogram defined by $\vec{D}$ and $\vec{E}$. This method for vector addition, which some of you may have learned, is called the parallelogram rule of vector addition.

**FIGURE 3.6** Two vectors can be added using the tip-to-tail rule or the parallelogram rule.

Vector addition is easily extended to more than two vectors. Figure 3.7 shows a hiker moving from initial position 0 to position 1, then position 2, then position 3, and finally arriving at position 4. These four segments are described by displacement vectors $\vec{D}_1$, $\vec{D}_2$, $\vec{D}_3$, and $\vec{D}_4$. The hiker’s net displacement, an arrow from position 0 to position 4, is the vector $\vec{D}_{net}$. In this case,

$$\vec{D}_{net} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4 \quad (3.5)$$

The vector sum is found by using the tip-to-tail method three times in succession.

**STOP TO THINK 3.1** Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?

(a) (b) (c) (d) (e)

**Multiplication by a Scalar**

Suppose a second bird flies twice as far to the east as the bird in Example 3.1. The first bird’s displacement was $\vec{A}_1 = (100 \text{ m, east})$, where a subscript has been added to denote the first bird. The second bird’s displacement will then certainly be $\vec{A}_2 = (200 \text{ m, east})$. The words “twice as” indicate a multiplication, so we can say

$$\vec{A}_2 = 2\vec{A}_1$$

Multiplying a vector by a positive scalar gives another vector of different magnitude but pointing in the same direction.

Let the vector $\vec{A}$ be

$$\vec{A} = (A, \theta_A) \quad (3.6)$$

where we’ve specified the vector’s magnitude $A$ and direction $\theta_A$. Now let $\vec{B} = c\vec{A}$, where $c$ is a positive scalar constant. We define the multiplication of a vector by a scalar such that

$$\vec{B} = c\vec{A} \text{ means that } (B, \theta_B) = (cA, \theta_A) \quad (3.7)$$
In other words, the vector is stretched or compressed by the factor \( c \) (i.e., vector \( \vec{B} \) has magnitude \( B = cA \)), but \( \vec{B} \) points in the same direction as \( \vec{A} \). This is illustrated in Figure 3.8 on the previous page.

We used this property of vectors in Chapter 1 when we asserted that vector \( \vec{a} \) points in the same direction as \( \vec{v} \). From the definition

\[
\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \left( \frac{1}{\Delta t} \right) \Delta \vec{v}
\]

where \( (1/\Delta t) \) is a scalar constant, we see that \( \vec{a} \) points in the same direction as \( \Delta \vec{v} \) but differs in length by the factor \( (1/\Delta t) \).

Suppose we multiply \( \vec{A} \) by zero. Using Equation 3.7,

\[
0 \cdot \vec{A} = \vec{0} = (0 \text{ m}, \text{ direction undefined})
\]

The product is a vector having zero length or magnitude. This vector is known as the **zero vector**, denoted \( \vec{0} \). The direction of the zero vector is irrelevant; you cannot describe the direction of an arrow of zero length.

What happens if we multiply a vector by a negative number? Equation 3.7 does not apply if \( c < 0 \) because vector \( \vec{B} \) cannot have a negative magnitude. Consider the vector \( -\vec{A} \), which is equivalent to multiplying \( \vec{A} \) by \(-1\). Because

\[
\vec{A} + (-\vec{A}) = \vec{0}
\]

the vector \(-\vec{A}\) must be such that, when it is added to \( \vec{A} \), the resultant is the zero vector \( \vec{0} \). In other words, the **tip** of \(-\vec{A}\) must return to the **tail** of \( \vec{A} \), as shown in Figure 3.9. This will be true only if \(-\vec{A}\) is equal in magnitude to \( \vec{A} \), but opposite in direction. Thus we can conclude that

\[
-\vec{A} = (A, \text{ direction opposite } \vec{A})
\]

That is, **multiplying a vector by \(-1\) reverses its direction without changing its length.**

As an example, Figure 3.10 shows vectors \( \vec{A}, 2\vec{A}, \) and \(-3\vec{A}\). Multiplication by 2 doubles the length of the vector but does not change its direction. Multiplication by \(-3\) stretches the length by a factor of 3 and reverses the direction. This addition of the three vectors is shown in Figure 3.11, using the tip-to-tail method. \( \Delta \vec{r}_{\text{net}} \) stretches from Carolyn’s initial position to her final position. The magnitude of her net displacement is found using the Pythagorean theorem:

\[
r_{\text{net}} = \sqrt{(120 \text{ km})^2 + (80 \text{ km})^2} = 144 \text{ km}
\]

The direction of \( \Delta r_{\text{net}} \) is described by angle \( \theta \), which is

\[
\theta = \tan^{-1} \left( \frac{80 \text{ km}}{120 \text{ km}} \right) = 33.7^\circ
\]

Thus Carolyn’s net displacement is \( \Delta r_{\text{net}} = (144 \text{ km}, 33.7^\circ \text{ north of east}) \).
Vector Subtraction

Figure 3.12a shows two vectors, \( \vec{P} \) and \( \vec{Q} \). What is \( \vec{R} = \vec{P} - \vec{Q} \)? Look back at Tactics Box 1.2 on page 11, which showed how to perform vector subtraction graphically. Figure 3.12b finds \( \vec{P} - \vec{Q} \) by writing \( \vec{R} = \vec{P} + (-\vec{Q}) \), then using the rules of vector addition.

STOP TO THINK 3.2 Which figure shows \( 2\vec{A} - \vec{B} \)?

3.3 Coordinate Systems and Vector Components

Thus far, our discussion of vectors and their properties has not used a coordinate system at all. Vectors do not require a coordinate system. We can add and subtract vectors graphically, and we will do so frequently to clarify our understanding of a situation. But the graphical addition of vectors is not an especially good way to find quantitative results. In this section we will introduce a coordinate description of vectors that will be the basis of an easier method for doing vector calculations.

Coordinate Systems

As we noted in the first chapter, the world does not come with a coordinate system attached to it. A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements. It may be helpful to think of drawing a grid on a piece of transparent plastic that you can then overlay on top of the problem. This conveys the idea that you choose:

- Where to place the origin, and
- How to orient the axes.

Different problem solvers may choose to use different coordinate systems; that is perfectly acceptable. However, some coordinate systems will make a problem easier to solve. Part of our goal is to learn how to choose an appropriate coordinate system for each problem.

We will generally use Cartesian coordinates. This is a coordinate system with the axes perpendicular to each other, forming a rectangular grid. The standard \( xy \)-coordinate system with which you are familiar is a Cartesian coordinate system. An \( xz \)-coordinate system would be a Cartesian coordinate system in three dimensions. There are other possible coordinate systems, such as polar coordinates, but we will not be concerned with those for now.

The placement of the axes is not entirely arbitrary. By convention, the positive y-axis is located \( 90^\circ \) counterclockwise (ccw) from the positive x-axis, as illustrated in Figure 3.13. Figure 3.13 also identifies the four quadrants of the coordinate system, I through IV. Notice that the quadrants are counted ccw from the positive x-axis.
Coordinate axes have a positive end and a negative end, separated by zero at the origin where the two axes cross. When you draw a coordinate system, it is important to label the axes. This is done by placing x and y labels at the positive ends of the axes, as in Figure 3.13. The purpose of the labels is twofold:

- To identify which axis is which, and
- To identify the positive ends of the axes.

This will be important when you need to determine whether the quantities in a problem should be assigned positive or negative values.

**Component Vectors**

Let's see how we can use a coordinate system to describe a vector. Figure 3.14 shows a vector \( \vec{A} \) and an xy-coordinate system that we've chosen. Once the directions of the axes are known, we can define two new vectors parallel to the axes that we call the **component vectors** of \( \vec{A} \). Vector \( \vec{A}_x \), called the *x-component vector*, is the projection of \( \vec{A} \) along the x-axis. Vector \( \vec{A}_y \), the *y-component vector*, is the projection of \( \vec{A} \) along the y-axis. Notice that the component vectors are perpendicular to each other.

You can see, using the parallelogram rule, that \( \vec{A} \) is the vector sum of the two component vectors:

\[
\vec{A} = \vec{A}_x + \vec{A}_y
\]  

(3.12)

In essence, we have broken vector \( \vec{A} \) into two perpendicular vectors that are parallel to the coordinate axes. This process is called the **decomposition** of vector \( \vec{A} \) into its component vectors.

**NOTE** You do not need the tail of \( \vec{A} \) to be at the origin. All we need to know is the orientation of the coordinate system so that we can draw \( \vec{A}_x \) and \( \vec{A}_y \) parallel to the axes.

**Components**

You learned in Chapter 2 to give the one-dimensional kinematic variable \( v_x \) a positive sign if the velocity vector \( \vec{v} \) points toward the positive end of the x-axis, a negative sign if \( \vec{v} \) points in the negative x-direction. The basis of that rule is that \( v_x \) is what we call the *x-component* of the velocity vector. We need to extend this idea to vectors in general.

Suppose vector \( \vec{A} \) has been decomposed into component vectors \( \vec{A}_x \) and \( \vec{A}_y \) parallel to the coordinate axes. We can describe each component vector with a single number (a scalar) called the **component**. The *x-component* and *y-component* of vector \( \vec{A} \), denoted \( A_x \) and \( A_y \), are determined as follows:

**TACTICS BOX 3.1 Determining the Components of a Vector**

1. The absolute value \( |A_x| \) of the x-component \( A_x \) is the magnitude of the component vector \( \vec{A}_x \).
2. The **sign** of \( A_x \) is positive if \( \vec{A}_x \) points in the positive x-direction, negative if \( \vec{A}_x \) points in the negative x-direction.
3. The *y-component* \( A_y \) is determined similarly.

In other words, the component \( A_x \) tells us two things: how big \( \vec{A}_x \) is and, with its sign, which end of the axis \( \vec{A}_x \) points toward. Figure 3.15 on the next page shows three examples of determining the components of a vector.
CHAPTER 3 · Vectors and Coordinate Systems

\[ \text{Magnitude} = \sqrt{A_x^2 + A_y^2} \]

\[ \text{Direction of } \vec{A} = \tan^{-1}(A_y/A_x) \]

\[ A_x = A \cos \theta \]

\[ A_y = A \sin \theta \]

\[ C_x = C \sin \phi \]

\[ C_y = -C \cos \phi \]

\[ \text{The role of sine and cosine is reversed from that in Equations 3.13 because we are using a different angle.} \]

**NOTE** Each decomposition requires that you pay close attention to the direction in which the vector points and the angles that are defined. The minus sign, when needed, must be inserted manually. ❄

We can also go in the opposite direction and determine the length and angle of a vector from its \( x \)- and \( y \)-components. Because \( A \) in Figure 3.16a is the hypotenuse of a right triangle, its length is given by the Pythagorean theorem:

\[ A = \sqrt{A_x^2 + A_y^2} \]
Similarly, the tangent of angle \( \theta \) is the ratio of the far side to the adjacent side, so

\[
\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)
\]

where \( \tan^{-1} \) is the inverse tangent function. Equations 3.15 and 3.16 can be thought of as the “reverse” of Equations 3.13.

Equation 3.15 always works for finding the length or magnitude of a vector because the squares eliminate any concerns over the signs of the components. But finding the angle, just like finding the components, requires close attention to how the angle is defined and to the signs of the components. For example, finding the angle of vector \( \vec{C} \) in Figure 3.16b requires the length of \( \vec{C} \), without the minus sign. Thus vector \( \vec{C} \) has magnitude and direction

\[
\vec{C} = \sqrt{C_x^2 + C_y^2}
\]

\[
\phi = \tan^{-1}\left(\frac{C_y}{C_x}\right)
\]

Notice that the roles of \( x \) and \( y \) differ from those in Equation 3.16.

**EXAMPLE 3.3 Finding the components of an acceleration vector**

Find the \( x \)- and \( y \)-components of the acceleration vector \( \vec{a} \) shown in Figure 3.17a.

**VISUALIZE** It’s important to draw vectors. Figure 3.17b shows the original vector \( \vec{a} \) decomposed into components parallel to the axes.

**SOLVE** The acceleration vector \( \vec{a} = (6 \text{ m/s}^2, 30^\circ \) below the negative \( x \)-axis) points to the left (negative \( x \)-direction) and down (negative \( y \)-direction), so the components \( a_x \) and \( a_y \) are both negative:

\[
a_x = -\cos 30^\circ = -(6 \text{ m/s}^2) \cos 30^\circ = -5.2 \text{ m/s}^2
\]

\[
a_y = -\sin 30^\circ = -(6 \text{ m/s}^2) \sin 30^\circ = -3.0 \text{ m/s}^2
\]

**ASSESS** The units of \( a_x \) and \( a_y \) are the same as the units of vector \( \vec{a} \). Notice that we had to insert the minus signs manually by observing that the vector is in the third quadrant.

**EXAMPLE 3.4 Finding the direction of motion**

Figure 3.18a shows a particle’s velocity vector \( \vec{v} \). Determine the particle’s speed and direction of motion.

**VISUALIZE** Figure 3.18b shows the components \( v_x \) and \( v_y \) and defines an angle \( \theta \) with which we can specify the direction of motion.
Solve We can read the components of \( \vec{v} \) directly from the axes: 
\( v_x = -6 \text{ m/s} \) and \( v_y = 4 \text{ m/s} \). Notice that \( v_y \) is negative. This is enough information to find the particle’s speed \( v \), which is the magnitude of \( \vec{v} \):
\[
\begin{align*}
    v &= \sqrt{v_x^2 + v_y^2} = \sqrt{(-6 \text{ m/s})^2 + (4 \text{ m/s})^2} = 7.2 \text{ m/s}
\end{align*}
\]
From trigonometry, angle \( \theta \) is
\[
\theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{4 \text{ m/s}}{-6 \text{ m/s}} \right) = 33.7^\circ
\]
The absolute value signs are necessary because \( v_y \) is a negative number. The velocity vector \( \vec{v} \) can be written in terms of the speed and the direction of motion as
\[
\vec{v} = (7.2 \text{ m/s}, 33.7^\circ \text{ above the negative x-axis})
\]

STOP TO THINK 3.3 What are the \( x \)- and \( y \)-components \( C_x \) and \( C_y \) of vector \( \vec{C} \)?

3.4 Vector Algebra

Vector components are a powerful tool for doing mathematics with vectors. In this section you’ll learn how to use components to add and subtract vectors. First, we’ll introduce an efficient way to write a vector in terms of its components.

**Unit Vectors**

The vectors \((1, +x\text{-direction})\) and \((1, +y\text{-direction})\), shown in Figure 3.19, have some interesting and useful properties. Each has a magnitude of 1, no units, and is parallel to a coordinate axis. A vector with these properties is called a **unit vector**. These unit vectors have the special symbols
\[
\hat{i} = (1, +x\text{-direction})
\]
\[
\hat{j} = (1, +y\text{-direction})
\]

The notation \( \hat{i} \) (read “i hat”) and \( \hat{j} \) (read “j hat”) indicates a unit vector with a magnitude of 1.

Unit vectors establish the directions of the positive axes of the coordinate system. Our choice of a coordinate system may be arbitrary, but once we decide to place a coordinate system on a problem we need something to tell us “That direction is the positive \( x \)-direction.” This is what the unit vectors do.

The unit vectors provide a useful way to write component vectors. The component vector \( A_x \) is the piece of vector \( A \) that is parallel to the \( x \)-axis. Similarly, \( A_y \) is parallel to the \( y \)-axis. Because, by definition, the vector \( \hat{i} \) points along the \( x \)-axis and \( \hat{j} \) points along the \( y \)-axis, we can write
\[
\begin{align*}
    \hat{A}_x &= A_x \hat{i} \\
    \hat{A}_y &= A_y \hat{j}
\end{align*}
\]  

(3.18)
Equations 3.18 separate each component vector into a scalar piece of length $A_x$ (or $A_y$) and a directional piece $\hat{i}$ (or $\hat{j}$). The full decomposition of vector $\vec{A}$ can then be written

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

(3.19)

Figure 3.20 shows how the unit vectors and the components fit together to form vector $\vec{A}$.

**NOTE** In three dimensions, the unit vector along the $+z$-direction is called $\hat{k}$, and to describe vector $\vec{A}$ we would include an additional component vector $A_z \hat{k}$.

You may have learned in a math class to think of vectors as pairs or triplets of numbers, such as $(4, -2, 5)$. This is another, and completely equivalent, way to write the components of a vector. Thus we could write, for a vector in three dimensions,

$$\vec{B} = 4\hat{i} + 2\hat{j} + 5\hat{k} = (4, -2, 5)$$

You will find the notation using unit vectors to be more convenient for the equations we will use in physics, but rest assured that you already know a lot about vectors if you learned about them as pairs or triplets of numbers.

**EXAMPLE 3.5 Run rabbit run!**

A rabbit, escaping a fox, runs $40^\circ$ north of west at 10 m/s. A coordinate system is established with the positive $x$-axis to the east and the positive $y$-axis to the north. Write the rabbit's velocity in terms of components and unit vectors.

**VISUALIZE** Figure 3.21 shows the rabbit's velocity vector and the coordinate axes. We're showing a velocity vector, so the axes are labeled $v_x$ and $v_y$, rather than $x$ and $y$.

**SOLVE** 10 m/s is the rabbit's *speed*, not its velocity. The velocity, which includes directional information, is

$$\vec{v} = (10 \text{ m/s}, 40^\circ \text{ north of west})$$

Vector $\vec{v}$ points to the left and up, so the components $v_x$ and $v_y$ are negative and positive, respectively. The components are

$$v_x = -(10 \text{ m/s}) \cos 40^\circ = -7.66 \text{ m/s}$$

$$v_y = +(10 \text{ m/s}) \sin 40^\circ = 6.43 \text{ m/s}$$

With $v_x$ and $v_y$ now known, the rabbit's velocity vector is

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = (-7.66\hat{i} + 6.43\hat{j}) \text{ m/s}$$

Notice that we've pulled the units to the end, rather than writing them with each component.

**ASSESS** Notice that the minus sign for $v_x$ was inserted manually. Signs don’t occur automatically; you have to set them after checking the vector’s direction.

---

**Working with Vectors**

You learned in Section 3.2 how to add vectors graphically, but it is a tedious problem in geometry and trigonometry to find precise values for the magnitude and direction of the resultant. The addition and subtraction of vectors becomes much easier if we use components and unit vectors.

To see this, let’s evaluate the vector sum $\vec{D} = \vec{A} + \vec{B} + \vec{C}$. To begin, write this sum in terms of the components of each vector:

$$\vec{D} = D_x \hat{i} + D_y \hat{j} = A_i + B_i + C_i + A_j + B_j + C_j$$

(3.20)
We can group together all the \(x\)-components and all the \(y\)-components on the right side, in which case Equation 3.20 is

\[
(D_x)i + (D_y)j = (A_x + B_x + C_x)i + (A_y + B_y + C_y)j
\]  
(3.21)

Comparing the \(x\)- and \(y\)-components on the left and right sides of Equation 3.21, we find:

\[
D_x = A_x + B_x + C_x
\]
\[
D_y = A_y + B_y + C_y
\]
(3.22)

Stated in words, Equation 3.22 says that we can perform vector addition by adding the \(x\)-components of the individual vectors to give the \(x\)-component of the resultant and by adding the \(y\)-components of the individual vectors to give the \(y\)-component of the resultant. This method of vector addition is called \textit{algebraic addition}.

\textbf{EXAMPLE 3.6 Using algebraic addition to find a displacement}

Example 3.1 was about a bird that flew 100 m to the east, then 200 m to the northwest. Use the algebraic addition of vectors to find the bird’s net displacement. Compare the result to Example 3.1.

\textbf{Visualize} Figure 3.22 shows displacement vectors \(\vec{A} = (100 \text{ m, east}) \) and \(\vec{B} = (200 \text{ m, northwest})\). We draw vectors tip-to-tail if we are going to add them graphically, but it’s usually easier to draw them all from the origin if we are going to use algebraic addition.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.22}
\caption{The net displacement is \(\vec{C} = \vec{A} + \vec{B}\).}
\end{figure}

\textbf{Solve} To add the vectors algebraically we must know their components. From the figure these are seen to be

\[
\vec{A} = 100 \hat{i} \text{ m}
\]
\[
\vec{B} = (-200 \cos 45^\circ \hat{i} + 200 \sin 45^\circ \hat{j}) \text{ m}
\]
\[
= (-141 \hat{i} + 141 \hat{j}) \text{ m}
\]

Notice that vector quantities must include units. Also notice, as you would expect from the figure, that \(\vec{B}\) has a negative \(x\)-component. Adding \(\vec{A}\) and \(\vec{B}\) by components gives

\[
\vec{C} = \vec{A} + \vec{B} = (100 \hat{i} m + (-141 \hat{i} + 141 \hat{j}) m
\]
\[
= (100 m - 141 m)\hat{i} + (141 m)\hat{j}
\]
\[
= (-41 \hat{i} + 141 \hat{j}) \text{ m}
\]

This would be a perfectly acceptable answer for many purposes. However, we need to calculate the magnitude and direction of \(\vec{C}\) if we want to compare this result to our earlier answer. The magnitude of \(\vec{C}\) is

\[
C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-41 \text{ m})^2 + (141 \text{ m})^2} = 147 \text{ m}
\]

The angle \(\theta\), as defined in Figure 3.22, is

\[
\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{141 \text{ m}}{41 \text{ m}}\right) = 74^\circ
\]

Thus \(\vec{C} = (147 \text{ m, 74}^\circ \text{ north of west})\), in perfect agreement with Example 3.1.

Vector subtraction and the multiplication of a vector by a scalar, using components, are very much like vector addition. To find \(\vec{R} = \vec{P} - \vec{Q}\) we would compute

\[
R_x = P_x - Q_x
\]
\[
R_y = P_y - Q_y
\]
(3.23)

Similarly, \(\vec{T} = c\vec{S}\) would be

\[
T_x = cS_x
\]
\[
T_y = cS_y
\]
(3.24)
The next few chapters will make frequent use of vector equations. For example, you will learn that the equation to calculate the force on a car skidding to a stop is

\[ \vec{F} = \vec{n} + \vec{w} + \mu \vec{f} \]  

(3.25)

The following general rule is used to evaluate such an equation:

The x-component of the left-hand side of a vector equation is found by doing scalar calculations (addition, subtraction, multiplication) with just the x-components of all the vectors on the right-hand side. A separate set of calculations uses just the y-components and, if needed, the z-components.

Thus Equation 3.25 is really just a shorthand way of writing three simultaneous equations:

\[ F_x = n_x + w_x + \mu f_x \]
\[ F_y = n_y + w_y + \mu f_y \]
\[ F_z = n_z + w_z + \mu f_z \]  

(3.26)

In other words, a vector equation is interpreted as meaning: Equate the x-components on both sides of the equals sign, then equate the y-components, and then the z-components. Vector notation allows us to write these three equations in a much more compact form.

**Tilted Axes and Arbitrary Directions**

As we’ve noted, the coordinate system is entirely your choice. It is a grid that you impose on the problem in a manner that will make the problem easiest to solve. We will soon meet problems where it will be convenient to tilt the axes of the coordinate system, such as those shown in Figure 3.23. Although you may not have seen such a coordinate system before, it is perfectly legitimate. The axes are perpendicular, and the y-axis is oriented correctly with respect to the x-axis. While we are used to having the x-axis horizontal, there is no requirement that it has to be that way.

Finding components with tilted axes is no harder than what we have done so far. Vector \( \vec{C} \) in Figure 3.23 can be decomposed \( \vec{C} = C_x \hat{i} + C_y \hat{j} \), where \( C_x = C \cos \theta \) and \( C_y = C \sin \theta \). Note that the unit vectors \( \hat{i} \) and \( \hat{j} \) correspond to the axes, not to “horizontal” and “vertical,” so they are also tilted.

Tilted axes are useful if you need to determine component vectors “parallel to” and “perpendicular to” an arbitrary line or surface. For example, we will soon need to decompose a force vector into component vectors parallel to and perpendicular to a surface.

Figure 3.24a shows a vector \( \vec{A} \) and a tilted line. Suppose we would like to find the component vectors of \( \vec{A} \) parallel and perpendicular to the line. To do so, establish a tilted coordinate system with the x-axis parallel to the line and the y-axis perpendicular to the line, as shown in Figure 3.24b. Then \( A_x \) is equivalent to vector \( \vec{A}_n \), the component of \( \vec{A} \) parallel to the line, and \( A_y \) is equivalent to the perpendicular component vector \( \vec{A}_l \). Notice that \( \vec{A} = \vec{A}_n + \vec{A}_l \).

If \( \phi \) is the angle between \( \vec{A} \) and the line, we can easily calculate the parallel and perpendicular components of \( \vec{A} \):

\[ A_n = A_x = A \cos \phi \]
\[ A_l = A_y = A \sin \phi \]  

(3.27)

It was not necessary to have the tail of \( \vec{A} \) on the line in order to find a component of \( \vec{A} \) parallel to the line. The line simply indicates a direction, and the component vector \( \vec{A}_n \) points in that direction.
EXAMPLE 3.7 Finding the force perpendicular to a surface

A horizontal force $\vec{F}$ with a strength of 10 N is applied to a surface. (You’ll learn in Chapter 4 that force is a vector quantity measured in units of newtons, abbreviated N.) The surface is tilted at a 20° angle. Find the component of the force vector perpendicular to the surface.

VISUALIZE Figure 3.25 shows a horizontal force $\vec{F}$ applied to the surface. A tilted coordinate system has its y-axis perpendicular to the surface, so the perpendicular component is $F_1 = F_y$.

SOLVE From geometry, the force vector $\vec{F}$ makes an angle $\phi = 20^\circ$ with the tilted x-axis. The perpendicular component of $\vec{F}$ is thus

$$F_1 = F \sin 20^\circ = (10 \text{ N}) \sin 20^\circ = 3.42 \text{ N}$$

STOP TO THINK 3.4 Angle $\phi$ that specifies the direction of $\vec{C}$ is given by

a. $\tan^{-1}(C_x/C_y)$

b. $\tan^{-1}(C_y/C_x)$

c. $\tan^{-1}(|C_x|/|C_y|)$

d. $\tan^{-1}(|C_y|/|C_x|)$

e. $\tan^{-1}(C_y/|C_x|)$

f. $\tan^{-1}(|C_x|/C_y)$
The goal of Chapter 3 has been to learn how vectors are represented and used.

GENERAL PRINCIPLES

A vector is a quantity described by both a magnitude and a direction.

Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors \( \hat{i} \) and \( \hat{j} \) define the directions of the \( x \)- and \( y \)-axes.

USING VECTORS

Components

The component vectors are parallel to the \( x \)- and \( y \)-axes.

\[
\vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}
\]

In the figure at the right, for example:

\[
A_x = A \cos \theta \quad A = \sqrt{A_x^2 + A_y^2}
\]

\[
A_y = A \sin \theta \quad \theta = \tan^{-1}(A_y/A_x)
\]

> Minus signs need to be included if the vector points down or left.

Working Graphically

Addition

\[
\vec{A} + \vec{B} = \vec{A} + \vec{B}
\]

Negative

\[
\vec{A} - \vec{B} = \vec{A} - \vec{B}
\]

Subtraction

\[
\vec{A} - \vec{B} = \vec{A} - \vec{B}
\]

Multiplication

\[
\vec{A} \times \vec{B} = \vec{A} \times \vec{B}
\]

Working Algebraically

Vector calculations are done component by component.

\[
\vec{C} = 2\vec{A} + \vec{B} \quad \text{means} \quad \begin{cases} 
C_x = 2A_x + B_x \\
C_y = 2A_y + B_y 
\end{cases}
\]

The magnitude of \( \vec{C} \) is then \( C = \sqrt{C_x^2 + C_y^2} \) and its direction is found using \( \tan^{-1} \).

TERMS AND NOTATION

- scalar quantity
- zero vector, \( \vec{0} \)
- vector quantity
- Cartesian coordinates
- magnitude
- quadrants
- resultant vector
- component vector
- graphical addition
- unit vector, \( \hat{i} \) or \( \hat{j} \)
- algebraic addition
- decomposition
Exercises

Section 3.2 Properties of Vectors

1. a. Can a vector have nonzero magnitude if a component is zero? If no, why not? If yes, give an example.
   b. Can a vector have zero magnitude and a nonzero component? If no, why not? If yes, give an example.

2. Suppose \( \vec{C} = \vec{A} + \vec{B} \).
   a. Under what circumstances does \( \vec{C} = \vec{A} - \vec{B} \)?
   b. Could \( \vec{C} = \vec{A} + \vec{B} \) if so, how? If not, why not?

3. Suppose \( \vec{C} = \vec{A} - \vec{B} \).
   a. Under what circumstances does \( \vec{C} = \vec{A} - \vec{B} \)?
   b. Could \( \vec{C} = \vec{A} + \vec{B} \) if so, how? If not, why not?

4. Trace the vectors in Figure Ex3.4 onto your paper. Then find (a) \( \vec{A} + \vec{B} \) and (b) \( \vec{A} - \vec{B} \).

5. Trace the vectors in Figure Ex3.5 onto your paper. Then find (a) \( \vec{A} + \vec{B} \) and (b) \( \vec{A} - \vec{B} \).

Section 3.3 Coordinate Systems and Vector Components

6. A position vector in the first quadrant has an x-component of 6 m and a magnitude of 10 m. What is the value of its y-component?

7. A velocity vector 40° below the positive x-axis has a y-component of 10 m/s. What is the value of its x-component?

8. a. What are the x- and y-components of vector \( \vec{E} \) in terms of the angle \( \theta \) and the magnitude \( E \) shown in Figure Ex3.8?
   b. For the same vector, what are the x- and y-components in terms of the angle \( \phi \) and the magnitude \( E \)?

9. Draw each of the following vectors, then find its x- and y-components.
   a. \( \vec{r} = (100 \text{ m}, 45° \text{ below } +x-axis) \)
   b. \( \vec{v} = (300 \text{ m/s}, 20° \text{ above } +x-axis) \)
   c. \( \vec{a} = (5.0 \text{ m/s}^2, -y\text{-direction}) \)
   d. \( \vec{F} = (50 \text{ N}, 36.9° \text{ above } -x-axis) \)

10. Draw each of the following vectors, then find its x- and y-components.
    a. \( \vec{r} = (2 \text{ km}, 30° \text{ left of } +y-axis) \)
    b. \( \vec{v} = (5 \text{ cm/s}, -x\text{-direction}) \)
    c. \( \vec{a} = (10 \text{ m/s}^2, 40° \text{ left of } -y-axis) \)
    d. \( \vec{F} = (50 \text{ N}, 36.9° \text{ right of } +y-axis) \)

11. Let \( \vec{C} = (3.15 \text{ m}, 15° \text{ above the negative } x-axis) \) and \( \vec{D} = (25.67, 30° \text{ to the right of the negative } y-axis) \). Find the magnitude, the x-component, and the y-component of each vector.

12. The quantity called the electric field is a vector. The electric field inside a scientific instrument is \( \vec{E} = (125 \hat{i} - 250 \hat{j}) \text{ V/m} \), where V/m stands for volts per meter. What are the magnitude and direction of the electric field?

Section 3.4 Vector Algebra

13. Draw each of the following vectors, label an angle that specifies the vector's direction, and find the vectors' magnitude and direction.
   a. \( \vec{A} = 4 \hat{i} - 6 \hat{j} \)
   b. \( \vec{F} = (50 \hat{i} + 80 \hat{j}) \text{ m/s} \)
   c. \( \vec{v} = (-20 \hat{i} + 40 \hat{j}) \text{ m/s} \)
   d. \( \vec{a} = (2 \hat{i} - 6 \hat{j}) \text{ m/s}^2 \)

14. Draw each of the following vectors, label an angle that specifies the vector's direction, and find its magnitude and direction.
   a. \( \vec{B} = -4 \hat{i} + 4 \hat{j} \)
   b. \( \vec{r} = (-2 \hat{i} - \hat{j}) \text{ cm} \)
   c. \( \vec{v} = (-10 \hat{i} - 100 \hat{j}) \text{ mph} \)
   d. \( \vec{a} = (20 \hat{i} + 10 \hat{j}) \text{ m/s}^2 \)

15. Let \( \vec{A} = 2 \hat{i} + 3 \hat{j}, \vec{B} = 4 \hat{i} - 2 \hat{j} \).
    a. Draw a coordinate system and on it show vectors \( \vec{A} \) and \( \vec{B} \).
    b. Use graphical vector subtraction to find \( \vec{C} = \vec{A} - \vec{B} \).
    c. What are the magnitude and direction of vector \( \vec{C} \)?

16. Let \( \vec{A} = 5 \hat{i} + 2 \hat{j}, \vec{B} = -3 \hat{i} - 5 \hat{j} \), and \( \vec{C} = \vec{A} + \vec{B} \).
    a. Write vector \( \vec{C} \) in component form.
    b. Draw a coordinate system and on it show vectors \( \vec{A}, \vec{B}, \) and \( \vec{C} \).
    c. What are the magnitude and direction of vector \( \vec{C} \)?

17. Let \( \vec{A} = 5 \hat{i} + 2 \hat{j}, \vec{B} = -3 \hat{i} - 5 \hat{j} \), and \( \vec{D} = \vec{A} - \vec{B} \).
    a. Write vector \( \vec{D} \) in component form.
    b. Draw a coordinate system and on it show vectors \( \vec{A}, \vec{B}, \) and \( \vec{D} \).
    c. What are the magnitude and direction of vector \( \vec{D} \)?

18. Let \( \vec{A} = 5 \hat{i} + 2 \hat{j}, \vec{B} = -3 \hat{i} - 5 \hat{j} \), and \( \vec{E} = 2 \hat{A} + 3 \hat{B} \).
    a. Write vector \( \vec{E} \) in component form.
    b. Draw a coordinate system and on it show vectors \( \vec{A}, \vec{B} \), and \( \vec{E} \).
    c. What are the magnitude and direction of vector \( \vec{E} \)?

19. Let \( \vec{A} = 5 \hat{i} + 2 \hat{j}, \vec{B} = -3 \hat{i} - 5 \hat{j} \), and \( \vec{F} = \vec{A} - 4 \vec{B} \).
    a. Write vector \( \vec{F} \) in component form.
    b. Draw a coordinate system and on it show vectors \( \vec{A}, \vec{B}, \) and \( \vec{F} \).
    c. What are the magnitude and direction of vector \( \vec{F} \)?

20. Are the following statements true or false? Explain your answer.
    a. The magnitude of a vector can be different in different coordinate systems.
    b. The direction of a vector can be different in different coordinate systems.
    c. The components of a vector can be different in different coordinate systems.

21. Let \( \vec{A} = (4.0 \text{ m}, \text{ vertically downward}) \) and \( \vec{B} = (5.0 \text{ m}, 120° \text{ clockwise from } A) \). Find the x- and y-components of \( \vec{A} \) and \( \vec{B} \) in each of the two coordinate systems shown in Figure Ex3.21.
Problems

22. What are the x- and y-components of the velocity vector shown in Figure Ex3.22?

\[ \vec{v} = (100 \text{ m/s, west}) \]

FIGURE EX3.22

Problems

23. Figure P3.23 shows vectors \( \vec{A} \) and \( \vec{B} \). Let \( \vec{C} = \vec{A} + \vec{B} \).
   a. Reproduce the figure on your page as accurately as possible, using a ruler and protractor. Draw vector \( \vec{C} \) on your figure, using the graphical addition of \( \vec{A} \) and \( \vec{B} \). Then determine the magnitude and direction of \( \vec{C} \) by measuring it with a ruler and protractor.
   b. Based on your figure of part a, use geometry and trigonometry to calculate the magnitude and direction of \( \vec{C} \).
   c. Decompose vectors \( \vec{A} \) and \( \vec{B} \) into components, then use these to calculate algebraically the magnitude and direction of \( \vec{C} \).

24. a. What is the angle \( \phi \) between vectors \( \vec{E} \) and \( \vec{F} \) in Figure P3.24?
   b. Use geometry and trigonometry to determine the magnitude and direction of \( \vec{G} = \vec{E} + \vec{F} \).
   c. Use components to determine the magnitude and direction of \( \vec{G} = \vec{E} + \vec{F} \).

25. For the three vectors shown above in Figure P3.25, \( \vec{A} + \vec{B} + \vec{C} = -2\vec{A} \). What is vector \( \vec{B} \)?
   a. Write \( \vec{B} \) in component form.
   b. Write \( \vec{B} \) as a magnitude and a direction.

26. Figure P3.26 shows vectors \( \vec{A} \) and \( \vec{B} \). Find vector \( \vec{C} \) such that \( \vec{A} + \vec{B} + \vec{C} = \vec{0} \). Write your answer in component form.

27. Figure P3.27 shows vectors \( \vec{A} \) and \( \vec{B} \). Find \( \vec{D} = 2\vec{A} + \vec{B} \). Write your answer in component form.

28. Let \( \vec{A} = (3.0 \text{ m, } 20^\circ \text{ south of east}), \vec{B} = (2.0 \text{ m, north}), \) and \( \vec{C} = (5.0 \text{ m, } 70^\circ \text{ south of west}) \).
   a. Draw and label \( \vec{A} \), \( \vec{B} \), and \( \vec{C} \) with their tails at the origin.
   b. Write \( \vec{A} \), \( \vec{B} \), and \( \vec{C} \) in component form, using unit vectors.
   c. Find the magnitude and the direction of \( \vec{D} = \vec{A} + \vec{B} + \vec{C} \).

29. Trace the vectors in Figure P3.29 onto your paper. Use the graphical method of vector addition and subtraction to find the following.
   a. \( \vec{D} + \vec{E} + \vec{F} \)
   b. \( \vec{D} + 2\vec{E} \)
   c. \( \vec{D} - 2\vec{E} + \vec{F} \)

30. Let \( \vec{E} = 2\vec{i} + 3\vec{j} \) and \( \vec{F} = 2\vec{i} - 2\vec{j} \). Find the magnitude of \( a. \vec{E} \) and \( \vec{F} \) \n   b. \( \vec{E} + \vec{F} \) \n   c. \( -\vec{E} - 2\vec{F} \)

31. Find a vector that points in the same direction as the vector \( \vec{1} + \vec{j} \) and whose magnitude is 1.

32. The position of a particle as a function of time is given by \( \vec{r} = (5\vec{i} + 4\vec{j})t^2 \text{ m} \), where \( t \) is in seconds.
   a. What is the particle’s distance from the origin at \( t = 0, 2, \) and \( 5 \) s?
   b. Find an expression for the particle’s velocity \( \vec{v} \) as a function of time.
   c. What is the particle’s speed at \( t = 0, 2, \) and \( 5 \) s?

33. While vacationing in the mountains you do some hiking. In the morning, your displacement is \( \vec{S}_{\text{morning}} = (2000 \text{ m, east}) + (3000 \text{ m, north}) + (200 \text{ m, vertical}) \). After lunch, your displacement is \( \vec{S}_{\text{lunch}} = (1500 \text{ m, west}) + (2000 \text{ m, north}) \) \( - (300 \text{ m, vertical}) \).
   a. At the end of the hike, how much higher or lower are you compared to your starting point?
   b. What is your total displacement?

34. The minute hand on a watch is 2.0 cm in length. What is the displacement vector of the tip of the minute hand?
   a. From 8:00 to 8:20 A.M.?
   b. From 8:00 to 9:00 A.M.?

35. Bob walks 200 m south, then jogs 400 m southwest, then walks 200 m in a direction 30° east of north.
   a. Draw an accurate graphical representation of Bob’s motion.
   b. Use either trigonometry or components to find the displacement that will return Bob to his starting point by the most direct route. Give your answer as a distance and a direction.
   c. Does your answer to part b agree with what you can measure on your diagram of part a?

36. Jim’s dog Sparky runs 50 m northeast to a tree, then 70 m west to a second tree, and finally 20 m south to a third tree.
   a. Draw a picture and establish a coordinate system.
   b. Calculate Sparky’s net displacement in component form.
   c. Calculate Sparky’s net displacement as a magnitude and an angle.

37. A field mouse trying to escape a hawk runs east for 5.0 m, darts southeast for 3.0 m, then drops 1.0 m down a hole into its burrow. What is the magnitude of the net displacement of the mouse?

38. Carlos runs with velocity \( \vec{v} = (5 \text{ m/s, } 25^\circ \text{ north of east}) \) for 10 minutes. How far to the north of his starting position does Carlos end up?

39. A cannon tilted upward at 30° fires a cannonball with a speed of 100 m/s. What is the component of the cannonball’s velocity parallel to the ground?
40. Jack and Jill ran up the hill at 3.0 m/s. The horizontal component of Jill’s velocity vector was 2.5 m/s.
   a. What was the angle of the hill?
   b. What was the vertical component of Jill’s velocity?

41. The treasure map in Figure P3.41 gives the following directions to the buried treasure: “Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig.” But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle 60° east of north. After walking 300 paces you see an opening through the woods. Which direction should you go, and how far, to reach the treasure?

42. Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 1.0 m/s. Mary can row the boat with a speed of 2.0 m/s relative to the water.
   a. If Mary rows straight north, how far downstream will she land?
   b. Draw a picture showing Mary’s displacement due to rowing, her displacement due to the river’s motion, and her net displacement.

43. A jet plane is flying horizontally with a speed of 500 m/s over a hill that slopes upward with a 3% grade (i.e., the “rise” is 3% of the “run”). What is the component of the plane’s velocity perpendicular to the ground?

44. A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. A wise elder duck finally realizes that the solution is to fly at an angle to the wind. If the ducks can fly at 8.0 m/s relative to the air, what direction should they head in order to move directly south?

45. A pine cone falls straight down from a pine tree growing on a 20° slope. The pine cone hits the ground with a speed of 10 m/s. What is the component of the pine cone’s impact velocity (a) parallel to the ground and (b) perpendicular to the ground?

46. The car in Figure P3.46 speeds up as it turns a quarter-circle curve from north to east. When exactly halfway around the curve, the car’s acceleration is \( \vec{a} = (2 \text{ m/s}^2, 15° \text{ south of east}) \). At this point, what is the component of \( \vec{a} \) (a) tangent to the circle and (b) perpendicular to the circle?

47. Figure P3.47 shows three ropes tied together in a knot. One of your friends pulls on a rope with 3 units of force and another pulls on a second rope with 5 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving?

48. Three forces are exerted on an object placed on a tilted floor in Figure P3.48. The forces are measured in newtons (N). Assuming that forces are vectors,
   a. What is the component of the net force \( \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \) parallel to the floor?
   b. What is the component of \( \vec{F}_{\text{net}} \) perpendicular to the floor?
   c. What are the magnitude and direction of \( \vec{F}_{\text{net}} \)?

49. Figure P3.49 shows four electrical charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly away from B) on charge A of 3 N. Charge C exerts an attractive force (directly toward C) on charge A of 6 N. Finally, charge D exerts an attractive force of 2 N on charge A. Assuming that forces are vectors, what is the magnitude and direction of the net force \( \vec{F}_{\text{net}} \) exerted on charge A?