Calculus Summer Review Packet

DUE THE FIRST DAY OF SCHOOL

The problems in this packet are designed to help you review topics that are important to your success in Calculus. All work must be shown for each problem. The problems should be done correctly, not just tried. You are expected to get each problem correct. Please DO NOT use your calculators to solve these problems. You must know how to do all these problems WITHOUT a calculator.

Note: Starred problems are for AP Calculus BC students only.

It is recommended that you work with one or more people, but each person must submit his/her own work. Before you leave school, write down the names, phone numbers, and/or email addresses for at least two people who are also taking calculus in the fall.

Name _____ Phone _____
Email _____

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Email _____

If you need help with any of the problems, check the Poolesville web site for links to on-line classes at Montgomery College, which are available to you free of charge. You may also e-mail Mrs. Loomis at Linda_L_Loomis@mcpsmd.org with questions.

All work should be completed to the best of your ability on the first day of school. After you have an opportunity to ask questions, you will be quizzed on this material during the first week of school!!

Have fun with the problems!

AP Calculus Summer Review

I. Simplify. Show the work that leads to your answer.

1. $\dfrac{x-4}{x^2-3x-4}$

2. $\dfrac{x^3-8}{x-2}$

3. $\dfrac{5-x}{x^2-25}$

4. $\dfrac{x^2-4x-32}{x^2-16}$

II. Complete the following identities.

1. $\sin^2 x + \cos^2 x =$ _____

2. $1 + \tan^2 x =$ _____

3. $\cot^2 x + 1 =$ _____

4. $\cos 2x =$ _____

5. $\sin 2x =$ _____

III. Simplify each expression.

1. $\dfrac{1}{x+h} - \dfrac{1}{x}$

2. $\dfrac{\dfrac{2}{x^2}}{\dfrac{10}{x^5}}$

3. $\dfrac{\dfrac{1}{3+x} - \dfrac{1}{3}}{x}$

4. $\dfrac{2x}{x^2-6x+9} - \dfrac{1}{x+1} - \dfrac{8}{x^2-2x-3}$

IV. Solve for **z**:

1. $4x + 10y\mathbf{z} = 0$

2. $y^2 + 3y\mathbf{z} - 8\mathbf{z} - 4x = 0$

V. If $f(x) = \{(3,5), (2,4), (1,7)\}$ $g(x) = \sqrt{x-3}$

$h(x) = \{(3,2), (4,3), (1,6)\}$ $k(x) = x^2 + 5$

determine each of the following:

1. $(f + h)(1) =$ _____

2. $(k - g)(5) =$ _____

3. $(f \circ h)(3) =$ _____

4. $(g \circ k)(7) =$ _____

5. $f^{-1}(x) =$ _____

6. $k^{-1}(x) =$ _____

7. $\frac{1}{f(x)} =$ _____

8. $(kg)(x) =$ _____

VI. Miscellaneous: Follow the directions for each problem.

1. Evaluate $\frac{f(x+h) - f(x)}{h}$ and simplify if $f(x) = x^2 - 2x$.

2. Expand $(x + y)^3$

3. Simplify: $x^{\frac{3}{2}}(x + x^{\frac{5}{2}} - x^2)$

* 4. Eliminate the parameter and write a rectangular equation for $x = t^2 + 3$, $y = 2t$

VII. Expand and simplify

* 1. $\sum_{n=0}^{4} \frac{n^2}{2}$

* 2. $\sum_{n=1}^{3} \frac{1}{n^3}$

VIII. Simplify

1. $\frac{\sqrt{x}}{x}$ _____

2. $e^{\ln 3}$ _____

3. $e^{(1+\ln x)}$ _____

4. ln 1 _____

5. $\ln e^7$ _____

6. $\log_3(1/3)$ _____

7. $\log_{1/2} 8$ _____

8. $\ln \frac{1}{2}$ _____

9. $e^{3\ln x}$ _____

10. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$ _____

11. $27^{2/3}$ _____

12. $(5a^{2/3})(4a^{3/2})$ _____

13. $(4a^{5/3})^{3/2}$ _____

* 14. $\frac{3(n+1)!}{5n!}$ _____

IX. Using the point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

1. with slope –2, containing the point (3, 4) 1. _____

2. containing the points (1, -3) and (-5, 2) 2. _____

3. with slope 0, containing the point (4, 2) 3. _____

4. perpendicular to the line in problem #1, containing the point (3, 4) 4. _____

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* X. Given the vectors **v** = -2**i** + 5**j** and w = 3**i** + 4**j**, determine

1. $\frac{1}{2}$**v** _____

2. **w** – **v** _____

3. length of **w** _____

4. the unit vector for **v** _____

XI. Determine the exact value of each expression.

1. sin 0 _____

2. sin $\frac{\pi}{2}$ _____

3. sin $\frac{3\pi}{4}$ _____

4. cos π _____

5. cos $\frac{3\pi}{4}$ _____

6. cos $\frac{\pi}{3}$ _____

7. tan $\frac{7\pi}{4}$ _____

8. tan $\frac{\pi}{6}$ _____

9. tan $\frac{2\pi}{3}$ _____

10. cos(Sin $^{-1}$ $\frac{1}{2}$) _____

11. Sin $^{-1}$ (sin $\frac{7\pi}{6}$) _____

XII. For each function, determine its domain and range.

Function	Domain	Range
1. $y = \sqrt{x-4}$	_____	_____
2. $y = \sqrt{x^2-4}$	_____	_____
3. $y = \sqrt{4-x^2}$	_____	_____
4. $y = \sqrt{x^2+4}$	_____	_____

XIII. Determine all points of intersection.

1. parabola $y = x^2 + 3x - 4$ and line $y = 5x + 11$

2. $r = \cos\theta$ and $r = \sin\theta$ for $0 \le \theta \le 2\pi$

XIV. Solve for x, where x is a real number. Show the work that leads to your solution.

1. $x^2 + 3x - 4 = 14$

2. $\frac{x^4 - 1}{x^3} = 0$

3. $(x - 5)^2 = 9$

4. $2x^2 + 5x = 8$

Solve for x, where x is a real number. Show the work that leads to your solution.

5. $(x + 3)(x – 3) > 0$

6. $x^2 – 2x - 15 \leq 0$

7. $12x^2 = 3x$

8. $\sin 2x = \sin x$, $0 \leq x \leq 2\pi$

9. $|x – 3| < 7$

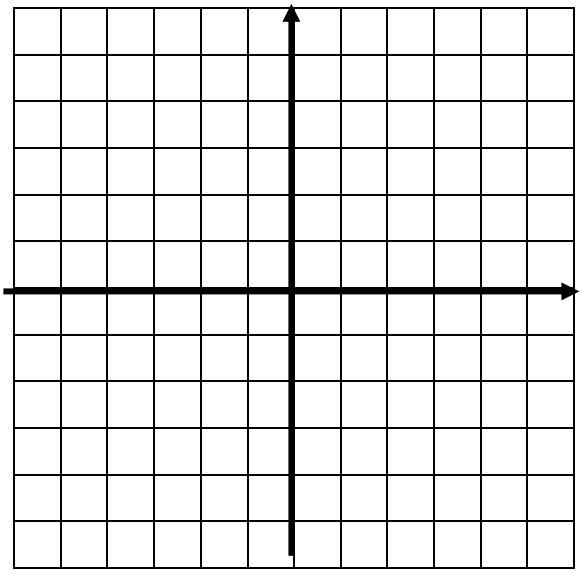
10. $(x + 1)^2(x – 2) + (x + 1)(x – 2)^2 = 0$

11. $27^{2x} = 9^{x-3}$

12. $\log x + \log(x – 3) = 1$

XV. Graph each function. Give its domain and range.

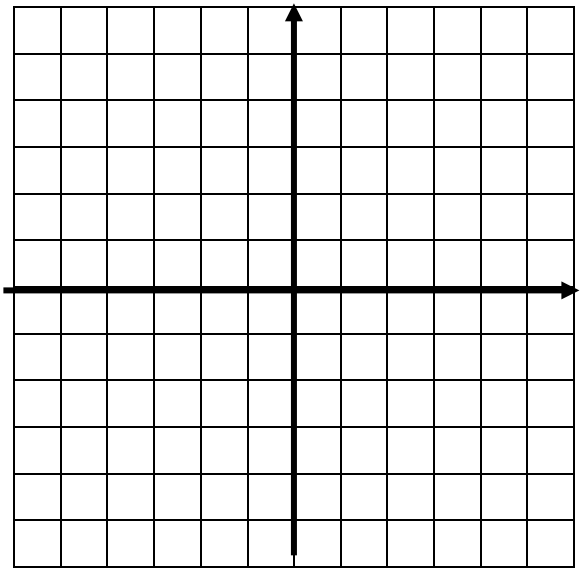
1. $y = \sin x$



Domain_____

Range _____

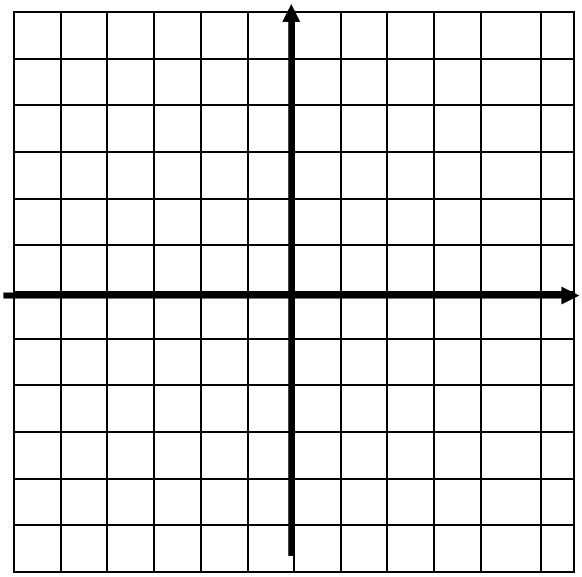
2. $y = e^x$



Domain_____

Range _____

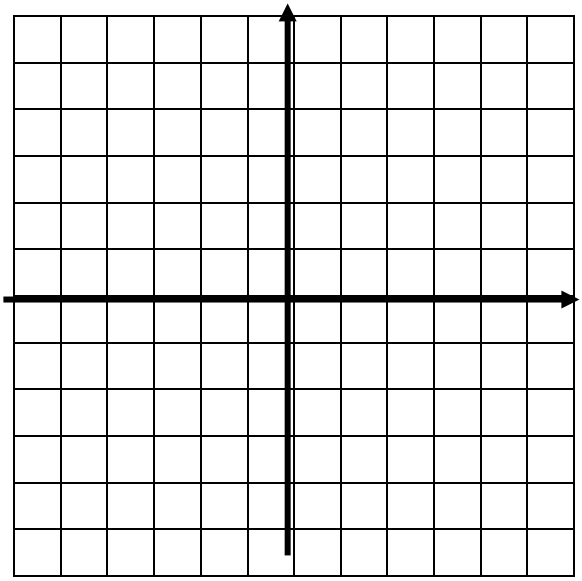
3. $y = \sqrt{x}$



Domain_____

Range _____

4. $y = \sqrt[3]{x}$

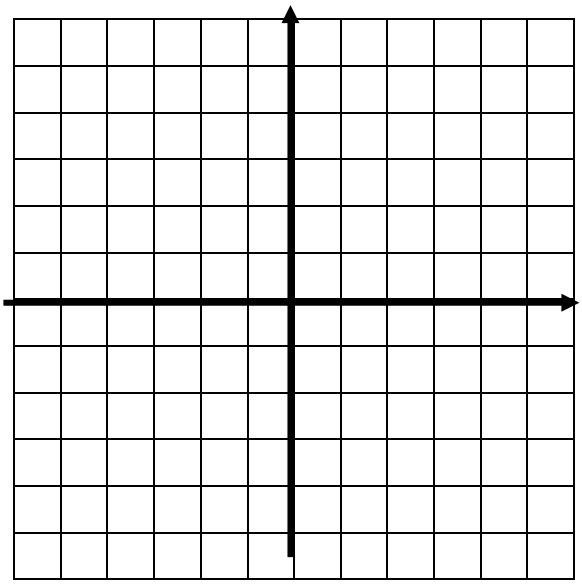


Domain_____

Range _____

Graph each function. Give its domain and range.

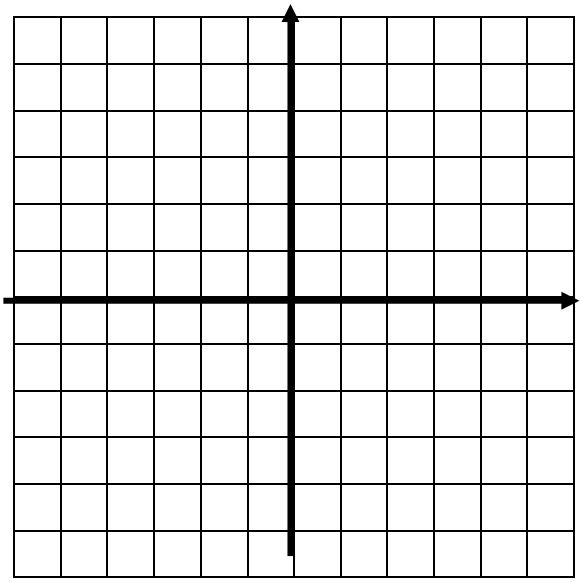
5. $y = \ln x$



Domain_____

Range _____

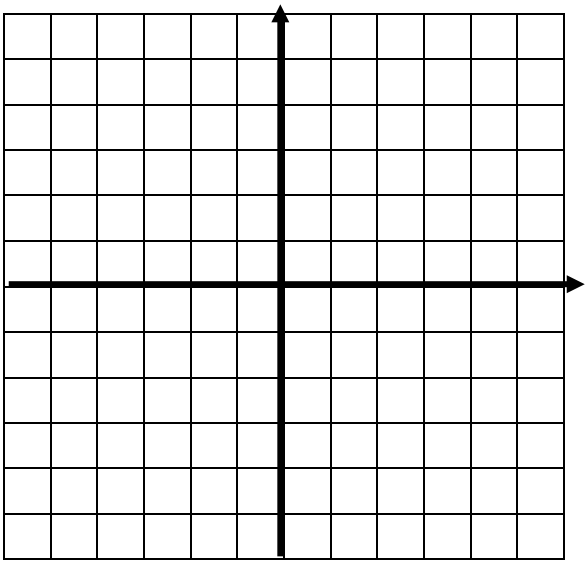
6. $y = |x + 3| - 2$



Domain_____

Range _____

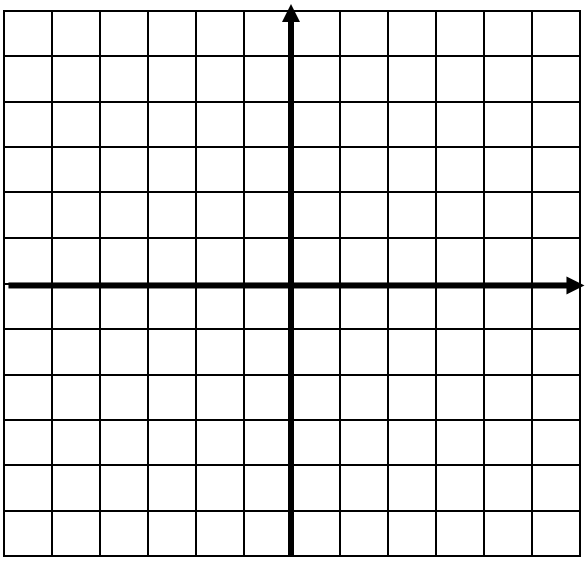
7. $y = \frac{1}{x}$



Domain_____

Range _____

8. $y = \begin{cases} x^2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \le x \le 3 \\ 4 & \text{if } x > 3 \end{cases}$



Domain_____

Range _____

XVI. Identify, by name, each polar graph. Give at least one characteristic of each graph (e.g. radius, location, length of petal, point (other than the pole) on the graph, etc.) Then sketch a graph of each.

1. r = 2 _____

2. r = 3sec θ _____

3. r = 1 + sin θ _____

4. r = 2cos 3θ _____

* XVII. Compute each of the following limits:

1. $\lim_{x\to-\pi}\frac{\cos x}{x}=$

2. $\lim_{x\to\infty}\left(\frac{x}{x+\sin x}\right)=$

3. $\lim_{x\to-1}\frac{x^3+1}{x+1}=$

4. $\lim_{x\to0^-}\frac{[x]}{x}=$

5. $\lim_{x\to\infty}\frac{25-4x^3}{5x^3-x^2}=$

6. $\lim_{x\to3^-}\frac{|x-3|}{x-3}=$

* XVIII.

1. Let f(x) = $\begin{cases}\frac{1}{x}, & x\le-2\\ \frac{x^2-4}{x-2}, & -2<x<2\\ 3, & x\ge2\end{cases}$

Compute each of the following limits:

a) $\lim_{x\to-\infty}f(x)=$ b) $\lim_{x\to-2^-}f(x)=$

c) $\lim_{x\to2^-}f(x)=$ d) $\lim_{x\to2^+}f(x)=$

e) $\lim_{x\to2}f(x)=$ f) $\lim_{x\to\infty}f(x)=$

2. Consider the function h defined below:

$$h(x)=\begin{cases}\frac{\tan5x}{4x}, & x\ne0\\ k, & x=0\end{cases}$$

What value should be assigned to k so that h will be continuous?