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Unit 2 Topic 2: Constructing and Comparing Linear and Exponential Functions

Algebra 1 Summit
Algebra Summit

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CONCEPT 1 Unit 2 Topic 2 : Comparing Linear and Exponential Functions

Dear Parents/Guardians,

Unit 2 Topic 2 is called Constructing and Comparing Linear and Exponential Functions. In this topic we will focus on the following :

- Analyze patterns to determine the next figure in a sequence
- Construct a recursive and explicit equation from an arithmetic/geometric sequence
- Construct an arithmetic and geometric sequence
- Identify and articulate the difference between an arithmetic and geometric sequence with increasing and decreasing common differences and common ratios
- Construct an explicit and recursive equation from a table
- Analyze patterns to determine the "arithmetic means" in an arithmetic sequence
- Analyze patterns to determine the "geometric means" in a geometric sequence
- Identify and state differences between discrete and continuous functions
- Identify a function as linear/exponential/neither based on their patters of growth in a table, graph, and equation.

In the last topic we discussed if a relation is a function or not. In this topic we will be determining if a relation is a function, based on how it is growing; is it *Linear*, *Exponential* or Neither.

What classifies something as an arithmetic or geometric sequence?

arithmetic sequence - An arithmetic sequences is a pattern with a common difference. A common difference is a common value that is added or subtracted each time in a list of consecutive terms. Find the common difference by subtracting the first term from the second term.

Example 1: 2, 4, 6, 8, 10 ~ common difference is +2

Example 2: 250, 245, 240, 235, 230 ~ common difference is -5

geometric sequence – A geometric sequence has a common ratio. A common ratio multiplies the same number each time in consecutive terms. This can be identified by taking the next term divided by the previous term and the result is the common ratio.

Example 1: 5, 25, 125, 525 ~ common ratio: times 5

Example 2: 10, 5, 2.5, 1.25, 0.625 ~ common ratio: times 0.5

Example 1:

In the pattern of below, determine the number of O's in Figure 5?

TABLE 1.1:

Start	Figure 1	Figure 2	Figure 3	Figure 4
O	OO O	OOO OO	OOOO OOO	OOOOO OOOO

In this example, the start figure has one O, the first figure has three O's, and each figure there on increases by two more O's.

How many O's would be in Figure 5?

TABLE 1.2:

Figure 5
OOOOOO OOOOO

There would be 11 O's in Figure 5. This type of pattern is arithmetic because the common difference each time is +2.

In Example 1 above make a table to look at the relationship between figure number, and number of O's. The figure number represents the *n* value, the independent variable; and the number of objects represents the f(n) value, the dependent variable.

TABLE 1.3:

Figure Number n	Number of O's $f(n)$
0	1
1	3
2	5
3	7
4	9

Construct a recursive and explicit equation from and arithmetic sequence.

Explicit Formula for Arithmetic Sequence	Recursive Formula (always 2 parts) for Arithmetic Sequence
$f(n) = f(0) + D \cdot n$	$f(\_) = initial\ value$ $f(n) = f(n-1) + D$
$f(0)$ *means replace the* y *value, when* $x = 0$ *where* $f(0)$ *is* $D \cdot n$ *means replace the* D, *with the common difference, then multiply it by* n.	$f(\_) =$ *initial value, mean replace where the words* initial value *are, with the first values of the* sequence, and in the ( )the term # The second line of the equation stays the same, except the D gets replaced with the common difference

To write an explicit formula follow the formation above.

Explicit Formula:

$f(n) = f(0) + D \cdot n$

Explicit Formula Example 1:

$f(n) = 1 + 2n$

f(0) was replaced with the number 1 because 1 is the f(n) value, when n = 0. This is found from the table that was created above (table 1.3). The 2 replaced the D, because positive two is the common difference. The explicit equation can be used to find the number of objects at any figure number, n.

The recursive formula will be modeled the same way.

Recursive Equation:

$f(0) = initialvalue$

$f(n) = f(n-1) + D$

Recursive Formula Example 1:

$f(0) = 1$

$f(n) = f(n-1) + 2$

The words "initial value" were replaced with the number 1 because it is the first value given in this example, when n=0. Since (0,1) was the first term given, this is what starts the recursive equation. The common difference of 2 replaces the D with the number 2. In the recursive formula, the first line will change depending on the situation. If the 7^{th} term is given first as (7, 19) then the first line would start with f(7)= 19.

Example 2:

TABLE 1.4:

Day 1	Day 2	Day 3	Day 4	Day 5
*	* *	* * * *	** ** ** **	**** **** **** ****

Is this an arithmetic or geometric sequence?

The sequence starts with 1, 2, 4, 8, 16, ... the common ratio each time is multiplying by 2. Since this has a common ratio and not a common difference this sequence is geometric.

According to the figure below, each day more stars appear. How many stars will there be on Day 7?

TABLE 1.5:

Day 7
**************** **************** **************** ****************

How many stars will there be at Day n?

An explicit or recursive formula can be used to represent how to find the number of stars on Day n. Explicit and Recursive equations are different depending on if a sequence is arithmetic or geometric. Below is how to write an explicit and recursive formula for a geometric sequence.

Explicit Formula for Geometric Sequence	Recursive Formula (always 2 parts) for Geometric
$f(n) = f(0) \cdot R^n$ or $f(n) = f(1) \cdot R^{n-1}$	$f(\_) = initial\ value$ $f(n) = R \cdot f(n-1)$
$f(0)$ *means replace with the y value, when x = 0* R^n *means replace the R, with the rate of change, then raise it to the power of n.*	$f(\_)$ *= initial value, meaning replace where the words* initial value *are, with the first term in the sequence* and the term number in the parenthesis. The second line of the equation stays the same, except the R gets replaced with the rate of change

Write an explicit and recursive formula to model this situation.

Explicit Formula:

$f(n) = f(0) \cdot R^n$

Explicit Formula Example 2:

$f(n) = .5 \cdot 2^n$

<u>or</u> **f(n) = 1 • 2**$^{n-1}$

The R is replaced, with the common ratio of 2. Finding the f(0) term for this equation will be more difficult in this case because it was not given. To find the zero term from the first term, we do the opposite of the common ratio to go backwards in our pattern. To go from the first day, to the second day, we multiply by 2. To go from the first day, to the zero day, we divide by two. The .5 came from the one star in day 1, divided by two equals .5.

The other way to write the explicit formula from using the f(1) day is listed above as well. The 1 replaces the f(1) because there was 1 start on day 1. The R is replaced with 2, the common ratio. The exponent stays n-1. Both of these formulas work to find the number of stars on day n. The second explicit is easier to write in this example since the f(0) term was never given. Both equations would be considered correct.

The recursive is modeled the same way:

Recursive Formula:

$f(0) = initialvalue$

$f(n) = R \cdot f(n-1)$

Recursive Formula Example 2:

$f(1) = 1$

$f(n) = 2 \cdot f(n-1)$

The initial value in this situation was at Day 1, there was one star. So the initial value was f(1) = 1. The common ratio replaces R in the second part of the equation. The common ratio in this problem is multiply by positive two, so 2 replaces R. The common ratio always replaces R, regardless of whether it is a positive, negative, decimal or whole number.

Example 3:

TABLE 1.6:

n	f(n)
0	0
1	-3
2	-6
3	-9

Write an explicit and recursive formula for the table above.

First determine if the sequence is arithmetic or geometric in order to choose the appropriate formula. Identify this by finding the change in the f(n) values. What is happening to f(n)? f(n) is decreasing by three at a time. Since it is constantly decreasing by three, the common difference is -3. Since there is a common difference and not a common ratio, our sequence is arithmetic. Follow the tables above to create an explicit and recursive formula for an arithmetic sequence.

Explicit Formula:

$f(n) = f(0) + D \cdot n$

Explicit Example 3:

$f(n) = 0 - 3 \cdot n$

Since the f(n) value when n = 0 is also 0, we put a zero into f(0). The common difference is -3, so replace the D with a -3 (negative three). This means that any value of n, can be substituted into this equation where n is multiplied by three, and subtract from zero to give us the corresponding f(n) value. The formula can also be written as $f(n) = -3 \cdot n$ or $f(n) = -3n$

Recursive Formula:

$f(0) = initial value$

$f(n) = f(n-1) + D$

Recursive Example 3:

$f(0) = 0$

$f(n) = f(n-1) - 3$

The first value given in this situation was (0,0), so the *n* value goes in the parenthesis, and substitute the initial value for where the f(n) value goes. The common difference is negative three -3 which replaces the D. This equation states that the initial value in this situation is (0,0). Take the term before it [f(n-1)] then subtract three, which will give the next term. **The recurisve formula only works when you have the term before it, which is why it is necessary to include the initial value.**

Find the missing values in the arithmetic sequence.

TABLE 1.7:

x 0	1	2	3	4	5
y 30					5

The question asks to find all of the missing values for y, when only knowing the first and last term in a specific sequence. Besides using a "guess and check" method, there is a more efficient way.

Use the rate of change, slope formula: change in y, over change in x: $\frac{\triangle y}{\triangle x}$.

First take the last y value and subtract the first one $\frac{5 - 30}{5 - 0} = \frac{-25}{5} = -5$.

Then subtract the last x value, minus the first one

Simplify the fraction: -25 divided by 5 is -5. -5 is our common difference/rate of change in this problem.

finished table:

TABLE 1.8:

0	1	2	3	4	5
30	25	20	15	10	5

Find the missing values in this arithmetic table:

TABLE 1.9:

1	2	3	4	5	6
9					44

Use the last y value and subtract my first y value $\frac{44 - 9}{6 - 1} = \frac{35}{5} = 7$

subtract the last x, and first x;

the common difference is + 7.

finished table

TABLE 1.10:

1	2	3	4	5	6
9	16	23	30	37	44

This equation will not work with a geometric sequence if there is more than one missing value. If there are more than one missing values, follow the example below.

Find the missing terms in the geometric table below:

TABLE 1.11:

1	2	3	4
5			320

5 times ____ = 2nd term

2nd term times ____ = 3rd term

3rd term times ____ = 320

There are 3 blanks to get from the start of the table to the end. Use n or any variable to represent the unknown in the blanks.

$5 \bullet n \bullet n \bullet n = 320$

$5n^3 = 320$

$5n^3/5 = 320/5$

$n^3 = 64$

What cubed is 64? The answer is 4. 4 is the common ratio in this problem. To check our work substitute 4 back into the blanks above to see if the problem stays true.

5 times 4 = 2nd term 20

2nd term 20 times 4 = 3rd term 80

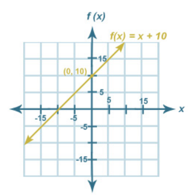
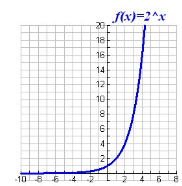
3rd 80 term times 4 = 320

The pattern worked! Four is the common ratio in this problem.

What is the difference between a linear and exponential function?

According to the table below there are many differences between a linear and exponential function. Always identify a function by the way it is changing. In the problems below, identify if they are linear or exponential and write a recursive and explicit formula for each.

TABLE 1.12:

	Linear	**Exponential**
Definition	A linear function has a common difference that is being added or subtracted each time. The common difference is also known as the slope.	An Exponential function has a common rate of a change. The rate at which this type of function is increasing or decrease is its rate of change.
How is y changing?	Arithmetic	Geometric
In a table	Common Difference of +7	Common Ratio -0.25
Graph	Line	Curve
Samples	 http://funinfunction.weebly.com/analyzing-linear-and-exponential-functions.html	 http://funinfunction.weebly.com/analyzing-linear-and-exponential-functions.html
Explicit Formula	$f(n) = f(0) + Dn$	$f(n) = f(0) \cdot R^n$ **or** **f(n) = f(1) * R**$^{n-1}$
Recursive Formula	$f() = initialvalue$ $f(n) = f(n-1) + D$	$f() = initialvalue$ $f(n) = R \cdot f(n-1)$
Calculator Equation	**Y = ax + b** **LinReg**	**ExpReg**
Other:	**May also be seen in the form of y=mx + b**	**May also be seen in the form of y = a(b)**^{x}