



### CK-12 FlexBook



## MCPS C2.0 Geometry Unit 1 Topic 1 FlexBook

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# SLT 1 & 2 Define, recognize, and describe the fundamental terms of Euclidean Geometry

What if you were given a picture of a figure or an object, like a map with cities and roads marked on it? How could you explain that picture geometrically? After completing this Concept, you'll be able to describe such a map using geometric terms.

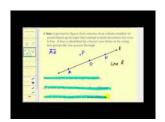
#### **Watch This**



#### MEDIA

Click image to the left for more content.

CK-12 Foundation: Chapter1BasicGeometricDefinitionsA



#### MEDIA

Click image to the left for more content.

James Sousa: Definitions of and Postulates Involving Points, Lines, and Planes

#### Guidance

A **point** is an exact location in space. A point describes a location, but has no size. Dots are used to represent points in pictures and diagrams. These points are said "Point A," "Point L", and "Point F." Points are labeled with a CAPITAL letter.

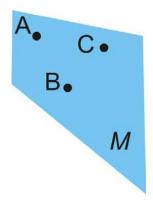


A **line** is a set of infinitely many points that extend forever in both directions. A line, like a point, does not take up space. It has direction, location and is <u>always straight</u>. Lines are one-dimensional because they only have length (no width). A line can by named or identified using any two points on that line or with a lower-case, italicized letter.



This line can be labeled  $\overrightarrow{PQ}$ ,  $\overrightarrow{QP}$  or just g. You would say "line PQ," "line QP," or "line g," respectively. Notice that the line over the  $\overrightarrow{PQ}$  and  $\overrightarrow{QP}$  has arrows over both the P and Q. The order of P and Q does not matter.

A **plane** is infinitely many intersecting lines that extend forever in all directions. Think of a plane as a huge sheet of paper that goes on forever. Planes are considered to be two-dimensional because they have a length and a width. A plane can be classified by any three points in the plane.



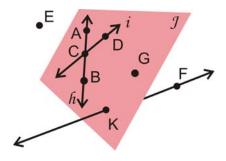
This plane would be labeled Plane ABC or Plane  $\mathcal{M}$ . Again, the order of the letters does not matter.

We can use **point**, **line**, and **plane** to define new terms. **Space** is the set of all points extending in *three* dimensions. Think back to the plane. It extended along two different lines: up and down, and side to side. If we add a third direction, we have something that looks like three-dimensional space, or the real-world.

Points that lie on the same line are **collinear** . P,Q,R,S , and T are collinear because they are all on line w . If a point U were located above or below line w , it would be **non-collinear** .



Points and/or lines within the same plane are **coplanar**. Lines h and i and points A,B,C,D,G, and K are **coplanar** in Plane  $\mathcal{I}$ . Line KF and point E are **non-coplanar** with Plane  $\mathcal{I}$ .



An **endpoint** is a point at the end of a line segment. Line segments are labeled by their endpoints,  $\overline{AB}$  or  $\overline{BA}$ . Notice that the bar over the endpoints has NO arrows. Order does not matter.

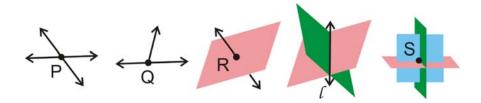


A **ray** is a part of a line with one endpoint that extends forever in the direction opposite that endpoint. A ray is labeled by its endpoint and one other point on the line.



Of lines, line segments and rays, rays are the only one where order matters. When labeling, always write the endpoint under the side WITHOUT the arrow,  $\overrightarrow{CD}$  or  $\overrightarrow{DC}$ .

An intersection is a point or set of points where lines, planes, segments, or rays cross each other.



#### **Postulates**

With these new definitions, we can make statements and generalizations about these geometric figures. This section introduces a few basic postulates. Throughout this course we will be introducing Postulates and Theorems so it is important that you understand what they are and how they differ.

**Postulates** are basic rules of geometry. We can assume that all postulates are true, much like a definition. **Theorems** are statements that can be proven true using postulates, definitions, and other theorems that have already been proven.

The only difference between a theorem and postulate is that a postulate is <u>assumed</u> true because it cannot be shown to be false, a theorem must be <u>proven</u> true. We will prove theorems later in this course.

**Postulate #1:** Given any two distinct points, there is exactly one (straight) line containing those two points.

Postulate #2: Given any three non-collinear points, there is exactly one plane containing those three points.

**Postulate #3:** If a line and a plane share two points, then the entire line lies within the plane.

**Postulate #4:** If two distinct lines intersect, the intersection will be one point.

**Postulate #5:** If two distinct planes intersect, the intersection will be a line.

When making geometric drawings, be sure to be clear and label all points and lines.

#### Example A

What best describes San Diego, California on a globe?

A. point

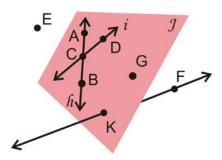
B. line

C. plane

Answer: A city is usually labeled with a dot, or point, on a globe.

#### **Example B**

Use the picture below to answer these questions.



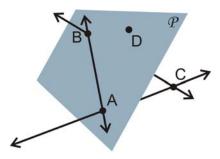
- a) List another way to label Plane  ${\mathcal I}$  .
- b) List another way to label line h.
- c) Are *K* and *F* collinear?
- d) Are E, B and F coplanar?

Answer:

- a) Plane BDG. Any combination of three coplanar points that are not collinear would be correct.
- b)  $\overrightarrow{AB}$ . Any combination of two of the letters A, B, or C would also work.
- c) Yes
- d) Yes

#### **Example C**

Describe the picture below using all the geometric terms you have learned.



#### Answer:

 $\overrightarrow{AB}$  and D are coplanar in Plane  $\mathcal{P}$ , while  $\overrightarrow{BC}$  and  $\overrightarrow{AC}$  intersect at point C which is non-coplanar. Watch this video for help with the Examples above.



MEDIA

Click image to the left for more content.

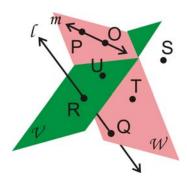
CK-12 Foundation: Chapter1BasicGeometricDefinitionsB

#### Vocabulary

A *point* is an exact location in space. A *line* is infinitely many points that extend forever in both directions. A *plane* is infinitely many intersecting lines that extend forever in all directions. *Space* is the set of all points extending in three dimensions. Points that lie on the same line are *collinear*. Points and/or lines within the same plane are *coplanar*. An *endpoint* is a point at the end of part of a line. A *line segment* is a part of a line with two endpoints. A *ray* is a part of a line with one endpoint that extends forever in the direction opposite that point. An *intersection* is a point or set of points where lines, planes, segments, or rays cross. A *postulate* is a basic rule of geometry is assumed to be true. A *theorem* is a statement that can be proven true using postulates, definitions, and other theorems that have already been proven.

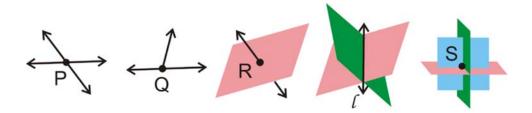
#### **Guided Practice**

- 1. What best describes the surface of a movie screen?
- A. point
- B. line
- C. plane
- 2. Answer the following questions about the picture.



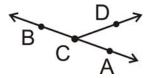
- a) Is line l coplanar with Plane  $\mathcal V$ , Plane  $\mathcal W$ , both, or neither?
- b) Are R and Q collinear?
- c) What point belongs to neither Plane V nor Plane W?
- d) List three points in Plane W.

- 3. Draw and label the intersection of line  $\overrightarrow{AB}$  and ray  $\overrightarrow{CD}$  at point C.
- 4. How do the figures below intersect?



#### **Answers:**

- 1. The surface of a movie screen is most like a plane.
- 2. a) Neither
- b) Yes
- c) S
- d) Any combination of P, O, T, and Q would work.
- 3. It does not matter the placement of A or B along the line nor the direction that  $\overrightarrow{CD}$  points.



4. The first three figures intersect at a point, P,Q and R, respectively. The fourth figure, two planes, intersect in a line, l. And the last figure, three planes, intersect at one point, S.

#### **Interactive Practice**



#### MEDIA

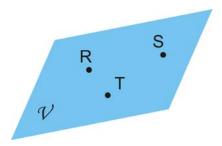
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#### **Practice**

1. Name this line in two ways.

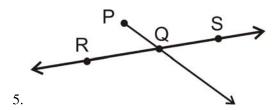


2. Name the geometric figure below in two different ways.



- 3. Draw three ways three different planes can (or cannot) intersect.
- 4. What type of geometric object is made by the intersection of a sphere (a ball) and a plane? Draw your answer.

Use geometric notation to explain each picture in as much detail as possible.



For 6-15, determine if the following statements are ALWAYS true, SOMETIMES true, or NEVER true.

- 6. Any two distinct points are collinear.
- 7. Any three points determine a plane.
- 8. A line is composed of two rays with a common endpoint.
- 9. A line segment has infinitely many points between two endpoints.
- 10. A point takes up space.
- 11. A line is one-dimensional.
- 12. Any four distinct points are coplanar.
- 13.  $\overrightarrow{AB}$  could be read "ray AB" or "ray BA."
- 14.  $\overrightarrow{AB}$  could be read "line AB" or "line BA."
- 15. Theorems are proven true with postulates.

In Algebra you plotted points on the coordinate plane and graphed lines. For 16-20, use graph paper and follow the steps to make the diagram on the same graph.

- 16. Plot the point (2, -3) and label it A.
- 17. Plot the point (-4, 3) and label it B.
- 18. Draw the segment  $\overline{AB}$ .
- 19. Locate point C, the intersection of this line with the x- axis.
- 20. Draw the ray  $\overrightarrow{CD}$  with point D(1,4).



# SLT 3 Copy a segment and copy an angle

#### **Watch This**



MEDIA

Click image to the left for more content.

https://www.youtube.com/watch?v=38dkb\_0egjU Construct a Copy of an Angle

#### Guidance

As you have studied math, you have often created **drawings**. Drawings are a great way to help communicate a visual idea. A **construction** is similar to a drawing in that it produces a visual outcome. However, while drawings are often just rough sketches that help to convey an idea, constructions are step-by-step processes used to create accurate geometric figures.

Constructions take us back over 2000 years to the ancient Greeks, before computers or other advanced technology. Using only the tools of a **compass** and a **straightedge**, they discovered how to copy segments, angles and shapes, how to create perfect regular polygons, and how to create perfect parallel and perpendicular lines. Today, learning constructions is a way to apply your knowledge of geometric principles. You can do constructions by hand, or with dynamic geometry software. In this concept, the focus is on hand constructions and making copies of segments, angles, and triangles.

#### To create a construction by hand, there are a few tools that you can use:

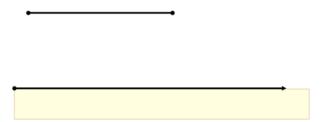
- 1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
- 2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
- 3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

#### Example A

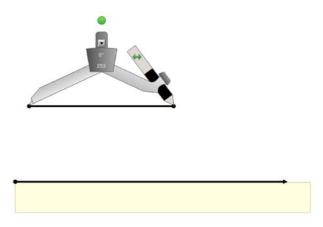
Use a straightedge to draw a line segment on your paper like the one shown below. Then, use your straightedge and compass to copy the line segment exactly.



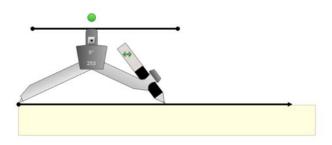
**Solution:** First use your straightedge and pencil to create a new ray.



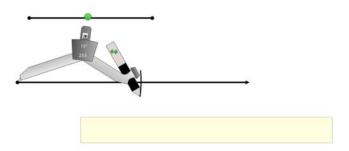
Now, you have one endpoint of your line segment. Your job is to figure out where the other endpoint should go on the ray. Use your compass to measure the width of the original line segment.



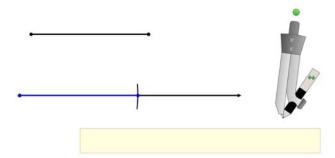
Now, move the compass so that the tip is on the endpoint of the ray.



You can now see where the endpoint of the segment should lie on the ray. Draw a little arc with the compass to mark where the endpoint should go.



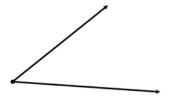
You can use your straightedge to draw the copied line segment in a different color if you wish.



Note that in this construction, the compass was used to copy a distance. This is one of the primary uses of a compass in constructions.

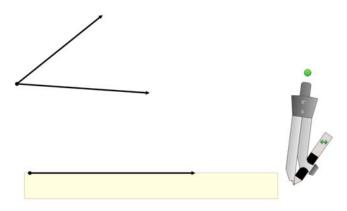
#### Example B

Use a straightedge to draw an angle on your paper like the one shown below. Then, use your straightedge and compass to copy the angle exactly.

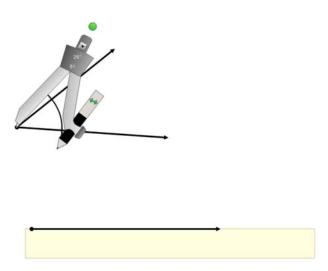


**Solution:** Keep in mind that what defines the angle is the opening between the two rays. The lengths of the rays are not relevant.

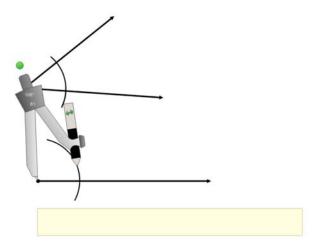
Start by using your straightedge and pencil to draw a new ray. This will be the bottom of the two rays used to create the angle.



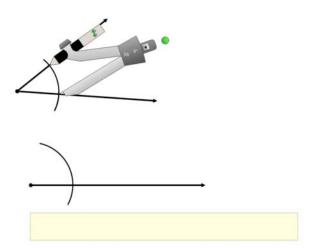
Next, use your compass to make an arc through the original angle. It does not matter how wide you open your compass for this.



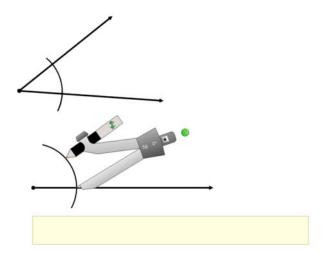
Next, leave your compass open to the same width, and make a similar arc through the new ray.



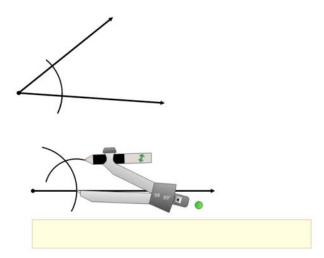
Now, you know that the second ray necessary to create the new angle will go somewhere through that arc. Measure the width of the arc on the original angle using the compass.



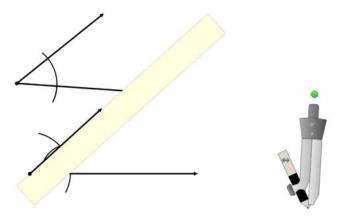
Leave the compass open to the same width, and move it to the new angle.



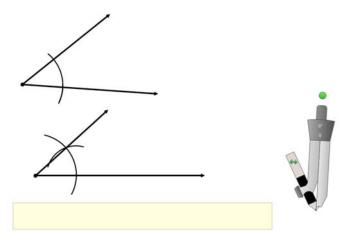
Make a mark to show where the pencil on the compass intersects the arc.



Use a straightedge to draw another ray that passes through the point of intersection of the two compass markings.

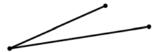


You have now copied the angle exactly.



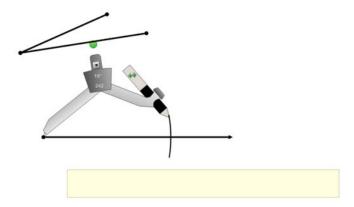
#### **Example C**

An angle is created from two line segments. Use a straightedge to draw a similar figure on your paper. Then, use the straightedge and compass to copy the figure exactly.

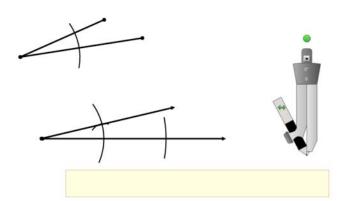


**Solution:** To copy this figure, you will need to copy both the line segments and the angle.

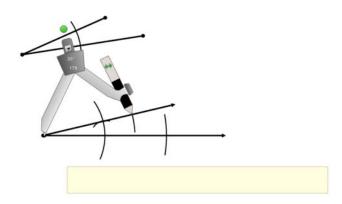
Start by copying the line segment on the bottom using the process outlined in Example A (draw a ray, use the compass to measure the width of the line segment, mark off the endpoint on the ray).



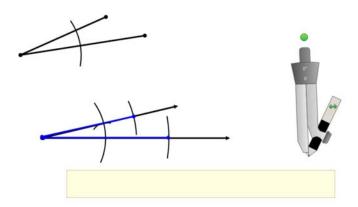
Next, copy the angle using the process outlined in Example B (draw an arc through the angle and draw the same arc through the new ray, measure the width of the arc, draw a new ray through the intersection of the two markings).



Finally, copy the second line segment by measuring its length using the compass and marking off the correct spot for the endpoint.



You can now draw the copied segments in a different color for emphasis.



#### Vocabulary

A *drawing* is a rough sketch used to convey an idea.

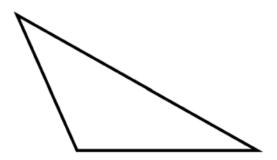
A *construction* is a step-by-step process used to create an accurate geometric figure.

A *compass* is a device that allows you to create a circle with a given radius. Compasses can also help you to copy distances.

A *straightedge* is anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.

#### **Guided Practice**

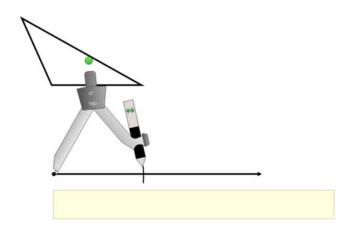
You drew a triangle similar to the one below for the concept problem.



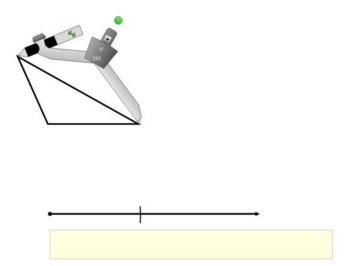
- 1. Copy your triangle using by using line segments.
- 2. Copy your triangle using two line segments and an angle.
- 3. Copy your triangle using two angles and a line segment.

#### **Answers:**

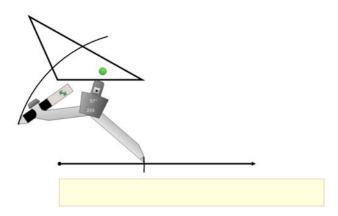
1. Start by copying one line segment. Here, the base line segment is copied.



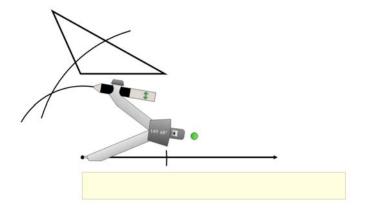
Next, use the compass to measure the length of one of the other sides of the triangle.



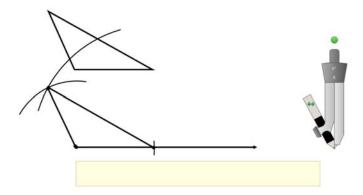
Move the compass to the location of the new triangle and make an arc to mark the length of the second side of the triangle from the correct endpoint.



Repeat with the third side of the triangle.

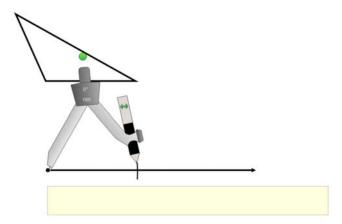


The point where the arcs intersect is the third vertex of the triangle. Connect to form the triangle.

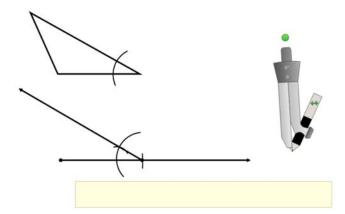


Note that with this method, you have only used the lengths of the sides of the triangle (as opposed to any angles) to construct the new triangle.

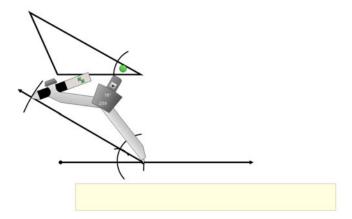
2. Start by copying one line segment. Here, the base of the triangle is copied.



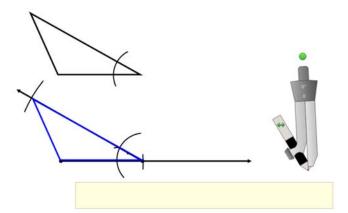
Next, copy the angle at one of the endpoints of the line segment.



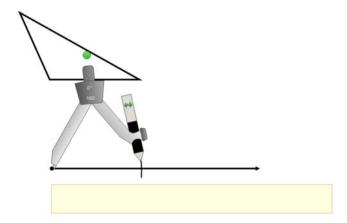
Copy the second side of the triangle (that creates the angle you copied) onto the ray that you just drew.



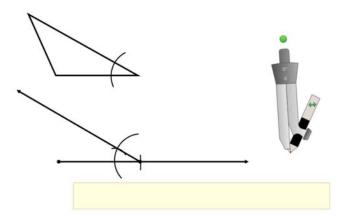
Connect to form the triangle.



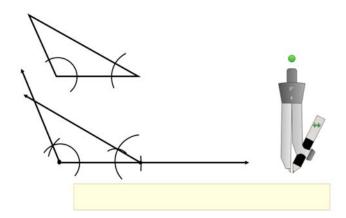
#### 3. Start by copying one line segment.



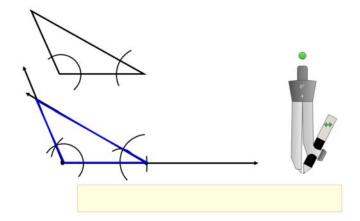
Next, copy the angle at one of the endpoints.



Copy the angle at the other endpoint of the line segment.



#### Connect to form the triangle.



#### **Practice**

- 1. What is the difference between a drawing and a construction?
- 2. What is the difference between a straightedge and a ruler?
- 3. Describe the steps for copying a line segment.
- 4. Describe the steps for copying an angle.
- 5. When copying an angle, do the lengths of the lines matter? Explain.
- 6. Explain the connections between copying a triangle and the triangle congruence criteria.
- 7. Draw a line segment and copy it with a compass and straightedge.
- 8. Draw another line segment and copy it with a compass and straightedge.
- 9. Draw an angle and copy it with a compass and straightedge.
- 10. Draw another angle and copy it with a compass and straightedge.

#### References

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## SLT 4 Construct a perpendicular bisector.

Watch This



MEDIA

Click image to the left for more content.

https://www.youtube.com/watch?v=RKk7EuLunQ8 Constructing a Perpendicular Bisector

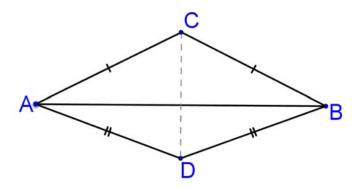
#### Guidance

A **construction** is similar to a **drawing** in that it produces a visual outcome. However, while drawings are often just rough sketches that help to convey an idea, constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

- 1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
- 2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
- 3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

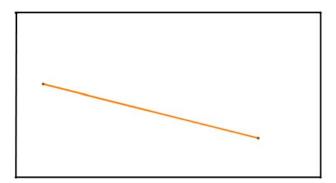
To **bisect** a segment or an angle means to divide it into two congruent parts. A bisector of a line segment will pass through the midpoint of the line segment. A **perpendicular bisector** of a segment passes through the midpoint of the line segment and is perpendicular to the line segment. In order to construct bisectors of segments and angles, it's helpful to remember some relevant theorems:

Any point on the perpendicular bisector of a line segment will be equidistant from the endpoints of the line segment. This means that one way to find the perpendicular bisector of a segment (such as  $\overline{AB}$  below) is to find two points that are equidistant from the endpoints of the line segment (such as  $\overline{C}$  and  $\overline{D}$  below) and connect them.



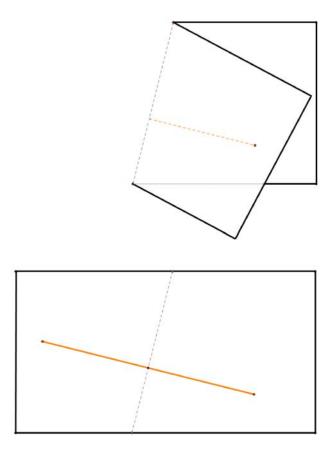
#### Example A

Draw a line segment like the orange one below on a piece of paper (the rectangle below represents the paper). Fold the paper in order to find the perpendicular bisector of the line segment.



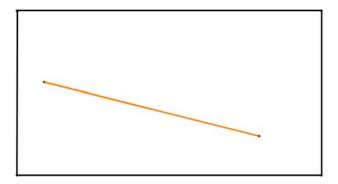
**Solution:** To find the perpendicular bisector, fold the paper so that the endpoints lie on top of each other. By doing this, you have matched the two halves of the line segment exactly.

Any point on the fold is now equidistant from both endpoints, because the endpoints are now in the same place! This means that the crease made by the fold will be the perpendicular bisector of the line segment.

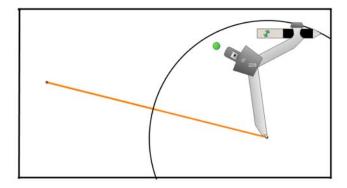


#### Example B

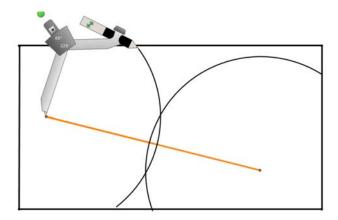
Draw a line segment like the orange one below on a piece of paper (the rectangle below represents the paper). Use a compass and straightedge to find the perpendicular bisector of the line segment.



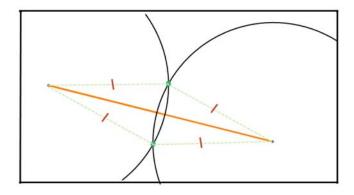
**Solution:** Use your compass to draw a circle centered at each endpoint with the **same radius**. Make sure the radius of the circles is large enough so that the circles will intersect. *If the circles are too big to fit on the paper, draw the portion of the circle that fits on the paper.* Here is the first circle:



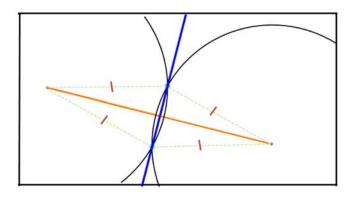
Here is the second circle:



The points of interest are the two points where the circles intersect. Because the radius of each circle is the same, each of these points are equidistant from the endpoints of the line segment.



Therefore, each of these points lies on the perpendicular bisector of the line segment. Use your straightedge to draw a line connecting the two intersection points. This is the perpendicular bisector of the line segment.



#### Example C

How can you use a compass and straightedge to find the midpoint of a line segment?

**Solution:** One way is to use the process from Example B to construct the perpendicular bisector. The midpoint of the line segment will be the point where the perpendicular bisector intersects the line segment.

#### Vocabulary

A *drawing* is a rough sketch used to convey an idea.

A *construction* is a step-by-step process used to create an accurate geometric figure.

A *compass* is a device that allows you to create a circle with a given radius. Compasses can also help you to copy distances.

A *straightedge* is anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.

A *bisector* divides a line segment or angle into two congruent parts.

A *perpendicular* bisector of a segment is a line that bisects the segment and meets the segment at a right angle.

#### **Practice**

- 1. What does it mean to bisect a segment or an angle?
- 2. Describe the steps for finding the perpendicular bisector of a line segment.
- 3. Describe how to use the perpendicular bisector of a line segment to find the midpoint of the line segment.
- 4. What's the difference between a bisector and a perpendicular bisector? How can you construct a non-perpendicular bisector of a line segment?
- 5. Draw a line segment on your paper and construct the perpendicular bisector of the segment.
- 6. Draw another line segment on your paper and construct the perpendicular bisector of that segment using another method.
- 7. Compare and contrast the two methods for finding a bisector-paper folding vs. compass and straightedge.

#### References

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# 4 SLT 5 Bisect a segment and bisect an angle

Here you will learn how to construct bisectors of line segments and angles.

See SLT 4 for bisecting a segment.

#### **Watch This**

http://www.mathopenref.com/constbisectangle.html Bisecting an Angle

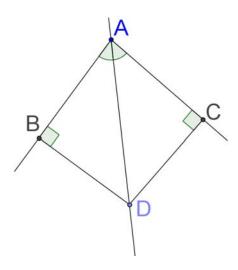
#### Guidance

A **construction** is similar to a **drawing** in that it produces a visual outcome. However, while drawings are often just rough sketches that help to convey an idea, constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

- 1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
- 2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
- 3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

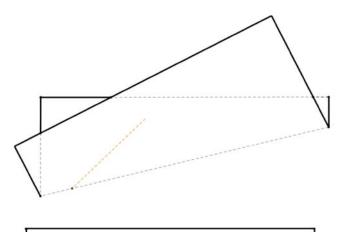
To **bisect** a segment or an angle means to divide it into two congruent parts. In order to construct bisectors of segments and angles, it's helpful to remember some relevant theorems:

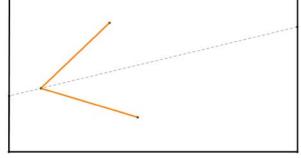
Any point on the angle bisector of an angle will be equidistant from the rays that create the angle. This means that one way to find the angle bisector of an angle (such as  $\angle BAC$  below) is to find two points that are equidistant from the rays that create the angle (such as points A and D below).



#### Example A

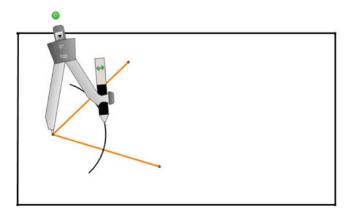
Fold the paper so that one segment overlaps the other segment. The crease will be the angle bisector.



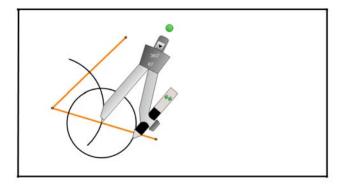


#### **Example B**

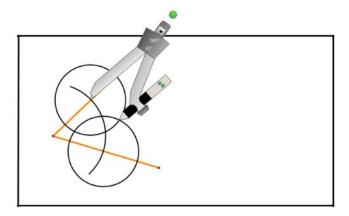
Use your compass to create a portion of a circle centered at the vertex of the angle that passes through both segments creating the angle:



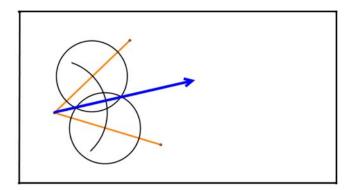
Create two circles with the same radius centered at each intersection point of the arc and the two segments. Make sure the radius is big enough so that the circles will overlap. Here is the first circle:



Here is the second circle:



The points where the circles intersect are equidistant from the segments creating the angle. Therefore, they define the bisector of the angle. Connect those intersection points to create the angle bisector:



#### Vocabulary

A *drawing* is a rough sketch used to convey an idea.

A *construction* is a step-by-step process used to create an accurate geometric figure.

A *compass* is a device that allows you to create a circle with a given radius. Compasses can also help you to copy distances.

A *straightedge* is anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.

A *bisector* divides a line segment or angle into two congruent parts.

An angle bisector of an angle is a line that bisects the angle.

#### **Practice**

- 1. Draw an angle on your paper and construct the bisector of the angle.
- 2. Draw another angle on your paper and construct the bisector of that angle using another method.
- 3. Compare and contrast the two methods for finding a bisector-paper folding vs. compass and straightedge.
- 4. Construct a 45° angle (look at the concept problem for help). Then, construct a 22.5° angle.
- 5. Construct an isosceles right triangle. *Hint: Start by creating a right angle by constructing a perpendicular bisector* .

#### References

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# SLT 6 Construct a line parallel to a given line through a point not on the line

#### **Watch This**



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Click image to the left for more content.

https://www.youtube.com/watch?v=hchk0UkE4BU Constructing Parallel Lines

#### Guidance

**Constructions** are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

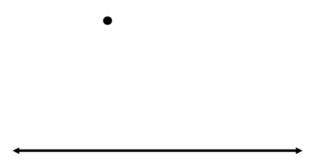
- 1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
- 2. Straightedge: Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
- 3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

To construct **parallel lines**, remember that *if corresponding angles are congruent then lines are parallel*. This means that if you can copy an angle to create congruent corresponding angles, you can create parallel lines. This will be explored in Example A.

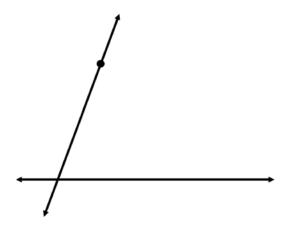
To construct **perpendicular lines**, remember that you already know how to construct a perpendicular bisector. You can use this method to construct a line perpendicular to another line through any given point.

#### Example A

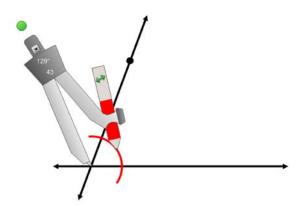
Use your straightedge to draw a line and a point like the one below. Then, construct a line through the point that is parallel to the original line.



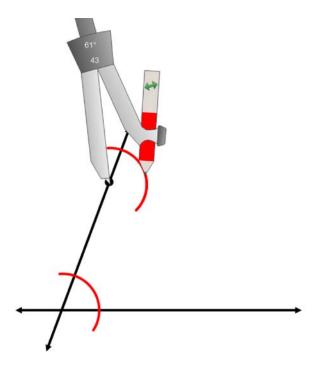
**Solution:** Start by using your straightedge to draw a line through the point that intersects the original line. This will become the transversal after you have constructed the parallel line.



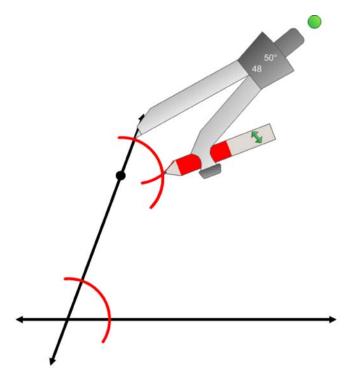
Now, your goal is to copy one of the four angles created at the intersection of the two lines. Draw an arc through the angle you will copy:



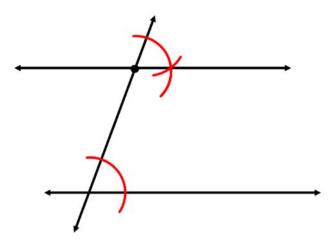
Draw an arc with the same radius in the corresponding location, with the original point as the vertex.



Continue to copy the angle by measuring its width and marking off the correct width for the new angle.

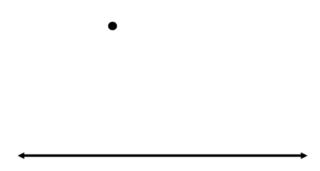


Use your straightedge to construct the parallel line.

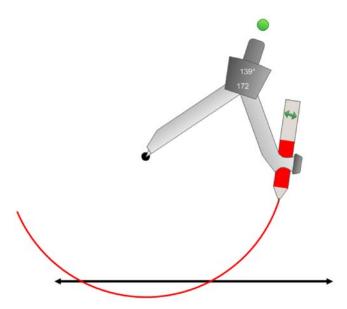


#### Example B

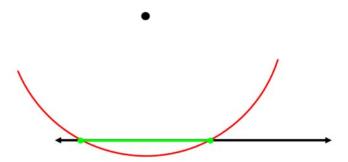
Use your straightedge to draw a line and a point like the one below. Then, construct a line through the point that is perpendicular to the original line.



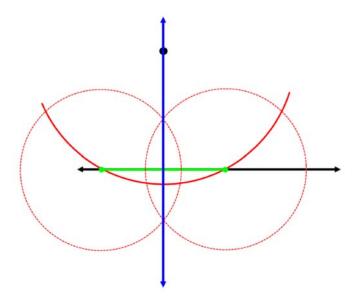
**Solution:** You already know how to construct the perpendicular bisector of a segment. First, find a segment whose perpendicular bisector will pass through the given point. Draw a partial circle centered at the point that passes through the given line two times.



The segment that connects the two points of intersection is the segment you will construct a perpendicular bisector for:



Construct the perpendicular bisector of the green segment:



The blue line is perpendicular to the original line and passes through the original point.

#### Vocabulary

A *drawing* is a rough sketch used to convey an idea.

A *construction* is a step-by-step process used to create an accurate geometric figure.

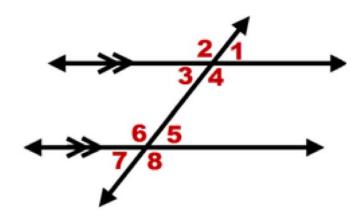
A *compass* is a device that allows you to create a circle with a given radius. Compasses can also help you to copy distances.

A *straightedge* is anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.

Two lines are *perpendicular* if they meet at a right angle.

Two lines are *parallel* if they never intersect.

In the diagram below,  $\angle 1$  and  $\angle 5$  are *corresponding angles* .  $\angle 2$  and  $\angle 6$ ,  $\angle 3$  and  $\angle 7$ ,  $\angle 4$  and  $\angle 8$  are also corresponding angles. If lines are parallel, then corresponding angles are congruent.

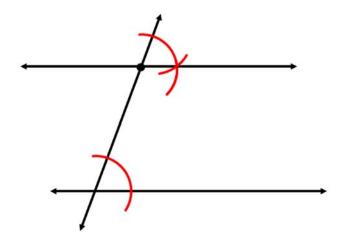


#### **Guided Practice**

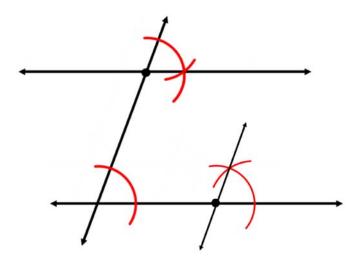
- 1. Draw a line segment. Construct a line parallel to the line segment.
- 2. Extend your construction from #1 to construct a parallelogram.
- 3. How can you be sure that your quadrilateral from #2 is a parallelogram?

#### **Answers:**

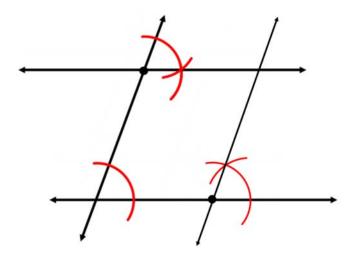
1. Follow the steps from Example A.



2. Draw a point on the original line. Copy the original angle with the new point as the vertex.



Extend the new parallel line. The quadrilateral formed is a parallelogram.



3. A parallelogram is a quadrilateral with two pairs of parallel sides. The construction created two pairs of parallel lines, so the quadrilateral must be a parallelogram.

#### **Practice**

Draw a line segment and a point.

- 1. Construct a line perpendicular to the line segment that passes through the point.
- 2. Construct a line parallel to the line segment that passes through the point.
- 3. Extend your construction to construct a rectangle. Explain what you did to construct the rectangle.

Draw another line segment and point.

- 4. Construct a line parallel to the line segment that passes through the point.
- 5. Extend your construction to construct a parallelogram. Explain what you did to construct the parallelogram.

Draw another line segment and point.

- 6. Construct a line parallel to the line segment that passes through the point.
- 7. Extend your construction to construct a trapezoid. Explain what you did to construct the trapezoid.

Draw another line segment and point.

- 8. Construct a line perpendicular to the line segment that passes through the point.
- 9. Extend your construction to construct a square. Explain what you did to construct the square.
- 10. Justify why the shape you created in #12 must be a square.
- 11. Explain why the method for constructing parallel lines involves copying an angle.
- 12. Explain why the method for constructing perpendicular lines relies on the method for constructing a perpendicular bisector.

#### References

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