





# 1 MEASUREMENTS AND UNCERTAINTIES

## Introduction

This topic is different from other topics in the course book. The content discussed here will be used in most aspects of your studies in physics. You will come across many aspects of this work

in the context of other subject matter. Although you may wish to do so, you would not be expected to read this topic in one go, rather you would return to it as and when it is relevant.

## 1.1 Measurements in physics

### Understanding

- Fundamental and derived SI units
- Scientific notation and metric multipliers
- Significant figures
- Orders of magnitude
- Estimation



### Applications and skills

- Using SI units in the correct format for all required measurements, final answers to calculations and presentation of raw and processed data
- Using scientific notation and metric multipliers
- Quoting and comparing ratios, values, and approximations to the nearest order of magnitude
- Estimating quantities to an appropriate number of significant figures



### Nature of science

In physics you will deal with the qualitative and the quantitative, that is, descriptions of phenomena using words and descriptions using numbers. When we use words we need to interpret the meaning and one person's interpretation will not necessarily be the same as another's. When we deal with numbers (or equations), providing we have learned the rules, there is no mistaking someone else's meaning. It is likely that some readers will be more comfortable

with words than symbols and vice-versa. It is impossible to avoid either methodology on the IB Diploma course and you must learn to be careful with both your numbers and your words. In examinations you are likely to be penalized by writing contradictory statements or mathematically incorrect ones. At the outset of the course you should make sure that you understand the mathematical skills that will make you into a good physicist.

## Quantities and units

Physicists deal with **physical quantities**, which are those things that are *measurable* such as mass, length, time, electrical current, etc. Quantities are related to one another by equations such as  $\rho = \frac{m}{V}$  which is the symbolic form of saying that density is the ratio of the mass of an object to its volume. Note that the symbols in the equation are all written in italic (sloping) fonts – this is how we can be sure that the symbols represent quantities. Units are always written in Roman (upright) font because they sometimes share the same symbol with a quantity. So “*m*” represents the quantity “mass” but “m” represents the unit “metre”. We will use this convention throughout the course book, and it is also the convention used by the IB.



### Nature of science

#### The use of symbols

The use of Greek letters such as rho ( $\rho$ ) is very common in physics. There are so many quantities that, even using the 52 Arabic letters (lower case and capitals), we soon run out of unique symbols. Sometimes symbols such as *d* and *x* have multiple uses, meaning that Greek letters have become just one way of trying to tie a symbol to a quantity uniquely. Of course, we must consider what happens when we run out of Greek letters too – we then use Russian ones from the Cyrillic alphabet.

| Greek |         |     | Russian |                      |  |
|-------|---------|-----|---------|----------------------|--|
| A α   | alpha   | N ν | nu      | Аа Әә Бб Вв Гг Дд Ее |  |
| B β   | beta    | Ξ ξ | ksi     | Жж Зз Ѕз Ии Йй Кк    |  |
| Г γ   | gamma   | Ο ο | omicron | Лл Мм Нн Њн Оо Пп    |  |
| Δ δ   | delta   | Π π | pi      | Рр Сс Тт Уу Ўу Фф    |  |
| E ε   | epsilon | Ρ ρ | rho     | Хх Цц Чч Шш Ъъ       |  |
| Z ζ   | zeta    | Σ σ | sigma   | Юю Яя                |  |
| H η   | eta     | Τ τ | tau     |                      |  |
| Θ θ   | theta   | Υ υ | upsilon |                      |  |
| I ι   | iota    | Φ φ | phi     |                      |  |
| K κ   | kappa   | X χ | chi     |                      |  |
| Λ λ   | lambda  | Ψ ψ | psi     |                      |  |
| M μ   | mu      | Ω ω | omega   |                      |  |

**Fundamental quantities** are those quantities that are considered to be so basic that all other quantities need to be expressed in terms of them. In the density equation  $\rho = \frac{m}{V}$  only mass is chosen to be fundamental (volume being the product of three lengths), density and volume are said to be **derived quantities**.

It is essential that all measurements made by one person are understood by others. To achieve this we use units that are understood to have unambiguous meaning. The worldwide standard for units is known as SI – *Système international d’unités*. This system has been developed from the metric system of units and means that, when values of scientific quantities are communicated between people, there should never be any confusion. The SI defines both units and prefixes – letters used to form decimal multiples or sub-multiples of the units. The units themselves are classified as being either fundamental (or *base*), derived, and supplementary.

There are only two supplementary units in SI and you will meet only one of these during the Diploma course, so we might as well mention them first. The two supplementary units are the radian (rad) – the unit of angular measurement and the steradian (sr) – the unit of “solid angle”. The radian is a useful alternative to the degree and is defined as *the angle subtended by an arc of a circle having the same length as the radius*,



as shown in figure 1. We will look at the radian in more detail in Sub-topic 6.1. The steradian is the three-dimensional equivalent of the radian and uses the idea of mapping a circle on to the surface of a sphere.

## Fundamental and derived units

In SI there are seven **fundamental units** and you will use six of these on the Diploma course (the seventh, the candela, is included here for completeness). The fundamental quantities are length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity. The units for these quantities have exact definitions and are precisely reproducible, given the right equipment. This means that any quantity can, in theory, be compared with the fundamental measurement to ensure that a measurement of that quantity is accurate. In practice, most measurements are made against more easily achieved standards so, for example, length will usually be compared with a standard metre rather than the distance travelled by light in a vacuum. You will not be expected to know the definitions of the fundamental quantities, but they are provided here to allow you to see just how precise they are.

**metre (m):** the length of the path travelled by light in a vacuum during a time interval of  $\frac{1}{299\,792\,458}$  of a second.

**kilogram (kg):** mass equal to the mass of the international prototype of the kilogram kept at the Bureau International des Poids et Mesures at Sèvres, near Paris.

**second (s):** the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

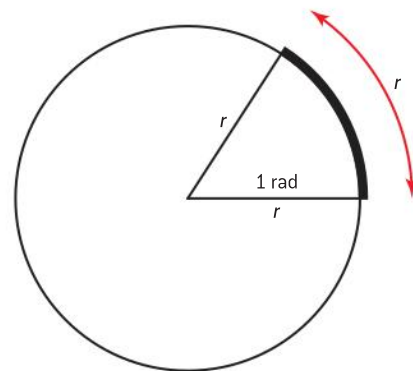
**ampere (A):** that constant current which, if maintained in two straight parallel conductors of infinite length, negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to  $2 \times 10^{-7}$  newtons per metre of length.

**kelvin (K):** the fraction  $\frac{1}{273.16}$  of the thermodynamic temperature of the triple point of water.

**mole (mol):** the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kg of carbon-12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

**candela (cd):** the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency  $540 \times 10^{12}$  hertz and that has a radiant intensity in that direction of  $\frac{1}{683}$  watt per steradian.

All quantities that are not fundamental are known as *derived* and these can always be expressed in terms of the fundamental quantities through a relevant equation. For example, speed is the rate of change of distance with respect to time or in equation form  $v = \frac{\Delta s}{\Delta t}$  (where  $\Delta s$  means the change in distance and  $\Delta t$  means the change in time). As both distance (and length) and time are fundamental quantities, speed is a derived quantity.



▲ Figure 1 Definition of the radian.



▲ Figure 2 The international prototype kilogram.

### TOK

#### Deciding on what is fundamental

Who has made the decision that the fundamental quantities are those of mass, length, time, electrical current, temperature, luminous intensity, and amount of substance? In an alternative universe it may be that the fundamental quantities are based on force, volume, frequency, potential difference, specific heat capacity, and brightness. Would that be a drawback or would it have meant that “humanity” would have progressed at a faster rate?



**Note**

If you are reading this at the start of the course, it may seem that there are so many things that you might not know; but, take heart, “Rome was not built in a day” and soon much will come as second nature. When we write units as  $\text{m s}^{-1}$  and  $\text{m s}^{-2}$  it is a more effective and preferable way to writing what you may have written in the past as  $\text{m/s}$  and  $\text{m/s}^2$ ; both forms are still read as “metres per second” and “metres per second squared.”

The units used for fundamental quantities are unsurprisingly known as fundamental units and those for derived quantities are known as derived units. It is a straightforward approach to be able to express the unit of any quantity in terms of its fundamental units, provided you know the equation relating the quantities. Nineteen fundamental quantities have their own unit but it is also valid, if cumbersome, to express this in terms of fundamental units. For example, the SI unit of pressure is the pascal (Pa), which is expressed in fundamental units as  $\text{m}^{-1} \text{kg s}^{-2}$ .

**Nature of science****Capitals or lower case?**

Notice that when we write the unit newton in full, we use a lower case n but we use a capital N for the symbol for the unit – unfortunately some word processors have default setting to correct this so take care! All units written in full should start with a lower case letter, but those that have been derived in honour of a scientist will have a symbol that is a capital letter. In this way there is no confusion between the scientist and the unit: “Newton” refers to Sir Isaac Newton but “newton” means the unit. Sometimes units are abbreviations of the scientist’s surname, so amp (which is a shortened form of ampère anyway) is named after Ampère, the volt after Volta, the farad, Faraday, etc.



▲ Figure 3 Choosing fundamental units in an alternative universe.

**Example of how to relate fundamental and derived units**

The unit of force is the newton (N). This is a derived unit and can be expressed in terms of fundamental units as  $\text{kg m s}^{-2}$ . The reason for this is that force can be defined as being the product of mass and acceleration or  $F = ma$ . Mass is a fundamental quantity but acceleration is not. Acceleration is the rate of change of velocity or  $a = \frac{\Delta v}{\Delta t}$  where  $\Delta v$  represents the change in velocity and  $\Delta t$  the change in time. Although time is a fundamental quantity, velocity is not so we need to take another step in defining velocity in fundamental quantities. Velocity is the rate of change of displacement (a quantity that we will discuss later in the topic but, for now, it simply means distance in a given direction). So the equation for velocity is  $v = \frac{\Delta s}{\Delta t}$  with  $\Delta s$  being the change in displacement and  $\Delta t$  again being the change in time. Displacement (a length) and time are both fundamental, so we are now in a position to put N into fundamental units. The unit of velocity is  $\text{m s}^{-1}$  and these are already fundamental – there is no shortened form of this. The units of acceleration will therefore be those of velocity divided by time and so will be  $\frac{\text{m s}^{-1}}{\text{s}}$  which is written as  $\text{m s}^{-2}$ . So the unit of force will be the unit of mass multiplied by the unit of acceleration and, therefore, be  $\text{kg m s}^{-2}$ . This is such a common unit that it has its own name, the newton, ( $\text{N} \equiv \text{kg m s}^{-2}$  – a mathematical way of expressing that the two units are identical). So if you are in an examination and forget the unit of force you could always write  $\text{kg m s}^{-2}$  (if you have time to work it out!).

**Significant figures**

Calculators usually give you many digits in an answer. How do you decide how many digits to write down for the final answer?



Scientists use a method of rounding to a certain number of significant figures (often abbreviated to s.f.). “Significant” here means meaningful.

Consider the number 84 072, the 8 is the most significant digit, because it tells us that the number is eighty thousand and something. The 4 is the next most significant telling us that there are also four thousand and something. Even though it is a zero, the next digit, the 0, is the third most significant digit here.

When we face a decimal number such as 0.00245, the 2 is the most significant digit because it tells us that the number is two thousandth and something. The 4 is the next most significant, showing that there are four ten thousandths and something.

If we wish to express this number to two significant figures we need to round the number from three to two digits. If the last number had been 0.00244 we would have rounded down to 0.0024 and if it had been 0.00246 we would have rounded up to 0.0025. However, it is a 5 so what do we do? In this case there is equal justification for rounding up and down, so all you really need to be is consistent with your choice for a set of figures – you can choose to round up or down. Often you will have further digits to help you, so if the number had been 0.002451 and you wanted it rounded to two significant figures it would be rounded up to 0.0025.

### Some rules for using significant figures

- A digit that is not a zero will always be significant – 345 is three significant figures (3 s.f.).
- Zeros that occur sandwiched between non-zero digits are always significant – 3405 (4 s.f.); 10.3405 (6 s.f.).
- Non-sandwiched zeros that occur to the left of a non-zero digit are not significant – 0.345 (3 s.f.); 0.034 (2 s.f.).
- Zeros that occur to the right of the decimal point are significant, provided that they are to the right of a non-zero digit – 1.034 (4 s.f.); 1.00 (3 s.f.); 0.34500 (5 s.f.); 0.003 (1 s.f.).
- When there is no decimal point, trailing zeros are not significant (to make them significant there needs to be a decimal point) – 400 (1 s.f.); 400. (3 s.f.) – but this is rarely written.

### Scientific notation

One of the fascinations for physicists is dealing with the very large (e.g. the universe) and the very small (e.g. electrons). Many physical constants (quantities that do not change) are also very large or very small. This presents a problem: how can writing many digits be avoided? The answer is to use scientific notation.

The speed of light has a value of  $299\,792\,458\text{ m s}^{-1}$ . This can be rounded to three significant figures as  $300\,000\,000\text{ m s}^{-1}$ . There are a lot of zeros in this and it would be easy to miss one out or add another. In scientific notation this number is written as  $3.00 \times 10^8\text{ m s}^{-1}$  (to three significant figures).

Let us analyse writing another large number in scientific notation. The mass of the Sun to four significant figures is 1 989 000 000 000 000 000 000 000 kg (that is 1989 and twenty-seven zeros). To convert

this into scientific notation we write it as  $1.989$  and then we imagine moving the decimal point 30 places to the left (remember we can write as many trailing zeros as we like to a decimal number without changing it). This brings our number back to the original number and so it gives the mass of the Sun as  $1.989 \times 10^{30}$  kg.

A similar idea is applied to very small numbers such as the charge on the electron, which has an accepted value of approximately 0.000 000 000 000 1602 coulombs. Again we write the coefficient as 1.602 and we must move the decimal point 19 places to the right in order to bring 0.000 000 000 000 000 000 1602 into this form. The base is always 10 and moving our decimal point to the right means the exponent is negative. We can write this number as  $1.602 \times 10^{-19}$  C.

Apart from avoiding making mistakes, there is a second reason why scientific notation is preferable to writing numbers in longhand. This is when we are dealing with several numbers in an equation. In writing the value of the speed of light as  $3.00 \times 10^8 \text{ m s}^{-1}$ , 3.00 is called the “coefficient” of the number and it will always be a number between 1 and 10. The 10 is called the “base” and the 8 is the “exponent”.

There are some simple rules to apply:

- When adding or subtracting numbers the exponent must be the same or made to be the same.
- When multiplying numbers we add the exponents.
- When dividing numbers we subtract one exponent from the other.
- When raising a number to a power we raise the coefficient to the power and multiply the exponent by the power.

### Worked examples

In these examples we are going to evaluate each of the calculations.

**1**  $1.40 \times 10^6 + 3.5 \times 10^5$

#### Solution

These must be written as  $1.40 \times 10^6 + 0.35 \times 10^6$  so that both numbers have the same exponents.

They can now be added directly to give  $1.75 \times 10^6$

**2**  $3.7 \times 10^5 \times 2.1 \times 10^8$

#### Solution

The coefficients are multiplied and the exponents are added, so we have:  $3.7 \times 2.1 = 7.77$  (which we round to 7.8 to be in line with the data – something we will discuss later in this topic) and:  $5 + 8 = 13$

So we write this product as:  $7.8 \times 10^{13}$

**3**  $3.7 \times 10^5 \times 2.1 \times 10^{-8}$

#### Solution

Again the coefficients are multiplied and the exponents are added, so we have:  $3.7 \times 2.1 = 7.8$

Here the exponents are subtracted (since the 8 is negative) to give:  $5 - 8 = -3$

So we write this product as:  $7.8 \times 10^{-3}$

**4**  $\frac{4.8 \times 10^5}{3.1 \times 10^2}$

#### Solution

The coefficients are divided and the exponents are subtracted so we have:  $4.8 \div 3.1 = 1.548$  (which we round to 1.5)

And  $5 - 2 = 3$

This makes the result of the division  $1.5 \times 10^3$

**5**  $(3.6 \times 10^7)^3$

#### Solution

We cube 3.6 and  $3.6^3 = 46.7$

And multiply 7 by 3 to give 21

This gives  $46.7 \times 10^{21}$ , which should become  $4.7 \times 10^{22}$  in scientific notation.





## Metric multipliers (prefixes)

Scientists have a second way of abbreviating units: by using metric multipliers (usually called “prefixes”). An SI prefix is a name or associated symbol that is written before a unit to indicate the appropriate power of 10. So instead of writing  $2.5 \times 10^{12}$  J we could alternatively write this as 2.5 TJ (terajoule). Figure 4 gives the 20 SI prefixes – these are provided for you as part of the data booklet used in examinations.

## Orders of magnitude

An important skill for physicists is to understand whether or not the physics being considered is *sensible*. When performing a calculation in which someone’s mass was calculated to be 5000 kg, this should ring alarm bells. Since average adult masses (“weights”) will usually be 60–90 kg, a value of 5000 kg is an impossibility.

A number rounded to the nearest power of 10 is called an **order of magnitude**. For example, when considering the average adult human mass: 60–80 kg is closer to 100 kg than 10 kg, making the order of magnitude  $10^2$  and not  $10^1$ . Of course, we are not saying that all adult humans have a mass of 100 kg, simply that their average mass is closer to 100 than 10. In a similar way, the mass of a sheet of A4 paper may be 3.8 g which, expressed in kg, will be  $3.8 \times 10^{-3}$  kg. Since 3.8 is closer to 1 than to 10, this makes the order of magnitude of its mass  $10^{-3}$  kg. This suggests that the ratio of adult mass to the mass of a piece of paper (should you wish to make this comparison) =  $\frac{10^2}{10^{-3}} = 10^{2-(-3)} = 10^5 = 100\,000$ . In other words, an adult human is 5 orders of magnitude (5 powers of 10) heavier than a sheet of A4 paper.

## Estimation

Estimation is a skill that is used by scientists and others in order to produce a value that is a useable approximation to a true value. Estimation is closely related to finding an order of magnitude, but may result in a value that is more precise than the nearest power of 10.

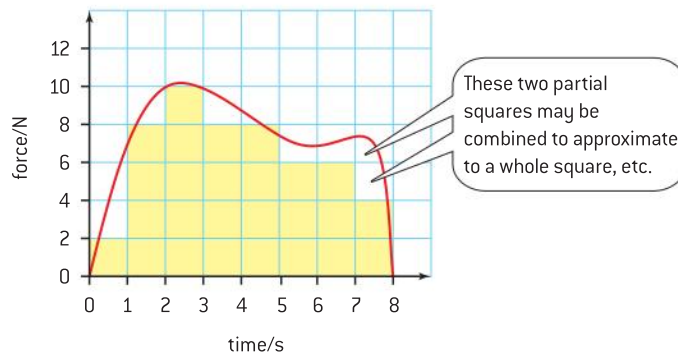
Whenever you measure a length with a ruler calibrated in millimetres you can usually see the whole number of millimetres but will need to estimate to the next  $\frac{1}{10}$  mm – you may need a magnifying glass to help you to do this. The same thing is true with most non-digital measuring instruments.

Similarly, when you need to find the area under a non-regular curve, you cannot truly work out the actual area so you will need to find the area of a rectangle and estimate how many rectangles there are. Figure 5 shows a graph of how the force applied to an object varies with time. The area under the graph gives the impulse (as you will see in Topic 2). There are 26 complete or nearly complete yellow squares under the curve and there are further partial squares totalling about four full squares in all. This gives about 30 full squares under the curve. Each curve has an area equivalent to  $2\text{ N} \times 1\text{ s} = 2\text{ N s}$ . This gives an estimate of about 60 N s for the total impulse.

| Factor     | Name  | Symbol |
|------------|-------|--------|
| $10^{24}$  | yotta | Y      |
| $10^{21}$  | zetta | Z      |
| $10^{18}$  | exa   | E      |
| $10^{15}$  | peta  | P      |
| $10^{12}$  | tera  | T      |
| $10^9$     | giga  | G      |
| $10^6$     | mega  | M      |
| $10^3$     | kilo  | k      |
| $10^2$     | hecto | h      |
| $10^1$     | deka  | da     |
| $10^{-1}$  | deci  | d      |
| $10^{-2}$  | centi | c      |
| $10^{-3}$  | milli | m      |
| $10^{-6}$  | micro | $\mu$  |
| $10^{-9}$  | nano  | n      |
| $10^{-12}$ | pico  | p      |
| $10^{-15}$ | femto | f      |
| $10^{-18}$ | atto  | a      |
| $10^{-21}$ | zepto | z      |
| $10^{-24}$ | yocto | y      |

▲ Figure 4 SI metric multipliers.

In an examination, estimation questions will always have a tolerance given with the accepted answer, so in this case it might be  $(60 \pm 2)$  N s.



▲ Figure 5

## 1.2 Uncertainties and errors

### Understanding

- Random and systematic errors
- Absolute, fractional, and percentage uncertainties
- Error bars
- Uncertainty of gradient and intercepts



### Nature of science

In Sub-topic 1.1 we looked at how we define the fundamental physical quantities. Each of these is measured on a scale by comparing the quantity with something that is “precisely reproducible”. By precisely reproducible do we mean “exact”? The answer to this is no. If we think about the definition of the ampere, we will measure a force of  $2 \times 10^{-7}$  N. If we measure it to be  $2.1 \times 10^{-7}$  it doesn’t invalidate the measurement since the definition is given to just one significant figure. All measurements have their limitations or uncertainties and it is important that both the measurer and the person working with the measurement understand what the limitations are. This is why we must always consider the uncertainty in any measurement of a physical quantity.



### Applications and skills

- Explaining how random and systematic errors can be identified and reduced
- Collecting data that include absolute and/or fractional uncertainties and stating these as an uncertainty range (using  $\pm$ )
- Propagating uncertainties through calculations involving addition, subtraction, multiplication, division, and raising to a power
- Using error bars to calculate the uncertainty in gradients and intercepts

### Equations

Propagation of uncertainties:

$$\text{If: } y = a \pm b$$

$$\text{then: } \Delta y = \Delta a + \Delta b$$

$$\text{If: } y = \frac{ab}{c}$$

$$\text{then: } \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

$$\text{If: } y = a^n$$

$$\text{then: } \frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$$



## Uncertainties in measurement

### Introduction

No experimental quantity can be absolutely accurate when measured – it is always subject to some degree of uncertainty. We will look at the reasons for this in this section.

There are two types of error that contribute to our uncertainty about a reading – systematic and random.

### Systematic errors

As the name suggests, these types of errors are due to the system being used to make the measurement. This may be due to faulty apparatus. For example, a scale may be incorrectly calibrated either during manufacture of the equipment, or because it has changed over a period of time.

Rulers warp and, as a result, the divisions are no longer symmetrical.

A timer can run slowly if its quartz crystal becomes damaged (not because the battery voltage has fallen – when the timer simply stops).



▲ Figure 1 Zero error on digital calliper.

When measuring distances from sealed radioactive sources or light-dependent resistors (LDRs), it is hard to know where the source is actually positioned or where the active surface of the LDR is.

The zero setting on apparatus can drift, due to usage, so that it no longer reads zero when it should – this is called a **zero error**.

Figure 1 shows a digital calliper with the jaws closed. This should read 0.000 mm but there is a zero error and it reads 0.01 mm. This means that all readings will be 0.01 mm bigger than they should be. The calliper can be reset to zero or 0.01 mm could be subtracted from any readings made.

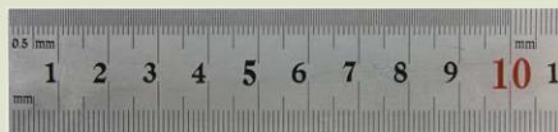
Often it is not possible to spot a systematic error and experimenters have to accept the reading on their instruments, or else spend significant effort in making sure that they are re-calibrated by checking the scale against a standard scale. Repeating a reading never removes the systematic error. The real problem with systematic errors is that it is only possible to check them by performing the same task with another apparatus. If the two sets give the same results, the likelihood is that they are both performing well; however, if there is disagreement in the results a third set may be needed to resolve any difference.

In general we deal with zero errors as well as we can and then move on with our experimentation. **When systematic errors are small, a measurement is said to be accurate.**

## Nature of science

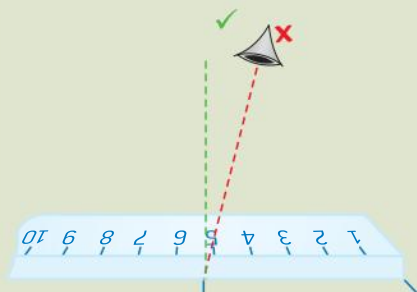
### Systematic errors

Uncertainty when using a 300 mm ruler may be quoted to  $\pm 0.5$  mm or  $\pm 1$  mm depending on your view of how precisely you can gauge the reading. To be on the safe side you might wish to use the larger uncertainty and then you will be sure that the reading lies within your bounds.



▲ Figure 2 Millimeter (mm) scale on ruler.

You should make sure you observe the scale from directly above and at right angles to the plane of the ruler in order to avoid parallax errors.



▲ Figure 3 Parallax error.

The meter in figure 4 shows an analogue ammeter with a fairly large scale – there is justification in giving this reading as being  $(40 \pm 5)$  A.



▲ Figure 4 Analogue scale.



▲ Figure 5 Digital scale.

The digital ammeter in figure 5 gives a value of 0.27 A which should be recorded as  $(0.27 \pm 0.01)$  A.

In each of these examples the uncertainty is quoted to the same **precision** (number of decimal places) as the reading – it is essential to do this as the number of decimal places is always indicative of precision. When we write an energy value as being 8 J we are implying that it is  $(8 \pm 1)$  J and if we write it as 8.0 J it implies a precision of  $\pm 0.1$  J.

## Random errors

Random errors can occur in any measurement, but crop up most frequently when the experimenter has to estimate the last significant figure when reading a scale. If an instrument is insensitive then it may be difficult to judge whether a reading would have changed in different circumstances. For a single reading the uncertainty could well be better than the smallest scale division available. But, since you are determining the maximum possible range of values, it is a sensible precaution to use this larger precision. Dealing with digital scales is a problem – the likelihood is that you have really no idea how precisely the scales are calibrated. Choosing the least significant digit on the scale may severely underestimate the uncertainty but, unless you know the manufacturer's data regarding calibration, it is probably the best you can do.

When measuring a time manually it is inappropriate to use the precision of the timer as the uncertainty in a reading, since your reaction time is likely to be far greater than this. For example, if you timed twenty oscillations of a pendulum to take 16.27 s this should be recorded as being  $(16.3 \pm 0.1)$  s. This is because your reaction time dominates the precision of the timer. If you know that your reaction time is greater than 0.1 s then you should quote that value instead of 0.1 s.



The best way of handling random errors is to take a series of repeat readings and find the average of each set of data. Half the range of the values will give a value that is a good approximation to the statistical value that more advanced error analysis provides. The range is the largest value minus the smallest value.

**Readings with small random errors are said to be precise** (this does not mean they are accurate, however).

### Worked examples

- 1** In measuring the angle of refraction at an air-glass interface for a constant angle of incidence the following results were obtained (using a protractor with a precision of  $\pm 1^\circ$ ):

$45^\circ, 47^\circ, 46^\circ, 45^\circ, 44^\circ$

How should we express the angle of refraction?

#### Solution

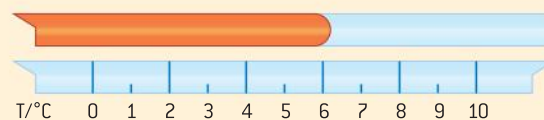
The mean of these values is  $45.4^\circ$  and the range is  $(47^\circ - 44^\circ) = 3^\circ$ .

Half the range is  $1.5^\circ$ .

How then do we record our overall value for the angle of refraction?

Since the precision of the protractor is  $\pm 1^\circ$ , we should quote our mean to a whole number (integral) value and it will round down to  $45^\circ$ . We should not minimize our uncertainty unrealistically and so we should round this up to  $2^\circ$ . This means that the angle of refraction should be recorded as  $45 \pm 2^\circ$ .

- 2** The diagram below shows the position of the meniscus of the mercury in a mercury-in-glass thermometer.



Express the temperature and its uncertainty to an appropriate number of significant figures.

#### Solution

The scale is calibrated in degrees but they are quite clear here, so it is reasonable to expect a precision of  $\pm 0.5^\circ\text{C}$ . The meniscus is closer to 6 than to 6.5 (although that is a judgement decision) so the values should be recorded as  $(6.0 \pm 0.5)^\circ\text{C}$ . Remember the measurement and the uncertainty should be to the same number of decimal places.

- 3** A student takes a series of measurements of a certain quantity. He then averages his measurements. What aspects of systematic and random uncertainties is he addressing by taking repeats and averages?

#### Solution

Systematic errors are not dealt with by means of repeat readings, but taking repeat readings and averaging them should cause the average value to be closer to the true value than a randomly chosen individual measurement.

## Absolute and fractional uncertainties

The values of uncertainties that we have been looking at are called **absolute uncertainties**. These values have the same units as the quantity and should be written to the same number of decimal places.

Dividing the uncertainty by the value itself leaves a dimensionless quantity (one with no units) and gives us the **fractional uncertainty**. Percentaging the fractional uncertainty gives the **percentage uncertainty**.



### Worked example

Calculate the absolute, fractional, and percentage uncertainties for the following measurements of a force,  $F$ :

2.5 N, 2.8 N, 2.6 N

### Solution

mean value =  $\frac{2.5 \text{ N} + 2.8 \text{ N} + 2.6 \text{ N}}{3} = 2.63 \text{ N}$ , this is rounded down to 2.6 N

range =  $(2.8 - 2.5) \text{ N} = 0.3 \text{ N}$ , giving an absolute uncertainty of 0.15 N that rounds up to 0.2 N

We would write our value for  $F$  as  $(2.6 \pm 0.2) \text{ N}$

the fractional uncertainty is  $\frac{0.2}{2.6} = 0.077$  and the percentage uncertainty will be  $0.077 \times 100\% = 7.7\%$

## Propagation of uncertainties

Often we measure quantities and then use our measurements to calculate other quantities with an equation. The uncertainty in the calculated value will be determined from a combination of the uncertainties in the quantities that we have used to calculate the value from. This is known as **propagation of uncertainties**.

There are some simple rules that we can apply when we are propagating uncertainties. In more advanced treatment of this topic we would demonstrate how these rules are developed, but we are going to focus on your application of these rules here (since you will never be asked to prove them and you can look them up in a text book or on the Internet if you want further information).

In the uncertainty equations discussed next,  $a$ ,  $b$ ,  $c$ , etc. are the quantities and  $\Delta a$ ,  $\Delta b$ ,  $\Delta c$ , etc. are the absolute uncertainties in these quantities.

### Addition and subtraction

This is the easiest of the rules because when we add or subtract quantities we always *add* their absolute uncertainties.

When  $a = b + c$  or  $a = b - c$  then  $\Delta a = \Delta b + \Delta c$

In order to use these relationships don't forget that the quantities being added or subtracted must have the same units.

So if we are combining two masses  $m_1$  and  $m_2$  then the total mass  $m$  will be the sum of the other two masses.

$m_1 = (200 \pm 10) \text{ g}$  and  $m_2 = (100 \pm 10) \text{ g}$  so  $m = 300 \text{ g}$  and  $\Delta m = 20 \text{ g}$  meaning we should write this as:

$$m = (300 \pm 20) \text{ g}$$

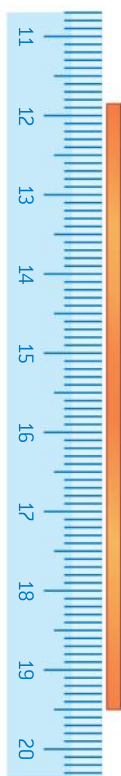
We use subtraction more often than we realise when we are measuring lengths. When we set the zero of our ruler against one end of an object we are making a judgement of where the zero is positioned and this really means that the value is  $(0.0 \pm 0.5) \text{ mm}$ .

A ruler is used to measure a metal rod as shown in figure 6. The length is found by subtracting the smaller measurement from the larger one. The uncertainty for each measurement is  $\pm 0.5 \text{ mm}$ .

Larger measurement = 195.0 mm

Smaller measurement = 118.5 mm

Length =  $(76.5 \pm 1.0) \text{ mm}$  as the uncertainty is  $0.5 \text{ mm} + 0.5 \text{ mm}$



▲ Figure 6 Measuring a length.



## Nature of science

### Subtracting values

When subtraction is involved in a relationship you need to be particularly careful. The resulting quantity becomes smaller in size (because of subtraction), while the absolute uncertainty becomes larger (because of addition). Imagine two values that are subtracted:  $b = 4.0 \pm 0.1$  and  $c = 3.0 \pm 0.1$ .

We won't concern ourselves with what these quantities actually are here.

If  $a = b - c$  then  $a = 1.0$  and since  $\Delta a = \Delta b + \Delta c$  then  $\Delta a = 0.2$

We have gone from two values in which the percentage uncertainty is 2.5% and 3.3% respectively to a calculated value with uncertainty of 20%. Now that really is propagation of uncertainties!

### Multiplication and division

When we multiply or divide quantities we *add* their fractional or percentage uncertainties, so:

when  $a = bc$  or  $a = \frac{b}{c}$  or  $a = \frac{c}{b}$

then  $\frac{\Delta a}{a} = \frac{\Delta b}{b} + \frac{\Delta c}{c}$

There are very few relationships in physics that do not include some form of multiplication or division.

We have seen that density  $\rho$  is given by the expression  $\rho = \frac{m}{V}$  where  $m$  is the mass of a sample of the substance and  $V$  is its volume. For a particular sample, the percentage uncertainty in the mass is 5% and for the volume is 12%.

The percentage uncertainty in the calculated value of the density will therefore be  $\pm 17\%$ .

If the sample had been cubical in shape and the uncertainty in each of the sides was 4% we can see how this brings about a volume with uncertainty of 12%:

For a cube the volume is the cube of the side length ( $V = l^3 = l \times l \times l$ )

so  $\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta l}{l} + \frac{\Delta l}{l} = 4\% + 4\% + 4\% = 12\%$

This example leads us to:

### Raising a quantity to a power

From the cube example you might have spotted that  $\frac{\Delta V}{V} = 3 \frac{\Delta l}{l}$

This result can be generalized so that when  $a = b^n$  (where  $n$  can be a positive or negative whole, integral, or decimal number)

then  $\frac{\Delta a}{a} = \left| n \frac{\Delta b}{b} \right|$

The modulus sign is included as an alternative way of telling us that the uncertainty can be either positive or negative.

### Worked example

The period  $T$  of oscillation of a mass  $m$  on a spring, having spring constant  $k$  is  $T = 2\pi\sqrt{\frac{m}{k}}$

*Don't worry about what these quantities actually mean at this stage.*

The uncertainty in  $k$  is 11% and the uncertainty in  $m$  is 5%.

Calculate the approximate uncertainty in a value for  $T$  of 1.20 s.

### Solution

First let's adjust the equation a little – we can write it as

$$T = 2\pi\left(\frac{m}{k}\right)^{\frac{1}{2}} \text{ which is of the form } a = 2\pi\left(\frac{b}{c}\right)^n$$

Although we will truncate  $\pi$ , we can really write it to as many significant figures as we wish and so the percentage uncertainty in  $\pi$ , as in 2 will be zero.

Using the division and power relationships:

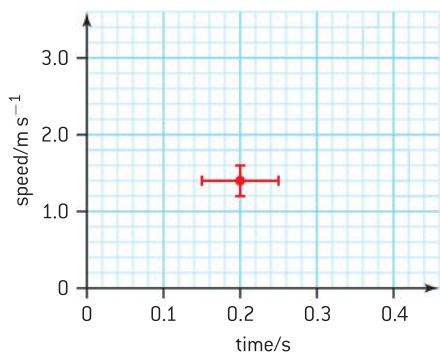
$$\frac{\Delta a}{a} = n \frac{\Delta b}{b} + n \frac{\Delta c}{c} \text{ or here } \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta m}{m} + \frac{1}{2} \frac{\Delta k}{k}$$

so the percentage uncertainty in  $T$  will be half that in  $m$  + half that in  $k$ .

This means that the percentage error in  $T = 0.5 \times 5\% + 0.5 \times 11\% = 8\%$

If the measured value of  $T$  is 1.20 s then the absolute uncertainty is  $1.20 \times \frac{8}{100} = 0.096$  This rounds up to 0.10 and so we quote  $T$  as being  $(1.20 \pm 0.10)$  s.

Remember that the quantity and the uncertainty must be to the same number of decimal places and so the zeros are important, as they give us the precision in the value.



▲ Figure 7 Error bars.

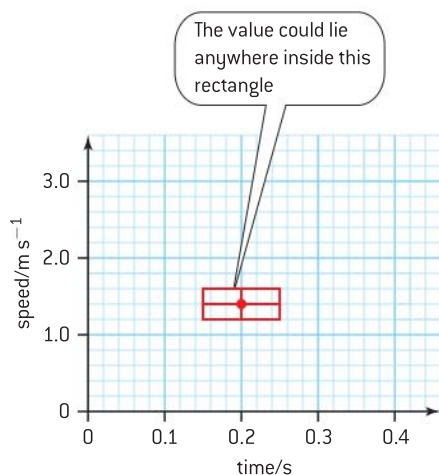
### Drawing graphs

An important justification for experimental work is to investigate the relationship between physical quantities. One set of values is rarely very revealing even if it can be used to calculate a physical constant, such as dividing the potential difference across a resistor by the current in the resistor to find the resistance. Although the calculation does tell you the resistance for one value of current, it says nothing about whether the resistance depends upon the current. Taking a series of values would tell you if the resistance was constant but, with the expected random errors, it would still not be definitive. By plotting a graph and drawing the line of best fit the pattern of results is far easier to spot, whether it is linear or some other relationship.

### Error bars

In plotting a point on a graph, uncertainties are recognized by adding error bars. These are vertical and horizontal lines that indicate the possible range of the quantity being measured. Suppose at a time of  $(0.2 \pm 0.05)$  s the speed of an object was  $(1.2 \pm 0.2)$  m s<sup>-1</sup> this would be plotted as shown in figure 7.

This means that the value could possibly be within the rectangle that touches the ends of the error bars as shown in figure 8. This is the **zone of uncertainty** for the data point. A line of best fit should be one that spreads the points so that they are evenly distributed on both sides of the line and also passes through the error bars.



▲ Figure 8 Zone of uncertainty.

### Uncertainties with gradients

Using a computer application, such as a spreadsheet, can allow you to plot a graph with data points and error bars. You can then read off the gradient and the intercepts from a linear graph directly. The application



will automatically draw the best trend line. You can then add the trend lines with the steepest and shallowest gradients that are just possible – while still passing through all the error bars. Students quite commonly, but incorrectly, use the extremes of the error bars that are furthest apart on the graphs. Although these could be appropriate, it is essential that all the trend lines you draw pass through all of the error bars.

In an experiment to measure the electromotive force (emf) and internal resistance of a cell, a series of resistors are connected across the cell. The currents in and potential differences across the resistors are then measured. A graph of potential difference,  $V$ , against current,  $I$ , should give a straight line of negative gradient. As you will see in Topic 5 the emf of a cell is related to the internal resistance  $r$  by the equation:

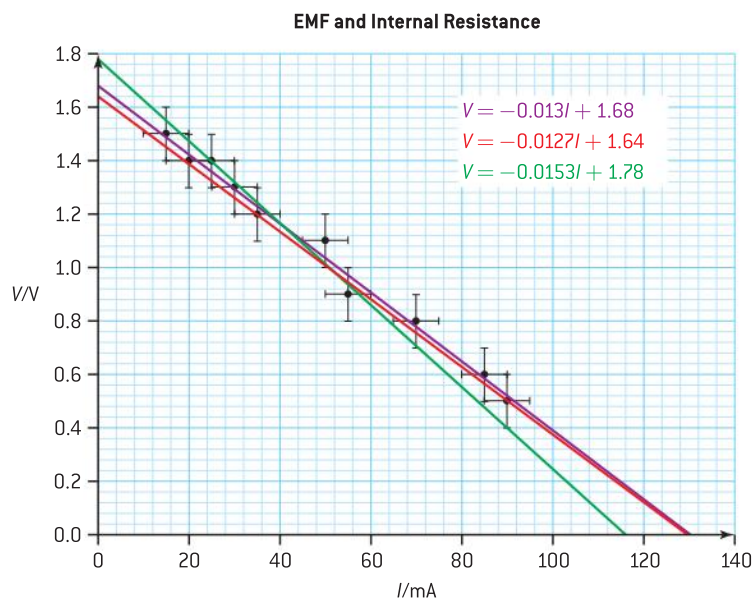
$$\varepsilon = I(R + r) = V + Ir$$

This can be rearranged to give  $V = \varepsilon - Ir$

So a graph of  $V$  against  $I$  is of gradient  $-r$  (the internal resistance) and intercept  $\varepsilon$  (the emf of the cell).

The table on the right shows a set of results from this experiment. With a milliammeter and voltmeter of low precision the repeat values are identical to the measurements given in the table.

The graph of figure 9 shows the line of best fit together with two lines that are just possible.



| $I \pm 5/\text{mA}$ | $V \pm 0.1/\text{V}$ |
|---------------------|----------------------|
| 15                  | 1.5                  |
| 20                  | 1.4                  |
| 25                  | 1.4                  |
| 30                  | 1.3                  |
| 35                  | 1.2                  |
| 50                  | 1.1                  |
| 55                  | 0.9                  |
| 70                  | 0.8                  |
| 85                  | 0.6                  |
| 90                  | 0.5                  |

▲ Figure 9 A graph of potential difference,  $V$ , against current,  $I$ , for a cell.

Converting from milliamps to amps, the equations of these lines suggest that the internal resistance (the gradient) is  $13.0 \, \Omega$  and the range is from  $12.7 \, \Omega$  to  $15.3 \, \Omega$  ( $= 2.6 \, \Omega$ ) meaning that half the range  $= 1.3 \, \Omega$ .

This leads to a value for  $r = (13.0 \pm 1.3) \, \Omega$ .

The intercept on the  $V$  axis of the line of best fit  $= 1.68$  which rounds to  $1.7 \, \text{V}$  (since the data is essentially to 2 significant figures). The range of the just possible lines gives  $1.6$  to  $1.8 \, \text{V}$  (when rounded to two significant figures). This means that  $\varepsilon = (1.6 \pm 0.1) \, \text{V}$ .

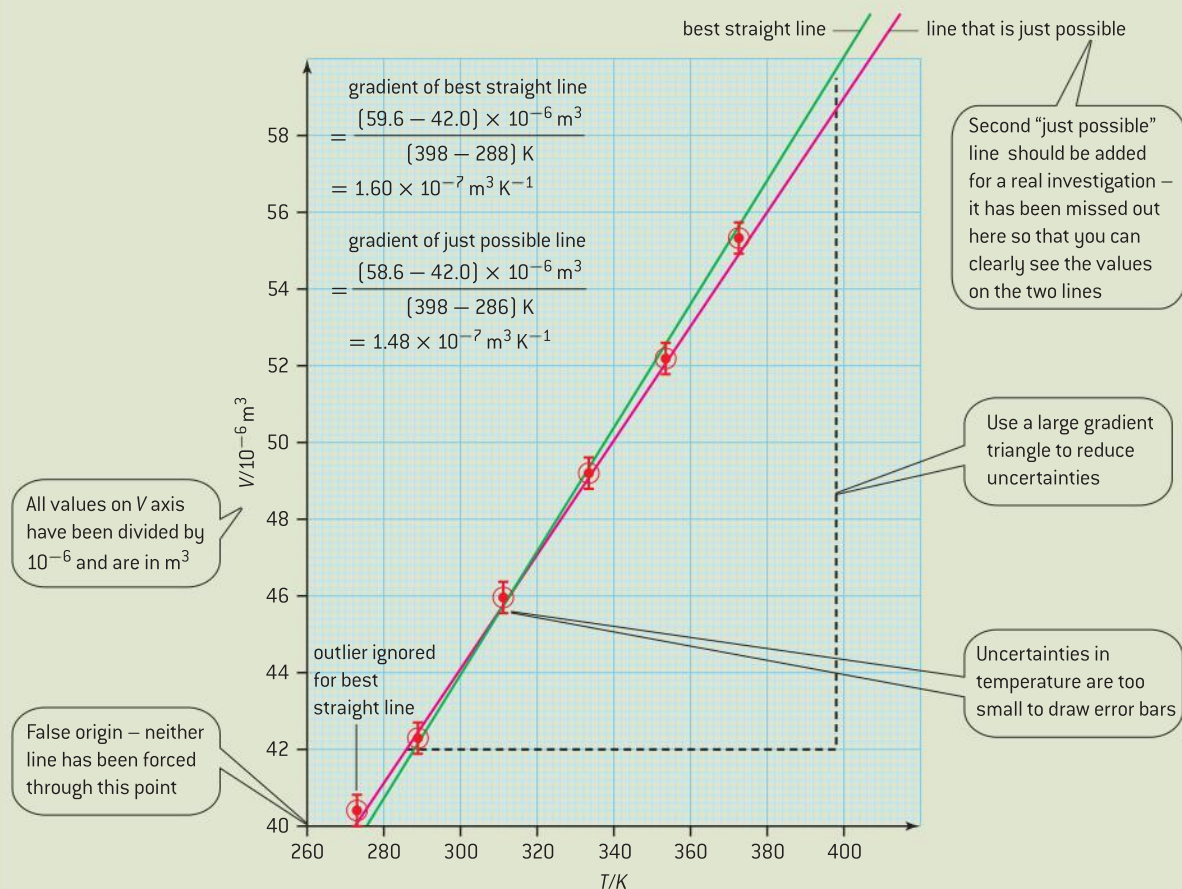


## Nature of science

### Drawing graphs manually

- One of the skills expected of physicists is to draw graphs by hand and you may well be tested on this in the data analysis question in Paper 3 of the IB Diploma Programme physics examination. You are also likely to need to draw graphs for your internal assessment.
- Try to look at your extreme values so that you have an idea of what scales to use. You will need a minimum of six points to give you a reasonable chance of drawing a valid line.
- Use scales that will allow you to spread your points out as much as possible (you should fill your page, but not overspill onto a second sheet as that would damage your line quality and lose you marks). You can always calculate an intercept if you need one; when you don't include the origin, your axes give you a **false origin** (which is fine).
- Use sensible scales that will make both plotting and your calculations clear-cut (avoid scales that are multiples of 3, 4, or 7 – stick to 2, 5, and 10).
- Try to plot your graph as you are doing the experiment – if apparently unusual values crop up, you will see them and can check that they are correct.
- Before you draw your line of best fit, you need to consider whether or not it is straight or a curve. There may well be anomalous points (outliers) that you can ignore, but if there is a definite trend to the curve then you should opt for a smooth curve drawn with a single line and not “sketched” artistically!

Figure 10 shows some of the key elements of a good hand-drawn graph. Calculating the gradients **on** the graph is very useful when checking values.



▲ Figure 10 Hand-drawn graph.





## Linearizing graphs

Many relationships between physical quantities are not directly proportional and a straight line cannot be obtained simply by plotting one quantity against the other. There are two approaches to dealing with non-proportional relationships: when we know the form of the relationship and when we do not.

If we do know the form of the relationship such as  $p \propto \frac{1}{V}$  (for a gas held at constant temperature) or  $T \propto \sqrt{l}$  (for a simple pendulum) we can plot a graph of one quantity against the power of the other quantity to obtain a straight-line origin graph. An alternative for the simple pendulum is to plot a graph of  $T^2$  against  $l$  which will give the same result.

**You should think about the propagation of errors when you consider the relative merits of plotting  $T$  against  $\sqrt{l}$  or  $T^2$  against  $l$ .**

*The following discussion applies to HL examinations, but offers such a useful technique that SL students may wish to utilize it when completing IAs or if they undertake an Extended Essay in a science subject.*

If we don't know the actual power involved in a relationship, but we suspect that one quantity is related to the other, we can write a general relationship in the form  $y = kx^n$  where  $k$  and  $n$  are constants.

By taking logs of this equation we obtain  $\log y = \log k + n \log x$  which we can arrange into  $\log y = n \log x + \log k$  and is of the form  $y = mx + c$ . This means that a graph of  $\log y$  against  $\log x$  will be linear of gradient  $n$  and have an intercept on the  $\log y$  axis of  $\log k$ .

This technique is very useful in carrying out investigations when a relationship between two quantities really is not known. The technique is also a useful way of dealing with exponential relationships by taking logs to base e, instead of base 10. For example, radioactive nuclides decay so that either the activity or the number of nuclei remaining falls according to the same general form. Writing the decay equation for the number of nuclei remaining gives:

$N = N_0 e^{-\lambda t}$  by taking logs to the base e we get  $\ln N = \ln N_0 - \lambda t$  (where  $\ln N$  is the usual way of writing  $\log_e N$ ).

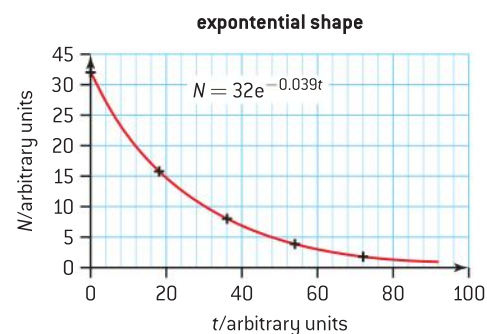
By plotting a graph of  $\ln N$  against  $t$  the gradient will be  $-\lambda$  and the intercept on the  $\ln N$  axis will be  $\ln N_0$ . This linearizes the graph shown in figure 11 producing the graph of figure 12. A linear graph is easier to analyse than a curve.

Capacitors also discharge through resistors using the same general mathematical relationship as that used for radioactive decay.

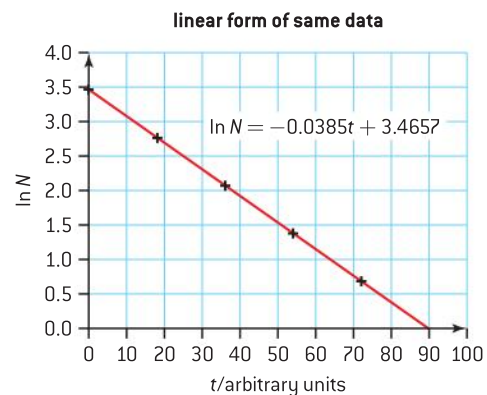
### Note

Relationships can never be “indirectly proportional” – this is a meaningless term since it is too vague. Consequently the term “proportional” means the same as “directly proportional”.

This topic is dealt with in more detail and with many further examples on the website.



▲ Figure 11



▲ Figure 12

## 1.3 Vectors and scalars

### Understanding

- Vector and scalar quantities
- Combination and resolution of vectors



### Applications and skills

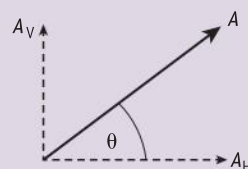
- Solving vector problems graphically and algebraically.

### Equations

The horizontal and vertical components of vector  $A$ :

$$\rightarrow A_H = A \cos \theta$$

$$\rightarrow A_V = A \sin \theta$$



### Nature of science

All physical quantities that you will meet on the course are classified as being vectors or scalars. It is important to know whether any quantity is a vector or a scalar since this will affect how the quantity is treated mathematically. Although the concept of adding forces is an intuitive application of vectors that has probably been used by sailors for millennia, the analytical aspect of it is a recent development. In the

*Philosophiæ Naturalis Principia Mathematica*, published in 1687, Newton used quantities which we now call vectors, but never generalized this to deal with the concepts of vectors. At the start of the 19th century vectors became an indispensable tool for representing three-dimensional space and complex numbers. Vectors are now used as a matter of course by physicists and mathematicians alike.

### Vector and scalar quantities

**Scalar** quantities are those that have magnitude (or *size*) but no direction. We treat scalar quantities as numbers (albeit with units) and use the rules of algebra when dealing with them. Distance and time are both scalars, as is speed. The average speed is simply the distance divided by the time, so if you travel 80 m in 10 s the speed will always be  $8 \text{ ms}^{-1}$ . There are no surprises.

**Vector** quantities are those which have both magnitude and direction. We must use vector algebra when dealing with vectors since we must take into account direction. The vector equivalent of distance is called displacement (i.e., it is a distance in a specified direction). The vector equivalent of speed is velocity (i.e., it is the speed in a specified direction). Time, as we have seen, is a scalar. Average velocity is defined as being displacement divided by time.



Dividing a vector by a scalar is the easiest operation that we need to do involving a vector. To continue with the example that we looked at with scalars, suppose the displacement was 80 m due north and the time was, again, 10 s. The average velocity would be  $8 \text{ ms}^{-1}$  due north. So, to generalize, when we divide a vector by a scalar we end up with a new vector that has the direction of the original one, but which will be of magnitude equal to that of the vector divided by that of the scalar.

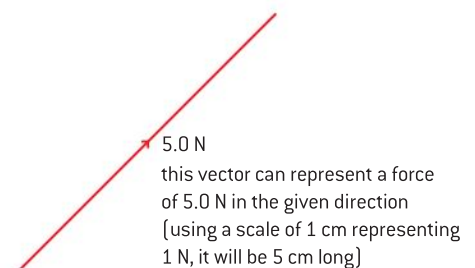
| Commonly used vectors and scalars    |                                     |                                                                                                                                                                                                                                                           |
|--------------------------------------|-------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Vectors                              | Scalars                             | Comments                                                                                                                                                                                                                                                  |
| force [ $F$ ]                        | mass [ $m$ ]                        | $\vec{F}$                                                                                                                                                                                                                                                 |
| displacement [ $s$ ]                 | length/distance [ $s$ , $d$ , etc.] | displacement used to be called “space” – now that means something else!                                                                                                                                                                                   |
| velocity [ $v$ or $u$ ]              | time [ $t$ ]                        |                                                                                                                                                                                                                                                           |
| momentum [ $p$ ]                     | volume [ $V$ ]                      |                                                                                                                                                                                                                                                           |
| acceleration [ $a$ ]                 | temperature [ $T$ ]                 |                                                                                                                                                                                                                                                           |
| gravitational field strength [ $g$ ] | speed [ $v$ or $u$ ]                | velocity and speed often have the same symbol                                                                                                                                                                                                             |
| electric field strength [ $E$ ]      | density [ $\rho$ ]                  | the symbol for density is the Greek “rho” not the letter “p”                                                                                                                                                                                              |
| magnetic field strength [ $B$ ]      | pressure [ $p$ ]                    |                                                                                                                                                                                                                                                           |
| area [ $A$ ]                         | energy/work [ $W$ , etc.]           | the direction of an area is taken as being at right angles to the surface                                                                                                                                                                                 |
|                                      | power [ $P$ ]                       |                                                                                                                                                                                                                                                           |
|                                      | current [ $I$ ]                     | with current having direction you might think that it should be a vector but it is not (it is the ratio of two scalars, charge and time, so it cannot be a vector). In more advanced work you might come across <i>current density</i> which is a vector. |
|                                      | resistance [ $R$ ]                  |                                                                                                                                                                                                                                                           |
|                                      | gravitational potential [ $V_g$ ]   | the subscripts tell us whether it is gravitational or electrical                                                                                                                                                                                          |
|                                      | electric potential [ $V_e$ ]        |                                                                                                                                                                                                                                                           |
|                                      | magnetic flux [ $\Phi$ ]            | flux is often thought as having a direction – it doesn't!                                                                                                                                                                                                 |

## Representing vector quantities

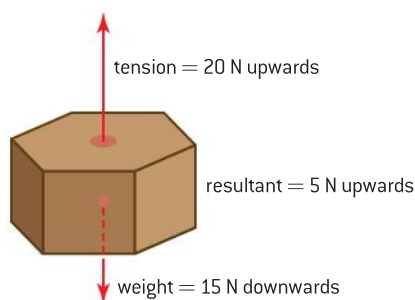
A vector quantity is represented by a line with an arrow.

- The direction the arrow points represents the direction of the vector.
- The length of the line represents the magnitude of the vector to a chosen scale.

When we are dealing with vectors that act in one dimension it is a simple matter to assign one direction as being positive and the opposite direction as being negative. Which direction is positive and which negative really doesn't matter as long as you are consistent. So, if one force acts upwards on an object and another force acts downwards, it is a simple matter to find the resultant by subtracting one from the other.



▲ Figure 1 Representing a vector.

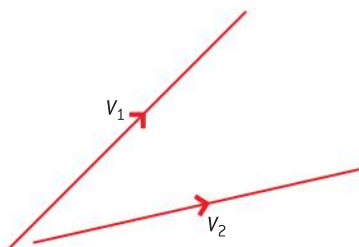


▲ Figure 2 Two vectors acting on an object.

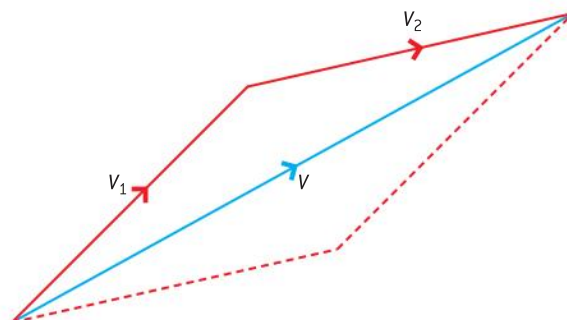
Figure 2 shows an upward tension and downward weight acting on an object – the upward line is longer than the downward line since the object is not in equilibrium and has an upward resultant.

## Adding and subtracting vectors

When adding and subtracting vectors, account has to be taken of their direction. This can be done either by a scale drawing (graphically) or algebraically.



▲ Figure 3 Two vectors to be added.



▲ Figure 4 Adding the vectors.

## Scale drawing (graphical) approach

Adding two vectors  $V_1$  and  $V_2$  which are not in the same direction can be done by forming a parallelogram to scale.

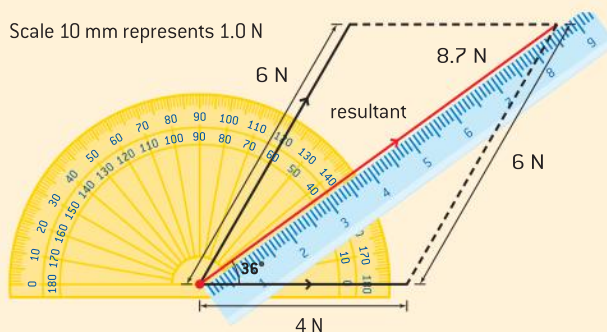
- Make a rough sketch of how the vectors are going to add together to give you an idea of how large your scale needs to be in order to fill the space available to you. This is a good idea when you are adding the vectors mathematically too.
- Having chosen a suitable scale, draw the scaled lines in the direction of  $V_1$  and  $V_2$  (so that they form two adjacent sides of the parallelogram).
- Complete the parallelogram by drawing in the remaining two sides.
- The blue diagonal represents the **resultant vector** in both magnitude and direction.

### Worked example

Two forces of magnitude 4.0 N and 6.0 N act on a single point. The forces make an angle of  $60^\circ$  with each other. Using a scale diagram, determine the resultant force.

### Solution

Don't forget that the vector must have a magnitude and a direction; this means that the angle is just as important as the size of the force.



length of resultant = 87 mm so the force = 8.7 N  
angle resultant makes with 4 N force =  $36^\circ$



## Algebraic approach

Vectors can act at any angle to each other but the most common situation that you are going to deal with is when they are at right angles to each other. We will deal with this first.

### Adding vector quantities at right angles

Pythagoras' theorem can be used to calculate a resultant vector when two perpendicular vectors are added (or subtracted). Assuming that the two vector quantities are horizontal and vertical *but the principle is the same as long as they are perpendicular*.

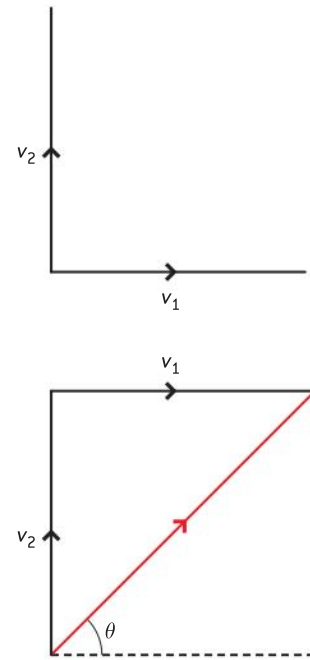
Figure 5 shows two perpendicular velocities  $v_1$  and  $v_2$ ; they form a parallelogram that is a rectangle.

The magnitude of the resultant velocity  $= \sqrt{v_1^2 + v_2^2}$

- The resultant velocity makes an angle  $\theta$  to the horizontal given by

$$\tan \theta = \left( \frac{v_1}{v_2} \right) \text{ so that } \theta = \tan^{-1} \left( \frac{v_1}{v_2} \right)$$

- Notice that the order of adding the two vectors makes no difference to the length or the direction of the resultant.



▲ Figure 5 Adding two perpendicular vectors.

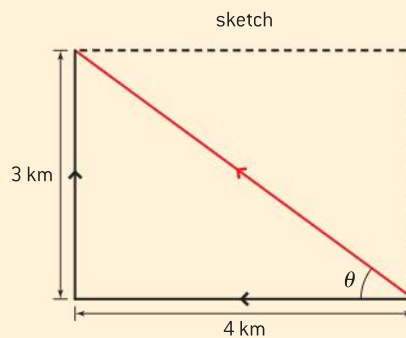
### Worked example

A walker walks 4.0 km due west from his starting point. He then stops before walking 3.0 km due north. At the end of his journey, how far is the walker from his starting point?

#### Solution

$$\text{resultant} = \sqrt{4^2 + 3^2} = 5 \text{ km}$$

$$\text{angle } \theta = \tan^{-1} \left( \frac{3}{4} \right) = 36.9^\circ$$

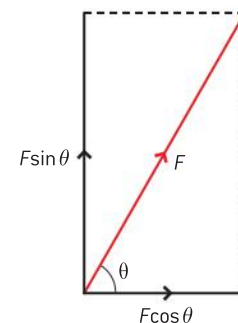


Before we look at adding vectors that are not perpendicular, we need to see how to resolve a vector – i.e. split it into two components.

### Resolving vectors

We have seen that adding two vectors together produces a **resultant vector**. It is sensible, therefore, to imagine that we could split the resultant into the two vectors from which it was formed. In fact this is true for any vector – it can be divided into components which, added together, make the resultant vector. There is no limit to the number of vectors that can be added together and, consequently, there is no limit to the number of components that a vector can be divided into. However, we most commonly divide a vector into two components that are perpendicular to one another. The reason for doing this is that perpendicular vectors have no effect on each other as we will see when we look at projectiles in Topic 2.

The force  $F$  in figure 6 has been resolved into the horizontal component equal to  $F \cos \theta$  and a vertical component equal to  $F \sin \theta$ . (The component opposite to the angle used is always the sine component.)



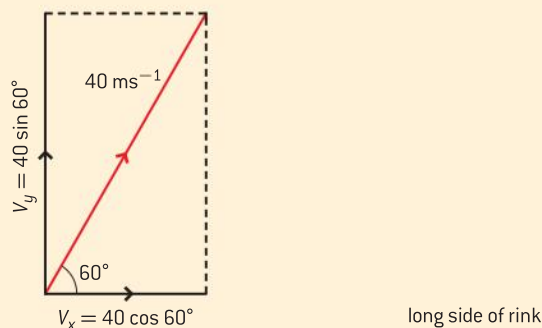
▲ Figure 6 Resolving a force.



### Worked example

An ice-hockey puck is struck at a constant speed of  $40 \text{ m s}^{-1}$  at an angle of  $60^\circ$  to the longer side of an ice rink. How far will the puck have travelled in directions **a)** parallel and **b)** perpendicular to the long side after  $0.5 \text{ s}$ ?

### Solution



**a)** Resolving parallel to longer side:

$$v_x = v \cos 60^\circ$$

$$v_x = 40 \cos 60^\circ = 20 \text{ m s}^{-1}$$

$$\text{distance travelled (x)} = v_x t = 20 \times 0.5 = 10 \text{ m}$$

**b)** Resolving parallel to shorter side:

$$v_y = v \sin 60^\circ$$

$$v_y = 40 \cos 60^\circ = 34.6 \text{ m s}^{-1}$$

$$\text{distance travelled (y)} = v_y t = 34.6 \times 0.5 = 17 \text{ m}$$

Just to demonstrate that resolving is the reverse of adding the components we can use Pythagoras' theorem to add together our two components giving:

$$\text{total speed} = \sqrt{20^2 + 34.6^2} = 39.96 \text{ m s}^{-1}$$

as the value for  $v_y$  was rounded this gives the expected  $40 \text{ m s}^{-1}$

### Adding vector quantities that are not at right angles

You are now in a position to add any vectors.

- Resolve each of the vectors in two directions at right angles – this will often be horizontally and vertically, but may be parallel and perpendicular to a surface.
- Add all the components in one direction to give a single component.
- Add all the components in the perpendicular direction to give a second single component.
- Combine the two components using Pythagoras' theorem, as for two vector quantities at right angles.

$V_1$  and  $V_2$  are the vectors to be added.

Each vector is resolved into components in the  $x$  and  $y$  directions. Note that since the  $x$  component of  $V_2$  is to the left it is treated as being negative (the  $y$  component of each vector is in the same direction... so upwards is treated as positive).

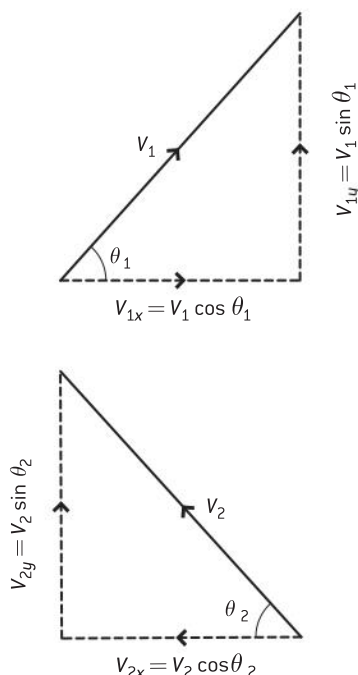
$$\text{Total } x \text{ component } V_x = V_{1x} + V_{2x} = V_1 \cos \theta_1 - V_2 \cos \theta_2$$

$$\text{Total } y \text{ component } V_y = V_{1y} + V_{2y} = V_1 \sin \theta_1 + V_2 \sin \theta_2$$

Having calculated  $V_x$  and  $V_y$  we can find the resultant by using

$$\text{Pythagoras so } V = \sqrt{V_x^2 + V_y^2}$$

$$\text{and the angle } \theta \text{ made with the horizontal} = \tan^{-1} \left( \frac{V_y}{V_x} \right).$$

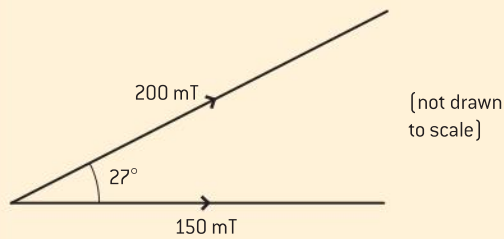


▲ Figure 7 Finding the resultant of two vectors that are not perpendicular.



### Worked example

Magnetic fields have strength 200 mT and 150 mT respectively. The fields act at  $27^\circ$  to one another as shown in the diagram.



Calculate the resultant magnetic field strength.

### Solution

The 150 mT field is horizontal and so has no vertical component.

Vertical component of the 200 mT field =  $200 \sin 27^\circ = 90.8 \text{ mT}$

This makes the total vertical component of the resultant field.

Horizontal component of the 200 mT field =  $200 \cos 27^\circ = 178.2 \text{ mT}$

Total horizontal component of resultant field =  $(150.0 + 178.2) \text{ mT} = 328.2 \text{ mT}$

Resultant field strength  $\sqrt{90.8^2 + 328.2^2} = 340 \text{ mT}$

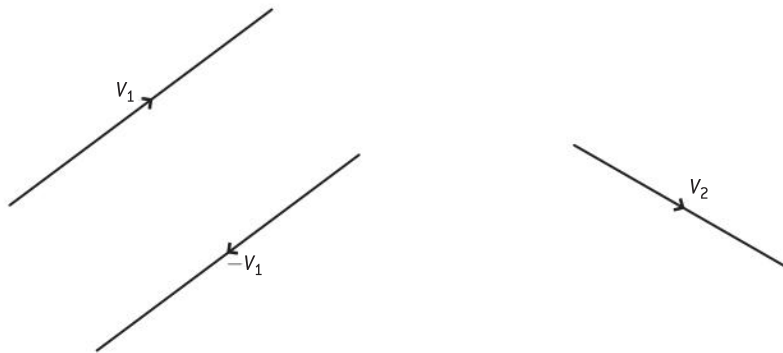
The resultant field makes an angle of:

$\tan^{-1}\left(\frac{90.8}{328.2}\right) = 15^\circ$  with the 150 mT field.

### Subtraction of vectors

Subtracting one vector from another is very simple – you just form the negative of the vector to be subtracted and add this to the other vector. The negative of a vector has the same magnitude but the opposite direction. Let's look at an example to see how this works:

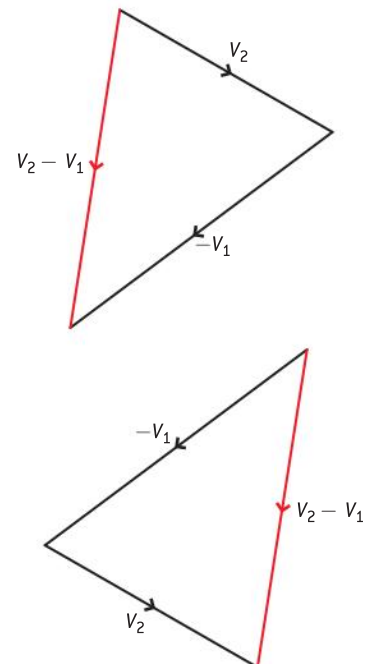
Suppose we wish to find the difference between two velocities  $v_1$  and  $v_2$  shown in figure 8.



▲ Figure 8 Positive and negative vectors.

In finding the difference between two values we subtract the first value from the second; so we need  $-v_1$ . We then add  $-v_1$  to  $v_2$  as shown in figure 9 to give the red resultant.

The order of combining the two vectors doesn't matter as can be seen from the two versions in figure 9. In each case the resultant is the same – it doesn't matter where the resultant is positioned as long as it has the same length and direction it is the same vector.



▲ Figure 9 Subtracting vectors.

## Questions

- 1 Express the following units in terms of the SI fundamental units.

- a) newton (N)
- b) watt (W)
- c) pascal (Pa)
- d) coulomb (C)
- e) volt (V)

(5 marks)

- 2 Express the following numbers to three significant figures.

- a) 257.52
- b) 0.002 347
- c) 0.1783
- d) 7873
- e) 1.997

(5 marks)

- 3 Complete the following calculations and express your answers to the most appropriate number of significant figures.

- a)  $1.34 \times 3.2$
- b)  $\frac{1.34 \times 10^2}{2.1 \times 10^3}$
- c)  $1.87 \times 10^2 + 1.97 \times 10^3$
- d)  $(1.97 \times 10^5) \times (1.0 \times 10^4)$
- e)  $(9.47 \times 10^{-2}) \times (4.0 \times 10^3)$

(5 marks)

- 4 Use the appropriate metric multiplier instead of a power of ten in the following.

- a)  $1.1 \times 10^4 \text{ V}$
- b)  $4.22 \times 10^{-4} \text{ m}$
- c)  $8.5 \times 10^{10} \text{ W}$
- d)  $4.22 \times 10^{-7} \text{ m}$
- e)  $3.5 \times 10^{-13} \text{ C}$

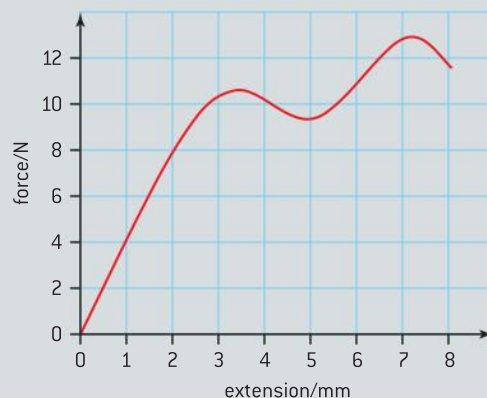
(5 marks)

- 5 Write down the order of magnitude of the following (you may need to do some research).

- a) the length of a human foot
- b) the mass of a fly
- c) the charge on a proton
- d) the age of the universe
- e) the speed of electromagnetic waves in a vacuum

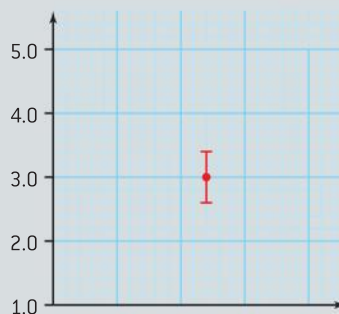
(5 marks)

- 6 a) Without using a calculator estimate to one significant figure the value of  $\frac{2\pi \times 4.9}{480}$ .
- b) When a wire is stretched, the area under the line of a graph of force against extension of the wire gives the elastic potential energy stored in the wire. Estimate the energy stored in the wire with the following characteristic:



(4 marks)

- 7 The grid below shows one data point and its associated error bar on a graph. The  $x$ -axis is not shown. State the  $y$ -value of the data point together with its absolute and percentage uncertainty.



(3 marks)



- 8 A ball falls freely from rest with an acceleration  $g$ . The variation with time  $t$  of its displacement  $s$  is given by  $s = \frac{1}{2}gt^2$ . The percentage uncertainty in the value of  $t$  is  $\pm 3\%$  and that in the value of  $g$  is  $\pm 2\%$ . Calculate the percentage uncertainty in the value of  $s$ .

(2 marks)

- 9 The volume  $V$  of a cylinder of height  $h$  and radius  $r$  is given by the expression  $V = \pi r^2 h$ . In a particular experiment,  $r$  is to be determined from measurements of  $V$  and  $h$ . The percentage uncertainty in  $V$  is  $\pm 5\%$  and that in  $h$  is  $\pm 2\%$ . Calculate the percentage uncertainty in  $r$ .

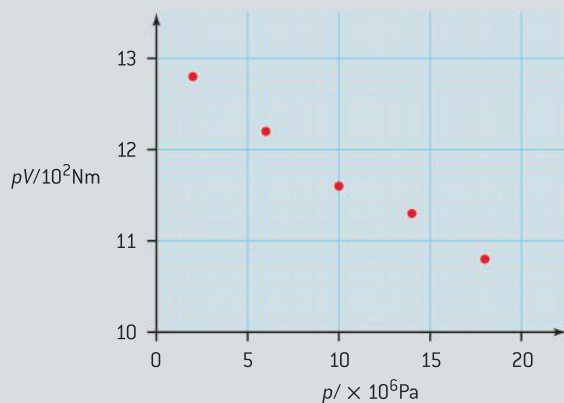
(3 marks)

10 (IB)

At high pressures, a real gas does not behave as an ideal gas. For a certain range of pressures, it is suggested that for one mole of a real gas at constant temperature the relation between the pressure  $p$  and volume  $V$  is given by the equation

$$pV = A + Bp \quad \text{where } A \text{ and } B \text{ are constants.}$$

In an experiment, 1 mole of nitrogen gas was compressed at a constant temperature of 150 K. The volume  $V$  of the gas was measured for different values of the pressure  $p$ . A graph of the product  $pV$  against  $p$  is shown in the diagram below.



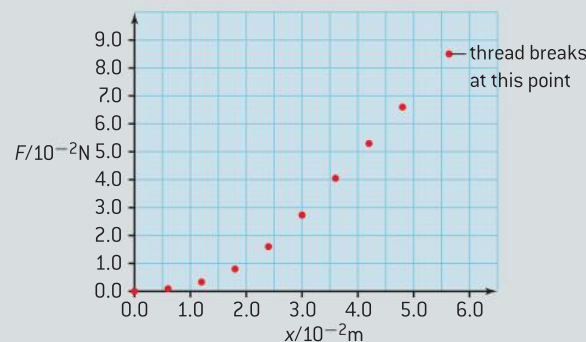
- a) Copy the graph and draw a line of best fit for the data points.
- b) Use your graph to determine the values of the constants  $A$  and  $B$  in the equation  $pV = A + Bp$

- c)  $p$  was measured to an accuracy of 5% and  $V$  was measured to an accuracy of 2%. Determine the absolute error in the value of the constant  $A$ .

(6 marks)

11 (IB)

An experiment was carried out to measure the extension  $x$  of a thread of a spider's web when a load  $F$  is applied to it.



- a) Copy the graph and draw a best-fit line for the data points.
- b) The relationship between  $F$  and  $x$  is of the form

$$F = kx^n$$

State and explain the graph you would plot in order to determine the value  $n$ .

- c) When a load is applied to a material, it is said to be *under stress*. The magnitude  $p$  of the stress is given by

$$p = \frac{F}{A}$$

where  $A$  is the cross-sectional area of the sample of the material.

Use the graph and the data below to deduce that the thread used in the experiment has a greater breaking stress than steel.

Breaking stress of steel =  $1.0 \times 10^9 \text{ N m}^{-2}$

Radius of spider web thread =  $4.5 \times 10^{-6} \text{ m}$

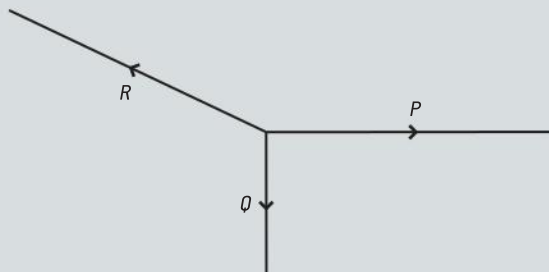
- d) The uncertainty in the measurement of the radius of the thread is  $\pm 0.1 \times 10^{-6} \text{ m}$ . Determine the percentage uncertainty in the value of the area of the thread.

(9 marks)

- 12** A cyclist travels a distance of 1200 m due north before going 2000 m due east followed by 500 m south-west. Draw a scale diagram to calculate the cyclist's final displacement from her initial position.

(4 marks)

- 13** The diagram shows three forces  $P$ ,  $Q$ , and  $R$  in equilibrium.  $P$  acts horizontally and  $Q$  vertically.



When  $P = 5.0 \text{ N}$  and  $Q = 3.0 \text{ N}$ , calculate the magnitude and direction of  $R$ .

(3 marks)

- 14** A boat, starting on one bank of a river, heads due south with a speed of  $1.5 \text{ m s}^{-1}$ . The river flows due east at  $0.8 \text{ m s}^{-1}$ .

- a) Calculate the resultant velocity of the boat relative to the bank of the river.  
b) The river is 50 m wide. Calculate the displacement from its initial position when the boat reaches the opposite bank.

(7 marks)

- 15** A car of mass 850 kg rests on a slope at  $25^\circ$  to the horizontal. Calculate the magnitude of the component of the car's weight which acts parallel to the slope.

(3 marks)





## Solutions for Topic 1 – Measurements and uncertainties

1. a)  $\text{kg ms}^{-2}$  d)  $\text{A s}$   
 b)  $\text{kg m}^2\text{s}^{-3}$  e)  $\text{kg m}^2\text{s}^{-1}\text{A}^{-1}$   
 c)  $\text{kg m}^{-1}\text{s}^{-2}$
2. a) 258 d) 7870  
 b) 0.00235 e) 2.00  
 c) 0.178
3. a) 4.3 d)  $2.0 \times 10^9$   
 b)  $6.4 \times 10^{-2}$  e)  $3.8 \times 10^2$   
 c)  $2.16 \times 10^3$
4. a) 11 kV d)  $0.422 \mu\text{m}$  or  $422 \text{ nm}$   
 b)  $0.422 \text{ mm}$  or  $422 \mu\text{m}$  e)  $0.35 \text{ pC}$  or  $350 \text{ fC}$   
 c) 85 GW
5. a)  $10^{-1} \text{ m}$  (a few 10s of cm)  
 b)  $10^{-4} - 10^{-2} \text{ kg}$  (flies come in many different shapes and sizes!)  
 c)  $10^{-19} \text{ C}$  ( $1.6 \times 10^{-19} \text{ C}$ )  
 d)  $10^{10} \text{ year}$  (13.7 billion years)  
 e)  $10^8 \text{ ms}^{-1}$  ( $3.0 \times 10^8 \text{ ms}^{-1}$ )
6. a)  $\frac{2 \times 3 \times 5}{500} = \frac{6}{100} = 0.06$   
 b) between 70 and 74 mJ
7.  $3.0 \pm 0.4$ ;  $\pm 13.3\%$  (or rounding up  $\pm 14\%$ )
8.  $\frac{\Delta s}{s} = \frac{\Delta g}{g} + 2 \frac{\Delta t}{t} = 0.02 + (2 \times 0.03) = 0.08 = \pm 8\%$
9.  $\frac{\Delta r}{r} = \frac{1}{2} \left( \frac{\Delta h}{h} + \frac{\Delta V}{V} \right) = 0.5 \times 0.02 \times 0.05 = 0.035 = \pm 4\%$
10. a) check line of best fit reasonable  
 b)  $A = y\text{-intercept}$ ;  $B = \text{gradient}$   
 c) use difference between steepest and shallowest gradients
11. a) check line of best fit reasonable  
 b)  $\log F = n \log x + \log k$ ; plot  $\log F$  against  $\log x$  and find gradient of graph which is  $n$   
 c)  $p = \frac{8.5 \times 10^{-2}}{\pi \times 4.5 \times 10^{-6}} = 1.3 \times 10^9 \text{ Nm}^{-2}$   
 d)  $\frac{\Delta A}{A} = 2 \frac{\Delta r}{r} = 2 \times \frac{0.1 \times 10^{-6}}{4.5 \times 10^{-6}} = 0.04 = \pm 4\%$
12.  $(1850 \pm 50) \text{ m}$  at angle of N  $(60 \pm 5)^\circ$  E from scale starting point
13. magnitude  $= \sqrt{5^2 + 3^2} = 5.8 \text{ N}$   
 $\tan \theta = \frac{5}{3}$ ;  $\theta = 31^\circ$  to horizontal
14. a) magnitude of velocity  $= \sqrt{1.5^2 + 0.8^2} = 1.7 \text{ m s}^{-1}$   
 $\tan \theta = \frac{0.8}{1.5}$ ;  $\theta = 28^\circ$  with original direction  
 b)  $s \cos 28 = 50$ ; displacement  $s = 57 \text{ m}$
15. component parallel to slope  $= mg \sin \theta = 850 \times 9.8 \sin 25 = 3500 \text{ N}$

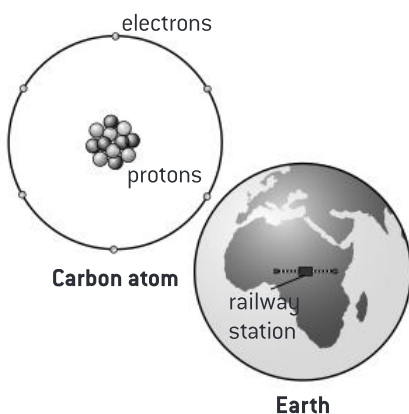
## The realm of physics – range of magnitudes of quantities in our universe

### ORDERS OF MAGNITUDE – INCLUDING THEIR RATIOS

Physics seeks to explain nothing less than the Universe itself. In attempting to do this, the range of the magnitudes of various quantities will be huge.

If the numbers involved are going to mean anything, it is important to get some feel for their relative sizes. To avoid 'getting lost' among the numbers it is helpful to state them to the nearest **order of magnitude** or power of ten. The numbers are just rounded up or down as appropriate.

Comparisons can then be easily made because working out the ratio between two powers of ten is just a matter of adding or subtracting whole numbers. The diameter of an atom,  $10^{-10}$  m, does not sound that much larger than the diameter of a proton in its nucleus,  $10^{-15}$  m, but the ratio between them is  $10^5$  or 100,000 times bigger. This is the same ratio as between the size of a railway station (order of magnitude  $10^2$  m) and the diameter of the Earth (order of magnitude  $10^7$  m).



For example, you would probably feel very pleased with yourself if you designed a new, environmentally friendly source of energy that could produce  $2.03 \times 10^3$  J from 0.72 kg of natural produce. But the meaning of these numbers is not clear – is this a lot or is it a little? In terms of orders of magnitudes, this new source produces  $10^3$  joules per kilogram of produce. This does not compare terribly well with the  $10^5$  joules provided by a slice of bread or the  $10^8$  joules released per kilogram of petrol.

You do NOT need to memorize all of the values shown in the tables, but you should try and develop a familiarity with them.

### RANGE OF MASSES

| Mass / kg                                   |
|---------------------------------------------|
| $10^{52}$ total mass of observable Universe |
| $10^{48}$                                   |
| $10^{44}$                                   |
| $10^{40}$ mass of local galaxy (Milky Way)  |
| $10^{36}$                                   |
| $10^{32}$                                   |
| $10^{28}$ mass of Sun                       |
| $10^{24}$ mass of Earth                     |
| $10^{20}$ total mass of oceans              |
| $10^{16}$ total mass of atmosphere          |
| $10^{12}$                                   |
| $10^8$ laden oil supertanker                |
| $10^4$ elephant                             |
| $10^0$ human                                |
| $10^{-4}$ mouse                             |
| $10^{-8}$ grain of sand                     |
| $10^{-12}$ blood corpuscle                  |
| $10^{-16}$ bacterium                        |
| $10^{-20}$                                  |
| $10^{-24}$ haemoglobin molecule             |
| $10^{-28}$ proton                           |
| $10^{-32}$ electron                         |

### RANGE OF LENGTHS

| Size / m                                            |
|-----------------------------------------------------|
| $10^{26}$ radius of observable Universe             |
| $10^{24}$                                           |
| $10^{22}$ radius of local galaxy (Milky Way)        |
| $10^{20}$                                           |
| $10^{18}$ distance to nearest star                  |
| $10^{16}$                                           |
| $10^{14}$                                           |
| $10^{12}$                                           |
| $10^{10}$ distance from Earth to Sun                |
| $10^8$ distance from Earth to Moon                  |
| $10^6$ radius of the Earth                          |
| $10^4$ deepest part of the ocean / highest mountain |
| $10^2$ tallest building                             |
| $10^0$                                              |
| $10^{-2}$ length of fingernail                      |
| $10^{-4}$ thickness of piece of paper               |
| $10^{-6}$ human blood corpuscle                     |
| $10^{-8}$ wavelength of light                       |
| $10^{-10}$                                          |
| $10^{-12}$ diameter of hydrogen atom                |
| $10^{-14}$ wavelength of gamma ray                  |
| $10^{-16}$ diameter of proton                       |

### RANGE OF TIMES

| Time / s                                       |
|------------------------------------------------|
| $10^{20}$ age of the Universe                  |
| $10^{18}$                                      |
| $10^{16}$ age of the Earth                     |
| $10^{14}$ age of species – <i>Homo sapiens</i> |
| $10^{12}$                                      |
| $10^{10}$ typical human lifespan               |
| $10^8$ 1 year                                  |
| $10^6$ 1 day                                   |
| $10^4$                                         |
| $10^2$                                         |
| $10^0$ heartbeat                               |
| $10^{-2}$ period of high-frequency sound       |
| $10^{-4}$                                      |
| $10^{-6}$ passage of light across a room       |
| $10^{-8}$                                      |
| $10^{-10}$                                     |
| $10^{-12}$ vibration of an ion in a solid      |
| $10^{-14}$ period of visible light             |
| $10^{-16}$                                     |
| $10^{-18}$ passage of light across an atom     |
| $10^{-20}$                                     |
| $10^{-22}$ passage of light across a nucleus   |
| $10^{-24}$                                     |

### RANGE OF ENERGIES

| Energy / J                                                              |
|-------------------------------------------------------------------------|
| $10^{44}$ energy released in a supernova                                |
| $10^{34}$                                                               |
| $10^{30}$ energy radiated by Sun in 1 second                            |
| $10^{26}$                                                               |
| $10^{22}$ energy released in an earthquake                              |
| $10^{18}$                                                               |
| $10^{14}$ energy released by annihilation of 1 kg of matter             |
| $10^{10}$ energy in a lightning discharge                               |
| $10^6$ energy needed to charge a car battery                            |
| $10^2$ kinetic energy of a tennis ball during game                      |
| $10^{-2}$ energy in the beat of a fly's wing                            |
| $10^{-6}$                                                               |
| $10^{-10}$                                                              |
| $10^{-14}$                                                              |
| $10^{-18}$ energy needed to remove electron from the surface of a metal |
| $10^{-22}$                                                              |
| $10^{-26}$                                                              |

# The SI system of fundamental and derived units

## FUNDAMENTAL UNITS

Any measurement and every quantity can be thought of as being made up of two important parts:

1. the number and
2. the units.

Without **both** parts, the measurement does not make sense. For example a person's age might be quoted as 'seventeen' but without the 'years' the situation is not clear. Are they 17 minutes, 17 months or 17 years old? In this case you would know if you saw them, but a statement like

$$\text{length} = 4.2$$

actually says nothing. Having said this, it is really surprising to see the number of candidates who forget to include the units in their answers to examination questions.

In order for the units to be understood, they need to be defined. There are many possible systems of measurement that have

been developed. In science we use the International System of units (SI). In SI, the **fundamental** or **base** units are as follows

| Quantity            | SI unit  | SI symbol |
|---------------------|----------|-----------|
| Mass                | kilogram | kg        |
| Length              | metre    | m         |
| Time                | second   | s         |
| Electric current    | ampere   | A         |
| Amount of substance | mole     | mol       |
| Temperature         | kelvin   | K         |
| (Luminous intensity | candela  | cd)       |

You do not need to know the precise definitions of any of these units in order to use them properly.

## DERIVED UNITS

Having fixed the fundamental units, all other measurements can be expressed as different combinations of the fundamental units. In other words, all the other units are **derived units**. For example, the fundamental list of units does not contain a unit for the measurement of speed. The definition of speed can be used to work out the derived unit.

$$\text{Since speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Units of speed} = \frac{\text{units of distance}}{\text{units of time}}$$

$$= \frac{\text{metres}}{\text{seconds}} \quad (\text{pronounced 'metres per second'})$$

$$= \frac{\text{m}}{\text{s}}$$

$$= \text{m s}^{-1}$$

Of the many ways of writing this unit, the last way ( $\text{m s}^{-1}$ ) is the best.

Sometimes particular combinations of fundamental units are so common that they are given a new derived name. For example, the unit of force is a derived unit – it turns out to be  $\text{kg m s}^{-2}$ . This unit is given a new name the newton (N) so that  $1\text{N} = 1 \text{ kg m s}^{-2}$ .

The great thing about SI is that, so long as the numbers that are substituted into an equation are in SI units, then the answer will also come out in SI units. You can always 'play safe' by converting all the numbers into proper SI units. Sometimes, however, this would be a waste of time.

There are some situations where the use of SI becomes awkward. In astronomy, for example, the distances involved

are so large that the SI unit (the metre) always involves large orders of magnitudes. In these cases, the use of a different (but non SI) unit is very common. Astronomers can use the astronomical unit (AU), the light-year (ly) or the parsec (pc) as appropriate. Whatever the unit, the conversion to SI units is simple arithmetic.

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$1 \text{ ly} = 9.5 \times 10^{15} \text{ m}$$

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$$

There are also some units (for example the hour) which are so common that they are often used even though they do not form part of SI. Once again, before these numbers are substituted into equations they need to be converted. Some common unit conversions are given on page 3 of the IB data booklet.

The table below lists the SI derived units that you will meet.

| SI derived unit  | SI base unit                                  | Alternative SI unit |
|------------------|-----------------------------------------------|---------------------|
| newton (N)       | $\text{kg m s}^{-2}$                          | —                   |
| pascal (Pa)      | $\text{kg m}^{-1} \text{ s}^{-2}$             | $\text{N m}^{-2}$   |
| hertz (Hz)       | $\text{s}^{-1}$                               | —                   |
| joule (J)        | $\text{kg m}^2 \text{ s}^{-2}$                | $\text{N m}$        |
| watt (W)         | $\text{kg m}^2 \text{ s}^{-3}$                | $\text{J s}^{-1}$   |
| coulomb (C)      | $\text{A s}$                                  | —                   |
| volt (V)         | $\text{kg m}^2 \text{ s}^{-3} \text{ A}^{-1}$ | $\text{W A}^{-1}$   |
| ohm ( $\Omega$ ) | $\text{kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$ | $\text{V A}^{-1}$   |
| weber (Wb)       | $\text{kg m}^2 \text{ s}^{-2} \text{ A}^{-1}$ | $\text{V s}$        |
| tesla (T)        | $\text{kg s}^{-2} \text{ A}^{-1}$             | $\text{Wb m}^{-2}$  |
| becquerel (Bq)   | $\text{s}^{-1}$                               | —                   |

## PREFIXES

To avoid the repeated use of scientific notation, an alternative is to use one of the list of agreed prefixes given on page 2 in the IB data booklet. These can be very useful but they can also lead to errors in calculations. It is very easy to forget to include the conversion factor.

For example,  $1 \text{ kW} = 1000 \text{ W}$ .  $1 \text{ mW} = 10^{-3} \text{ W}$  (in other words,  $\frac{1\text{W}}{1000}$ )

# Estimation

## ORDERS OF MAGNITUDE

It is important to develop a 'feeling' for some of the numbers that you use. When using a calculator, it is very easy to make a simple mistake (eg by entering the data incorrectly). A good way of checking the answer is to first make an estimate before resorting to the calculator. The multiple-choice paper (paper 1) does not allow the use of calculators.

Approximate values for each of the fundamental SI units are given below.

|       |                                                                                                                                             |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------|
| 1 kg  | A packet of sugar, 1 litre of water. A person would be about 50 kg or more                                                                  |
| 1 m   | Distance between one's hands with arms outstretched                                                                                         |
| 1 s   | Duration of a heart beat (when resting – it can easily double with exercise)                                                                |
| 1 amp | Current flowing from the mains electricity when a computer is connected. The maximum current to a domestic device would be about 10 A or so |

|          |                                                                                                          |
|----------|----------------------------------------------------------------------------------------------------------|
| 1 kelvin | 1K is a very low temperature. Water freezes at 273 K and boils at 373 K. Room temperature is about 300 K |
| 1 mol    | 12 g of carbon-12. About the number of atoms of carbon in the 'lead' of a pencil                         |

The same process can happen with some of the derived units.

|                     |                                                                                               |
|---------------------|-----------------------------------------------------------------------------------------------|
| 1 m s <sup>-1</sup> | Walking speed. A car moving at 30 m s <sup>-1</sup> would be fast                             |
| 1 m s <sup>-2</sup> | Quite a slow acceleration. The acceleration of gravity is 10 m s <sup>-2</sup>                |
| 1 N                 | A small force – about the weight of an apple                                                  |
| 1 V                 | Batteries generally range from a few volts up to 20 or so, the mains is several hundred volts |
| 1 Pa                | A very small pressure. Atmospheric pressure is about 10 <sup>5</sup> Pa                       |
| 1 J                 | A very small amount of energy – the work done lifting an apple off the ground                 |

## POSSIBLE REASONABLE ASSUMPTIONS

Everyday situations are very complex. In physics we often simplify a problem by making simple assumptions. Even if we know these assumptions are not absolutely true they allow us to gain an understanding of what is going on. At the end of the calculation it is often possible to go back and work out what would happen if our assumption turned out not to be true.

The table below lists some common assumptions. Be careful not to assume too much! Additionally we often have to assume that some quantity is constant even if we know that in reality it is varying slightly all the time.

| Assumption                                    | Example                                                     |
|-----------------------------------------------|-------------------------------------------------------------|
| Object treated as point particle              | Mechanics: Linear motion and translational equilibrium      |
| Friction is negligible                        | Many mechanics situations – but you need to be very careful |
| No thermal energy ("heat") loss               | Almost all thermal situations                               |
| Mass of connecting string, etc. is negligible | Many mechanics situations                                   |
| Resistance of ammeter is zero                 | Circuits                                                    |
| Resistance of voltmeter is infinite           | Circuits                                                    |
| Internal resistance of battery is zero        | Circuits                                                    |
| Material obeys Ohm's law                      | Circuits                                                    |
| Machine 100% efficient                        | Many situations                                             |
| Gas is ideal                                  | Thermodynamics                                              |
| Collision is elastic                          | Only gas molecules have perfectly elastic collisions        |
| Object radiates as a perfect black body       | Thermal equilibrium, e.g. planets                           |

## SCIENTIFIC NOTATION

Numbers that are too big or too small for decimals are often written in **scientific notation**:

$$a \times 10^b$$

where  $a$  is a number between 1 and 10 and  $b$  is an integer.

e.g.  $153.2 = 1.532 \times 10^2$ ;  $0.00872 = 8.72 \times 10^{-3}$

## SIGNIFICANT FIGURES

Any experimental measurement should be quoted with its uncertainty. This indicates the possible range of values for the quantity being measured. At the same time, the number of **significant figures** used will act as a guide to the amount of uncertainty. For example, a measurement of mass which is quoted as 23.456 g implies an uncertainty of  $\pm 0.001$  g (it has five significant figures), whereas one of 23.5 g implies an uncertainty of  $\pm 0.1$  g (it has three significant figures).

A simple rule for calculations (multiplication or division) is to quote the answer to the same number of significant digits as the LEAST precise value that is used.

For a more complete analysis of how to deal with uncertainties in calculated results, see page 5.

# Uncertainties and error in experimental measurement

## ERRORS – RANDOM AND SYSTEMATIC (PRECISION AND ACCURACY)

An experimental error just means that there is a difference between the recorded value and the 'perfect' or 'correct' value. Errors can be categorized as **random** or **systematic**.

Repeating readings does not reduce systematic errors.

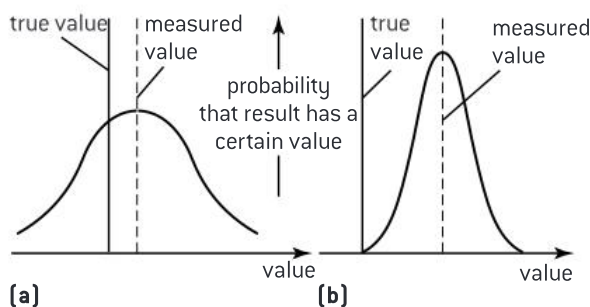
Sources of random errors include

- The readability of the instrument.
- The observer being less than perfect.
- The effects of a change in the surroundings.

Sources of systematic errors include

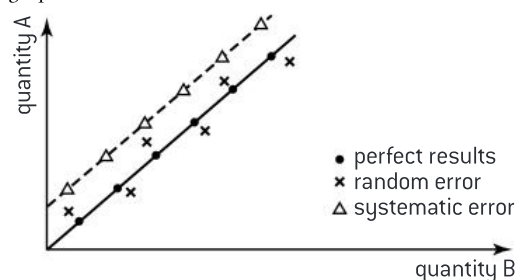
- An instrument with **zero error**. To correct for zero error the value should be subtracted from every reading.
- An instrument being wrongly **calibrated**.
- The observer being less than perfect in the same way every measurement.

An **accurate** experiment is one that has a small systematic error, whereas a **precise** experiment is one that has a small random error.



Two examples illustrating the nature of experimental results:  
(a) an accurate experiment of low precision  
(b) a less accurate but more precise experiment.

Systematic and random errors can often be recognized from a graph of the results.



Perfect results, random and systematic errors of two proportional quantities.

## ESTIMATING THE UNCERTAINTY RANGE

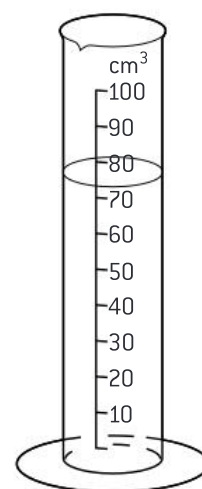
An **uncertainty range** applies to any experimental value. The idea is that, instead of just giving one value that implies perfection, we give the likely range for the measurement.

### 1. Estimating from first principles

All measurement involves a readability error. If we use a measuring cylinder to find the volume of a liquid, we might think that the best estimate is 73 cm<sup>3</sup>, but we know that it is not exactly this value (73.000000000000 cm<sup>3</sup>).

Uncertainty range is  $\pm 5$  cm<sup>3</sup>. We say volume =  $73 \pm 5$  cm<sup>3</sup>.

Normally the uncertainty range due to readability is estimated as below.



| Device         | Example                             | Uncertainty                              |
|----------------|-------------------------------------|------------------------------------------|
| Analogue scale | Rulers, meters with moving pointers | $\pm$ (half the smallest scale division) |
| Digital scale  | Top-pan balances, digital meters    | $\pm$ (the smallest scale division)      |

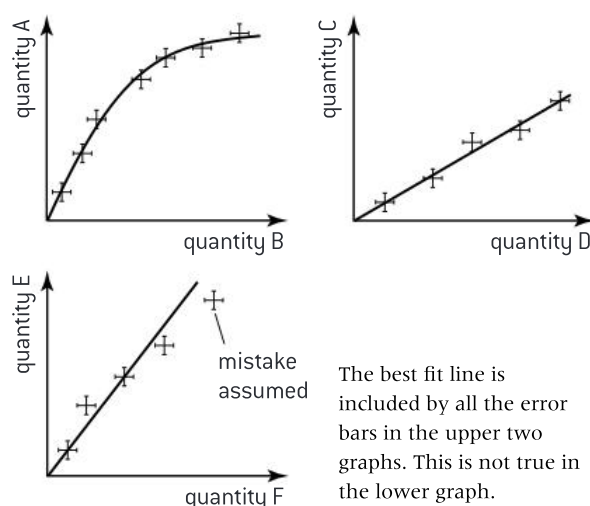
### 2. Estimating uncertainty range from several repeated measurements

If the time taken for a trolley to go down a slope is measured five times, the readings in seconds might be 2.01, 1.82, 1.97, 2.16 and 1.94. The average of these five readings is 1.98 s. The deviation of the largest and smallest readings can be calculated ( $2.16 - 1.98 = 0.18$ ;  $1.98 - 1.82 = 0.16$ ). The largest value is taken as the uncertainty range. In this example the time is  $1.98 \pm 0.18$  s. It would also be appropriate to quote this as  $2.0 \pm 0.2$  s.

## GRAPHICAL REPRESENTATION OF UNCERTAINTY

In many situations the best method of presenting and analysing data is to use a graph. If this is the case, a neat way of representing the uncertainties is to use **error bars**. The graphs below explain their use.

Since the error bar represents the uncertainty range, the 'best-fit' line of the graph should pass through ALL of the rectangles created by the error bars.



The best fit line is included by all the error bars in the upper two graphs. This is not true in the lower graph.

## SIGNIFICANT FIGURES IN UNCERTAINTIES

In order to be cautious when quoting uncertainties, final values from calculations are often rounded up to one significant figure, e.g. a calculation that finds the value of a force to be 4.264 N with an uncertainty of  $\pm 0.362$  N is quoted as  $4.3 \pm 0.4$  N. This can be unnecessarily pessimistic and it is also acceptable to express uncertainties to two significant figures. For example, the charge on an electron is  $1.602176565 \times 10^{-19}$  C  $\pm 0.000000035 \times 10^{-19}$  C. In data booklets this is sometimes expressed as  $1.602176565(35) \times 10^{-19}$  C.



# Uncertainties in calculated results

## MATHEMATICAL REPRESENTATION OF UNCERTAINTIES

For example if the mass of a block was measured as  $10 \pm 1$  g and the volume was measured as  $5.0 \pm 0.2$  cm<sup>3</sup>, then the full calculations for the density would be as follows.

$$\text{Best value for density} = \frac{\text{mass}}{\text{volume}} = \frac{10}{5} = 2.0 \text{ g cm}^{-3}$$

$$\text{The largest possible value of density} = \frac{11}{4.8} = 2.292 \text{ g cm}^{-3}$$

$$\text{The smallest possible value of density} = \frac{9}{5.2} = 1.731 \text{ g cm}^{-3}$$

$$\text{Rounding these values gives density} = 2.0 \pm 0.3 \text{ g cm}^{-3}$$

We can express this uncertainty in one of three ways – using **absolute**, **fractional** or **percentage uncertainties**.

If a quantity  $p$  is measured then the absolute uncertainty would be expressed as  $\pm \Delta p$ .

Then the fractional uncertainty is

$$\frac{\pm \Delta p}{p},$$

which makes the percentage uncertainty

$$\frac{\pm \Delta p}{p} \times 100\%.$$

In the example above, the fractional uncertainty of the density is  $\pm 0.15$  or  $\pm 15\%$ .

Thus equivalent ways of expressing this error are

$$\text{density} = 2.0 \pm 0.3 \text{ g cm}^{-3}$$

$$\text{OR density} = 2.0 \text{ g cm}^{-3} \pm 15\%$$

Working out the uncertainty range is very time consuming. There are some mathematical ‘short-cuts’ that can be used. These are introduced in the boxes below.

## MULTIPLICATION, DIVISION OR POWERS

Whenever two or more quantities are multiplied or divided and they each have uncertainties, the overall uncertainty is approximately equal to the **addition** of the **percentage** (fractional) uncertainties.

Using the same numbers from above,

$$\Delta m = \pm 1 \text{ g}$$

$$\frac{\Delta m}{m} = \pm \left( \frac{1 \text{ g}}{10 \text{ g}} \right) = \pm 0.1 = \pm 10\%$$

$$\Delta V = \pm 0.2 \text{ cm}^3$$

$$\frac{\Delta V}{V} = \pm \left( \frac{0.2 \text{ cm}^3}{5 \text{ cm}^3} \right) = \pm 0.04 = \pm 4\%$$

$$\begin{aligned} \text{The total \% uncertainty in the result} &= \pm (10 + 4)\% \\ &= \pm 14\% \end{aligned}$$

$$14\% \text{ of } 2.0 \text{ g cm}^{-3} = 0.28 \text{ g cm}^{-3} \approx 0.3 \text{ g cm}^{-3}$$

So density =  $2.0 \pm 0.3 \text{ g cm}^{-3}$  as before.

In symbols, if  $y = \frac{ab}{c}$

$$\text{Then } \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} \quad [\text{note this is ALWAYS added}]$$

Power relationships are just a special case of this law.

$$\text{If } y = a^n$$

$$\text{Then } \frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right| \quad (\text{always positive})$$

For example if a cube is measured to be  $4.0 \pm 0.1$  cm in length along each side, then

$$\% \text{ Uncertainty in length} = \pm \frac{0.1}{4.0} = \pm 2.5\%$$

$$\text{Volume} = (\text{length})^3 = (4.0)^3 = 64 \text{ cm}^3$$

$$\begin{aligned} \% \text{ Uncertainty in [volume]} &= \% \text{ uncertainty in } [(\text{length})^3] \\ &= 3 \times (\% \text{ uncertainty in [length]}) \\ &= 3 \times (\pm 2.5\%) \\ &= \pm 7.5\% \end{aligned}$$

$$\begin{aligned} \text{Absolute uncertainty} &= 7.5\% \text{ of } 64 \text{ cm}^3 \\ &= 4.8 \text{ cm}^3 \approx 5 \text{ cm}^3 \end{aligned}$$

$$\text{Thus volume of cube} = 64 \pm 5 \text{ cm}^3$$

## OTHER MATHEMATICAL OPERATIONS

If the calculation involves mathematical operations other than multiplication, division or raising to a power, then one has to find the highest and lowest possible values.

### Addition or subtraction

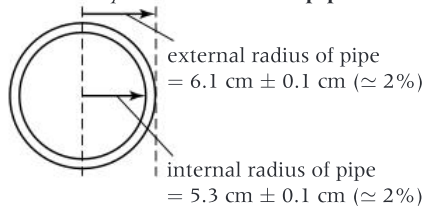
Whenever two or more quantities are added or subtracted and they each have uncertainties, the overall uncertainty is equal to the **addition** of the **absolute** uncertainties.

In symbols

$$\text{If } y = a \pm b$$

$$\Delta y = \Delta a + \Delta b \quad (\text{note ALWAYS added})$$

### uncertainty of thickness in a pipe wall



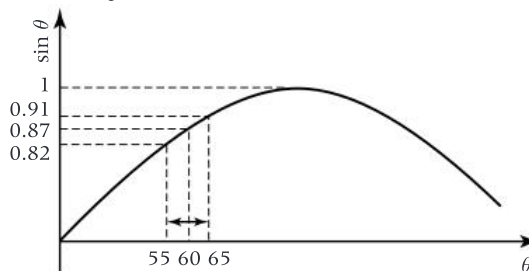
$$\begin{aligned} \text{thickness of pipe wall} &= 6.1 - 5.3 \text{ cm} \\ &= 0.8 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{uncertainty in thickness} &= \pm (0.1 + 0.1) \text{ cm} \\ &= 0.2 \text{ cm} \\ &= \pm 25\% \end{aligned}$$

## Other functions

There are no ‘short-cuts’ possible. Find the highest and lowest values.

e.g. uncertainty of  $\sin \theta$  if  $\theta = 60^\circ \pm 5^\circ$



$$\text{if } \theta = 60^\circ \pm 5^\circ$$

$$\text{best value to } \sin \theta = 0.87$$

$$\text{max. } \sin \theta = 0.91$$

$$\text{min. } \sin \theta = 0.82$$

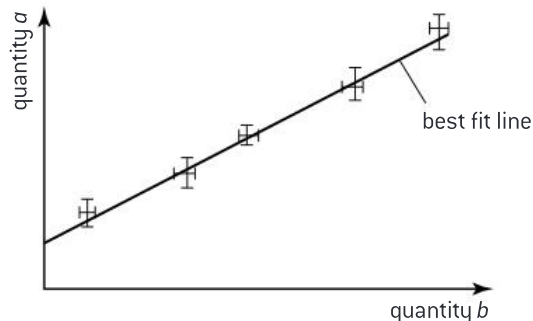
$$\therefore \sin \theta = 0.87 \pm 0.05$$

worst value used

# Uncertainties in graphs

## ERROR BARS

Plotting a graph allows one to visualize all the readings at one time. Ideally all of the points should be plotted with their error bars. In principle, the size of the error bar could well be different for every single point and so they should be individually worked out.



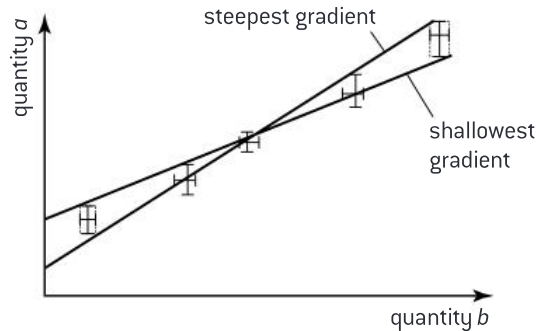
A full analysis in order to determine the uncertainties in the gradient of a best straight-line graph should **always make use of the error bars for all of the data points.**

In practice, it would often take too much time to add all the correct error bars, so some (or all) of the following short-cuts could be considered.

- Rather than working out error bars for each point – use the worst value and assume that all of the other error bars are the same.
- Only plot the error bar for the ‘worst’ point, i.e. the point that is furthest from the line of best fit. If the line of best fit is within the limits of this error bar, then it will probably be within the limits of all the error bars.
- Only plot the error bars for the first and the last points. These are often the most important points when considering the uncertainty ranges calculated for the gradient or the intercept (see right).
- Only include the error bars for the axis that has the worst uncertainty.

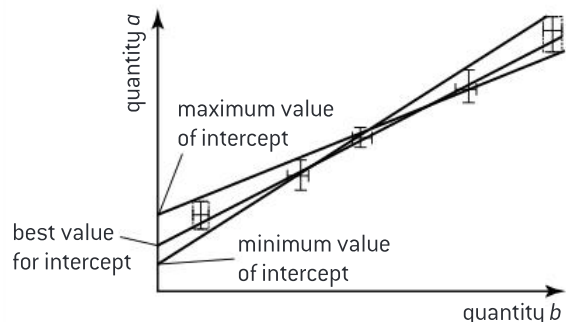
## UNCERTAINTY IN SLOPES

If the gradient of the graph has been used to calculate a quantity, then the uncertainties of the points will give rise to an uncertainty in the gradient. Using the steepest and the shallowest lines possible (i.e. the lines that are still consistent with the error bars) the uncertainty range for the gradient is obtained. This process is represented below.



## UNCERTAINTY IN INTERCEPTS

If the intercept of the graph has been used to calculate a quantity, then the uncertainties of the points will give rise to an uncertainty in the intercept. Using the steepest and the shallowest lines possible (i.e. the lines that are still consistent with the error bars) we can obtain the uncertainty in the result. This process is represented below.



# Vectors and scalars

## DIFFERENCE BETWEEN VECTORS AND SCALARS

If you measure any quantity, it must have a number AND a unit. Together they express the **magnitude** of the quantity. Some quantities also have a direction associated with them. A quantity that has magnitude and direction is called a **vector** quantity whereas one that has only magnitude is called a **scalar** quantity. For example, all forces are vectors.

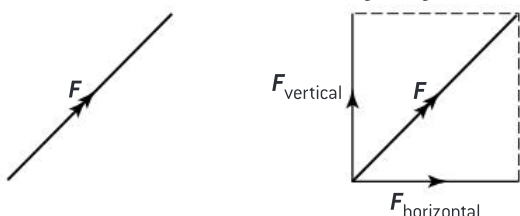
The table lists some common quantities. The first two quantities in the table are linked to one another by their definitions (see page 9). All the others are in no particular order.

| Vectors                      | Scalars                           |
|------------------------------|-----------------------------------|
| Displacement                 | Distance                          |
| Velocity                     | Speed                             |
| Acceleration                 | Mass                              |
| Force                        | Energy (all forms)                |
| Momentum                     | Temperature                       |
| Electric field strength      | Potential or potential difference |
| Magnetic field strength      | Density                           |
| Gravitational field strength | Area                              |

Although the vectors used in many of the given examples are forces, the techniques can be applied to all vectors.

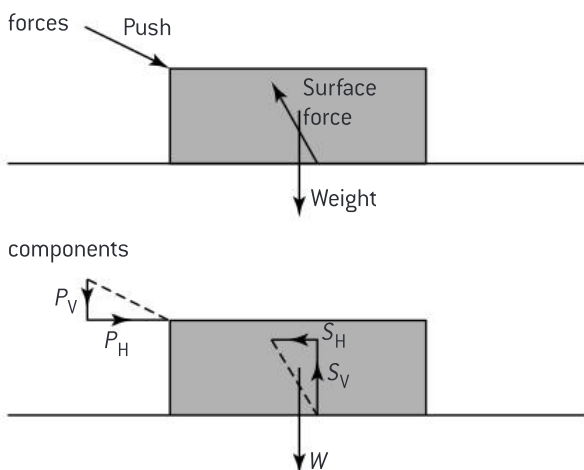
## COMPONENTS OF VECTORS

It is also possible to 'split' one vector into two (or more) vectors. This process is called **resolving** and the vectors that we get are called the **components** of the original vector. This can be a very useful way of analysing a situation if we choose to resolve all the vectors into two directions that are at right angles to one another.



Splitting a vector into components

These 'mutually perpendicular' directions are totally independent of each other and can be analysed separately. If appropriate, both directions can then be combined at the end to work out the final resultant vector.



Pushing a block along a rough surface

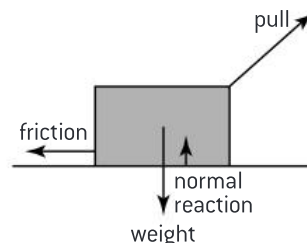
## REPRESENTING VECTORS

In most books a bold letter is used to represent a vector whereas a normal letter represents a scalar. For example ***F*** would be used to represent a force in magnitude AND direction. The list below shows some other recognized methods.

$\vec{F}$ ,  $\overline{F}$  or  $\underline{F}$

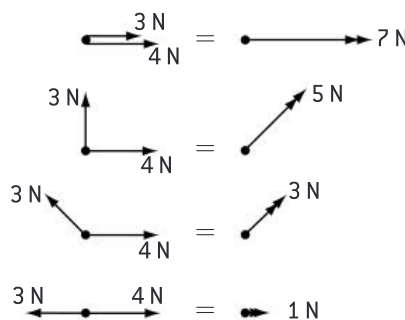
Vectors are best shown in diagrams using arrows:

- the relative magnitudes of the vectors involved are shown by the relative length of the arrows
- the direction of the vectors is shown by the direction of the arrows.



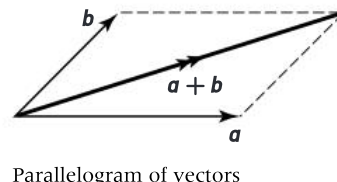
## ADDITION / SUBTRACTION OF VECTORS

If we have a 3 N and a 4 N force, the overall force (resultant force) can be anything between 1 N and 7 N depending on the directions involved.



The way to take the directions into account is to do a scale diagram and use the parallelogram law of vectors.

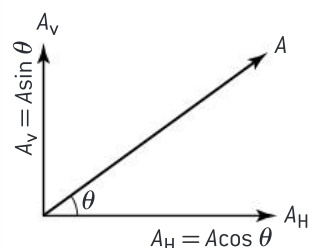
This process is the same as adding vectors in turn – the 'tail' of one vector is drawn starting from the head of the previous vector.



Parallelogram of vectors

## TRIGONOMETRY

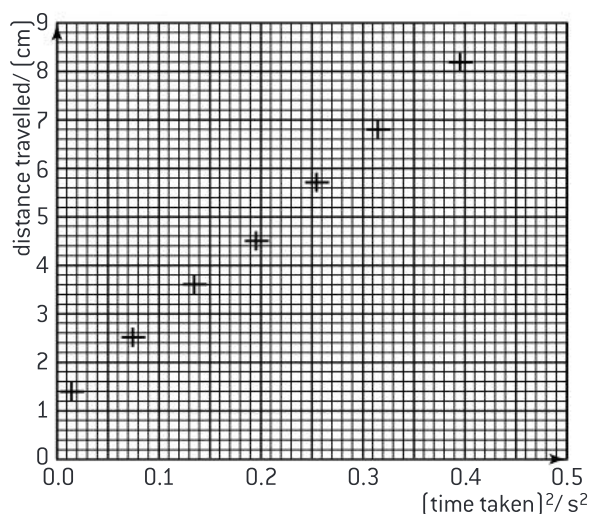
Vector problems can always be solved using scale diagrams, but this can be very time consuming. The mathematics of trigonometry often makes it much easier to use the mathematical functions of sine or cosine. This is particularly appropriate when resolving. The diagram below shows how to calculate the values of either of these components.



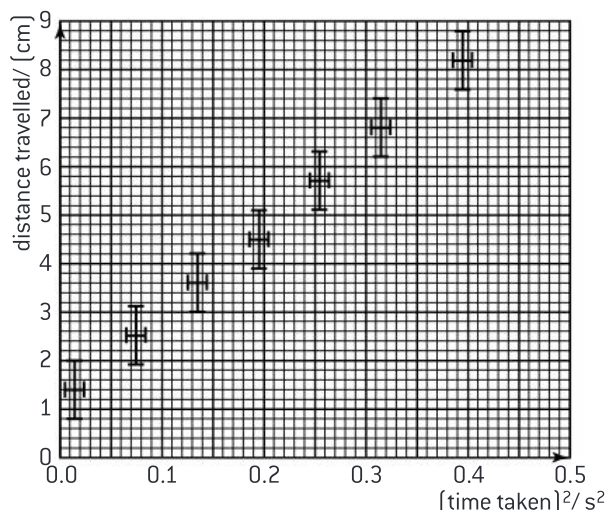
See page 14 for an example.

# IB Questions – measurement and uncertainties

1. An object is rolled from rest down an inclined plane. The distance travelled by the object was measured at seven different times. A graph was then constructed of the distance travelled against the (time taken)<sup>2</sup> as shown below.

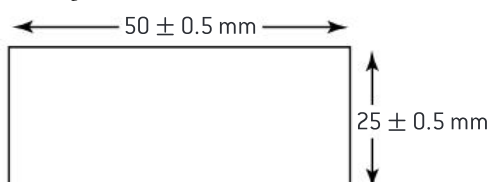


- a) (i) What quantity is given by the gradient of such a graph? [2]  
 (ii) Explain why the graph suggests that the collected data is valid but includes a systematic error. [2]  
 (iii) Do these results suggest that distance is proportional to (time taken)<sup>2</sup>? Explain your answer. [2]  
 (iv) Making allowance for the systematic error, calculate the acceleration of the object. [2]
- b) The following graph shows that same data after the uncertainty ranges have been calculated and drawn as error bars.



Add two lines to show the range of the possible acceptable values for the gradient of the graph. [2]

2. The lengths of the sides of a rectangular plate are measured, and the diagram shows the measured values with their uncertainties.



Which one of the following would be the best estimate of the percentage uncertainty in the calculated area of the plate?

- A.  $\pm 0.02\%$                       C.  $\pm 3\%$   
 B.  $\pm 1\%$                               D.  $\pm 5\%$

3. A stone is dropped down a well and hits the water 2.0 s after it is released. Using the equation  $d = \frac{1}{2}gt^2$  and taking  $g = 9.81 \text{ m s}^{-2}$ , a calculator yields a value for the depth  $d$  of the well as 19.62 m. If the time is measured to  $\pm 0.1 \text{ s}$  then the best estimate of the absolute error in  $d$  is

- A.  $\pm 0.1 \text{ m}$                               C.  $\pm 1.0 \text{ m}$   
 B.  $\pm 0.2 \text{ m}$                               D.  $\pm 2.0 \text{ m}$

4. In order to determine the density of a certain type of wood, the following measurements were made on a **cube** of the wood.

Mass = 493 g

Length of **each** side = 9.3 cm

The percentage uncertainty in the measurement of mass is  $\pm 0.5\%$  and the percentage uncertainty in the measurement of length is  $\pm 1.0\%$ .

The best estimate for the uncertainty in the density is

- A.  $\pm 0.5\%$                               C.  $\pm 3.0\%$   
 B.  $\pm 1.5\%$                               D.  $\pm 3.5\%$

5. Astronauts wish to determine the gravitational acceleration on Planet X by dropping stones from an overhanging cliff. Using a steel tape measure they measure the height of the cliff as  $s = 7.64 \text{ m} \pm 0.01 \text{ m}$ . They then drop three similar stones from the cliff, timing each fall using a hand-held electronic stopwatch which displays readings to one-hundredth of a second. The recorded times for three drops are 2.46 s, 2.31 s and 2.40 s.

- a) Explain why the time readings vary by more than a tenth of a second, although the stopwatch gives readings to one hundredth of a second. [1]  
 b) Obtain the average time  $t$  to fall, and write it in the form (value  $\pm$  uncertainty), to the appropriate number of significant digits. [1]  
 c) The astronauts then determine the gravitational acceleration  $a_g$  on the planet using the formula  $a_g = \frac{2s}{t^2}$ . Calculate  $a_g$  from the values of  $s$  and  $t$ , and determine the uncertainty in the calculated value. Express the result in the form  $a_g = (\text{value} \pm \text{uncertainty})$ , to the appropriate number of significant digits. [3]



6. This question is about finding the relationship between the forces between magnets and their separations.

In an experiment, two magnets were placed with their North-seeking poles facing one another. The force of repulsion,  $f$ , and the separation of the magnets,  $d$ , were measured and the results are shown in the table below.

| Separation $d/\text{m}$ | Force of repulsion $f/\text{N}$ |
|-------------------------|---------------------------------|
| 0.04                    | 4.00                            |
| 0.05                    | 1.98                            |
| 0.07                    | 0.74                            |
| 0.09                    | 0.32                            |

- a) Plot a graph of  $\log(\text{force})$  against  $\log(\text{distance})$ . [3]  
 b) The law relating the force to the separation is of the form

$$f = kd^n$$

- (i) Use the graph to find the value of  $n$ . [2]  
 (ii) Calculate a value for  $k$ , giving its units. [3]