

Name: _____

Richard Montgomery High School Department of Mathematics

Summer Math Packet

for students entering

IB AAF

IB Analysis and Applications of Functions

Below you will find a timeline for completing the packet. **RM teachers have recorded the worked-out solutions and posted them in a YouTube Playlist here: <http://bit.ly/1GKbu2U>.** You can type in the link, or use the QR Code at the bottom corner of this page. You are not required to watch these videos. Only watch them if you feel you need to. There are lots of problems, but we want you to be as prepared and as ready as possible for the upcoming challenges of your new math class 😊.

You may work with a classmate or a peer. You may use textbooks or other sources to help you complete the packet, however, each student must submit their own packet, and each student is responsible for understanding the material contained in the packet. Calculators are not necessary for any of the problems in this packet. Write your work on this packet, and make sure to show an appropriate amount of work for each question. If you need another sheet, you may attach it. These problems are a review of skills you should have already learned, but that may need revisiting. Your fluency (think accuracy with speed) with these skills will help you be successful in ALL of your IB math courses.

Suggested Timeline for Completing IB AAF Summer Packet

Estimated Time to Complete – 2 Hours Per Week

Third Week of July

Problems 1-8

Fourth Week of July

Problems 9-16

First Week of August

Problems 17-24

Second Week of August

Problems 25-32

Third Week of August

Problems 33-40



Formulas

Formulas you should be familiar with.

Equations of Lines

Slope-Intercept Form:	$y = mx + b$
Point-Slope Form:	$y - y_1 = m(x - x_1)$
Standard Form:	$Ax + By = C$

Exponential Function $y = a \cdot b^x$

Explicit Formula for an Arithmetic Sequence: $f(n) = f(0) + nd$ $f(n) = f(1) + (n - 1)d$
Explicit Formula for a Geometric Sequence: $f(n) = f(0) \cdot r^n$ $f(n) = f(1) \cdot r^{n-1}$

Forms of quadratic functions

Vertex Form:	$f(x) = a(x - h)^2 + k$	Axis of symmetry $x = h$
Standard Form:	$f(x) = ax^2 + bx + c$	$x = -\frac{b}{2a}$
Factored Form:	$f(x) = (x - d)(x - e)$	$x = \frac{d+e}{2}$

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Zero-product property: If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

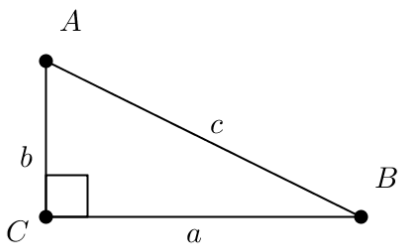
Pythagorean Theorem:

In a triangle with sides a , b , and longest side c , the equation $a^2 + b^2 = c^2$ holds if and only if the triangle is a right triangle.

Distance between two points (x_1, y_1) and (x_2, y_2) : $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope of a line containing two points (x_1, y_1) and (x_2, y_2) : $\frac{y_2 - y_1}{x_2 - x_1}$

Right triangles

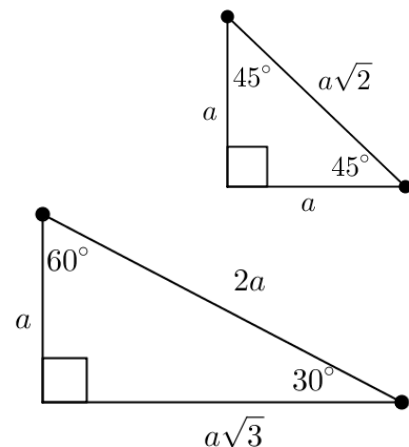


$$c^2 = a^2 + b^2$$

$$\sin A = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$



1. Gino knows that the formula for converting degrees Celsius (C) to degrees Fahrenheit (F) is $F = \frac{9}{5}C + 32$. He also knows how to transform an equation into an equivalent equation. Which of the following is correctly solved for C ?

A. $C = \frac{5F-160}{9}$ B. $C = 5F - \frac{160}{9}$ C. $C = \frac{5F+160}{9}$ D. $C = \frac{5}{9}F + 160$

2. Let a and b represent two numbers such that $a > b$. State whether each statement below is true or false. Justify your answer.

Statement	True or False?	Justification
$a + 8 > b + 8$		
$a - 7 > b - 7$		
$-7a > -7b$		
$\frac{a}{-10} < \frac{b}{-10}$		

3. Below is the solution to an inequality, represented graphically.



Which of the following inequalities has the solution graphed above?

A. $-3x > 6$ B. $-4x - 9 > -17$ C. $2x + 10 \leq 14$

4. Solve the following exponential equations.

(a) $3^{x+2} = 3^7$ (b) $25^{x+1} = 5^{x-3}$ (c) $9^{x+2} = 27^x$

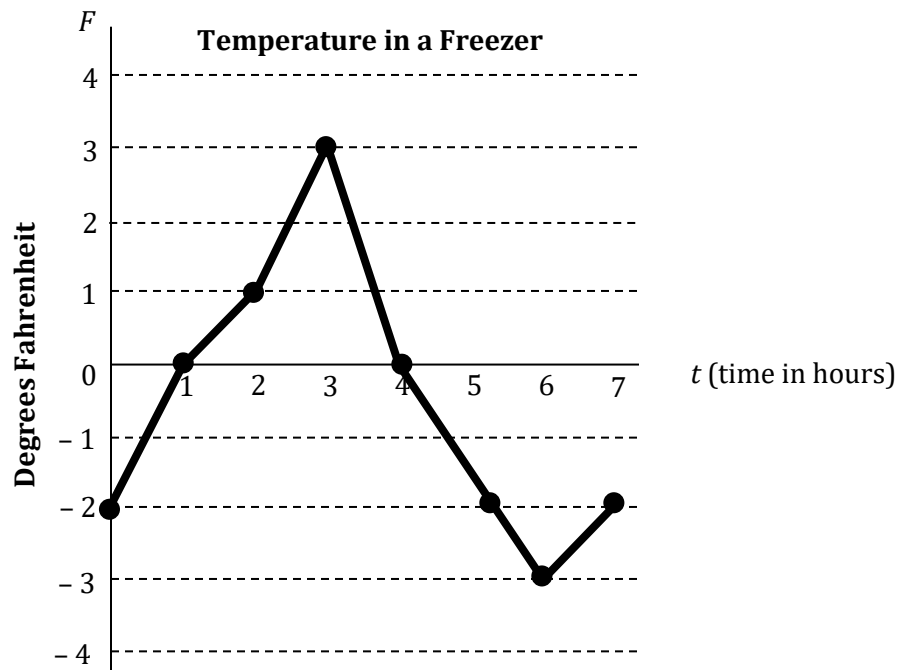
5. Vlad wished to solve the equation $64^x = 16^{x+7}$. There are many correct ways to solve this equation. Which of the following is **not** a correct first step in solving this equation?

A. $2^{6x} = 2^{4(x+7)}$ B. $4^{3x} = 4^{2(x+7)}$ C. $16^{4x} = 16^{x+7}$

6. Which of the following expressions is equal to 5^{2x} ?

A. 25^x B. 10^x C. 2^{5x}

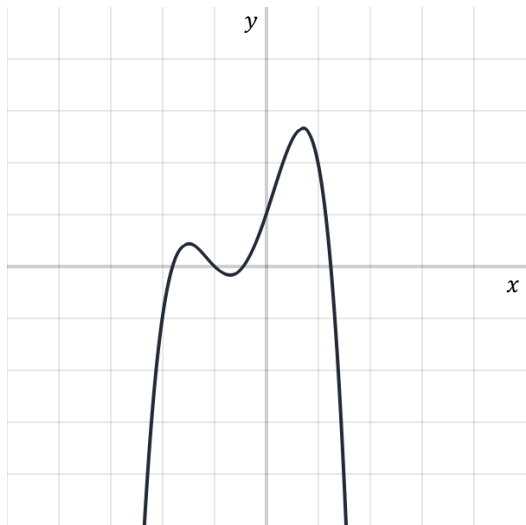
7. The graph below represents the temperature (F) in degrees Fahrenheit inside of a freezer as a function of time. The variable t represents the time, in hours, since midnight.



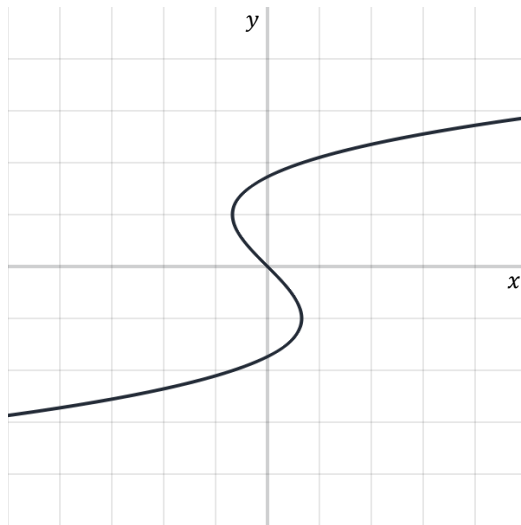
- (a) What is the domain of the function?
- (b) What is the range of the function?
- (c) What is the meaning of the F -intercept?
- (d) At what time t is the minimum temperature reached?
- (e) On what interval of time is the temperature decreasing?
- (f) What does the statement $F(5) = -2$ represent in the context of the situation?

8. For each relation below, determine whether or not the relation represents a function. Justify your answer.

(a)



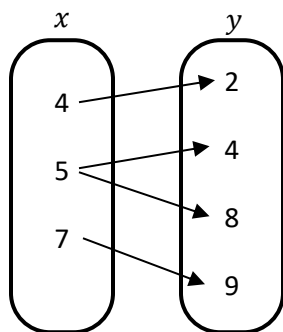
(b)



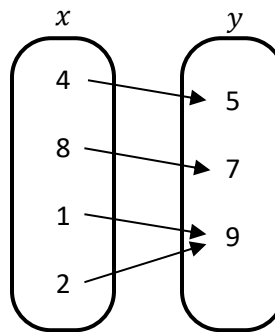
(c) $\{(2, 7), (4, -7), (2, 12), (6, 11)\}$

(d) $\{(4, 9), (6, 11), (11, 15), (10, 9)\}$

(e)



(f)

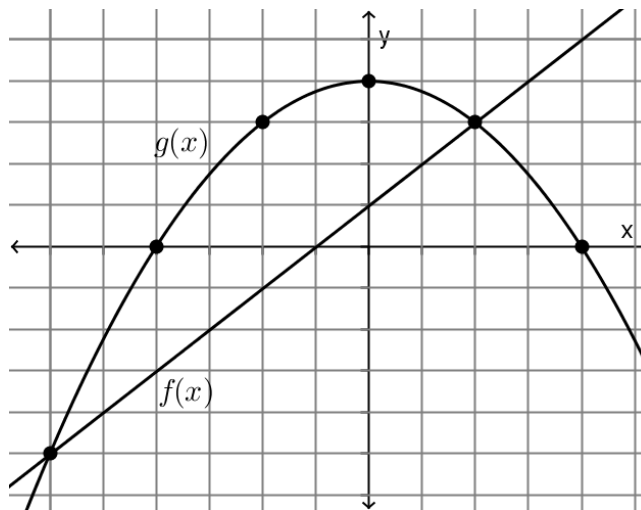


9. What is the number of solutions for this system of equations below?

$$\begin{cases} y = \frac{4}{5}x + 7 \\ y = \frac{3}{4}x + 7 \end{cases}$$

- A. 0 solutions
- B. exactly 1 solution
- C. exactly 2 solutions
- D. an infinite number of solutions

10. The graph of two functions, $f(x)$ and $g(x)$, are shown below.



For each problem, write the correct symbol, $<$, $=$, or $>$ in the box between the two statements.

- (a) $f(2)$ $g(-2)$
- (b) The x -intercept of $f(x)$ The y -intercept of $g(x)$
- (c) $f(1) \cdot f(3)$ $g(-2) + g(2)$

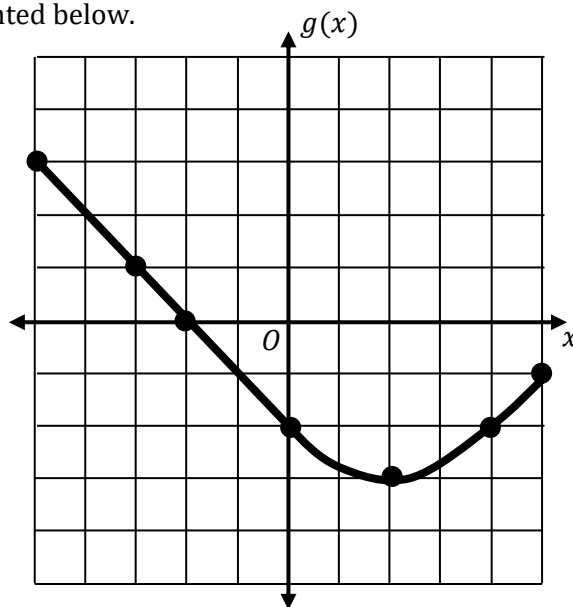
11. A rectangle has a length that is 6 inches longer than its width. If w represents, the width, write an expression, in terms of w , for the area of the rectangle.

12. Fill in the number that completes the square.

- (a) $x^2 + 6x + \underline{\hspace{2cm}}$
- (b) $x^2 - 12x + \underline{\hspace{2cm}}$
- (c) $x^2 + 7x + \underline{\hspace{2cm}}$

13. Two functions, $f(x)$ and $g(x)$ are represented below.

x	$f(x)$
-8	5
-6	6
-3	9
0	8
1	2
4	0
7	-10



Compare the statements below in columns A and B. In the empty space at the end of each row in the table, write the column letter (A or B) of the expression that has the **greater** value. If both expressions are equal, write =.

	Column A	Column B	Which is greater (or are they equal)?
(a)	$f(1)$	$g(-3)$	
(b)	The maximum value of $f(x)$	The x -intercept of $g(x)$	
(c)	$\frac{f(0) - f(4)}{4}$	$g(2) \cdot g(4)$	
(d)	The value of x that makes $f(x) = 0$	$g(-5) + g(-3)$	
(e)	$a + b, f(a) = 5$ and $f(b) = -10$	$g(4)$	

14. Find the slope of the line that passes through $(-5, 1)$ and $(7, -3)$.

Sketch the graph of the function, then give the information requested.

15. $f(x) = -x^2 + 4x + 5$

Vertex:

Line of Symmetry:

x -intercepts:

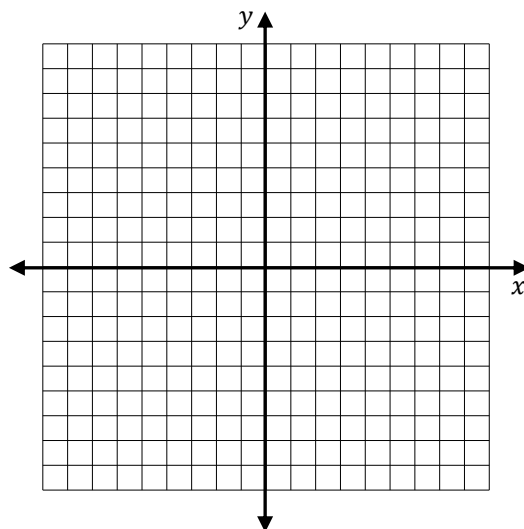
y -intercept:

Domain:

Range:

Interval on which $f(x)$ is increasing:

Interval on which $f(x)$ is decreasing:



16. Use the answer bank below to complete the following statements.

ANSWER BANK

difference		ratio
	linear	
arithmetic		exponential
	geometric	

(a) An explicit rule for an arithmetic sequence would be $a(n)$ _____ function.

(b) The recursive formula $f(0) = 20$, $f(n) = 30 \cdot f(n - 1)$ represents a(n) _____ sequence;
its explicit rule would be $a(n)$ _____ function.

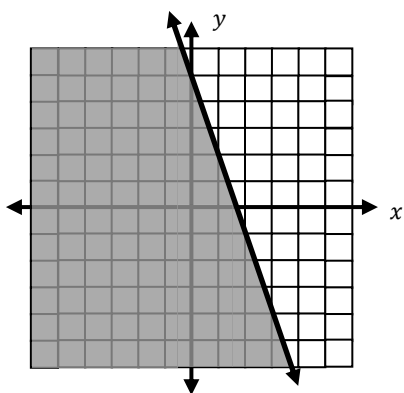
(c) The sequence 10, 12, 14, 16, 18, 20 has a constant _____ of 2.

(d) The sequence 10, 20, 40, 80, 160 has a constant _____ of 2

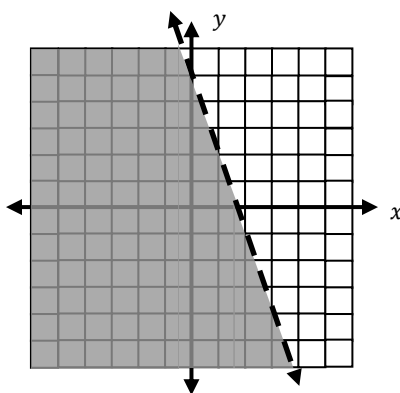
17. Match the following graphs to its corresponding equation.

- 1) $y > -3x + 5$ 2) $y < -3x + 5$ 3) $y \geq -3x + 5$ 4) $y \leq -3x + 5$

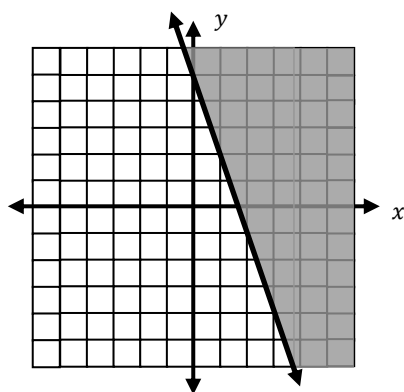
A.



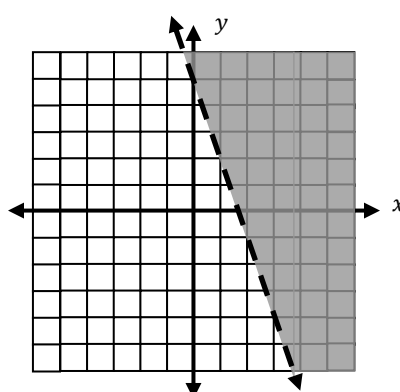
B.



C.

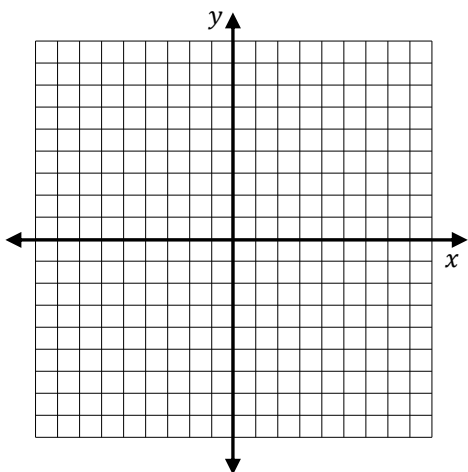


D.

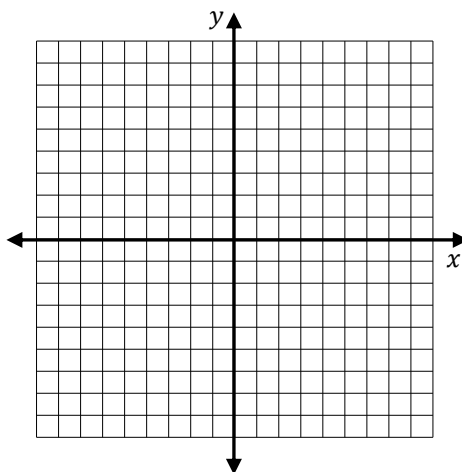


18. Graph the lines. Accurately plot and label at least 3 points on each graph.

(a) $y = 3x + 1$



(b) $3x + 2y = 6$



For items **19 and 20**, a function of time is given. In each item, determine the average rate of change on the given interval. *Give the units for your answer.*

19.

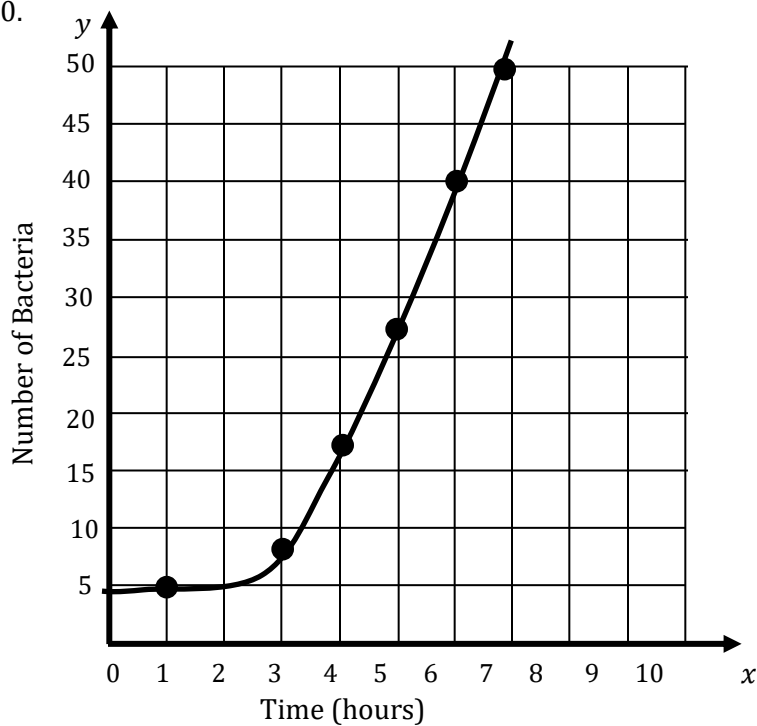
Time (seconds)	Distance (feet)
0	0
1	5
2	20
3	45
4	80
5	125

Average speed on the interval

$2 \leq t \leq 5$:

Units:

20.



Average rate of bacteria growth on the interval $1 \leq t \leq 6$.

Units:

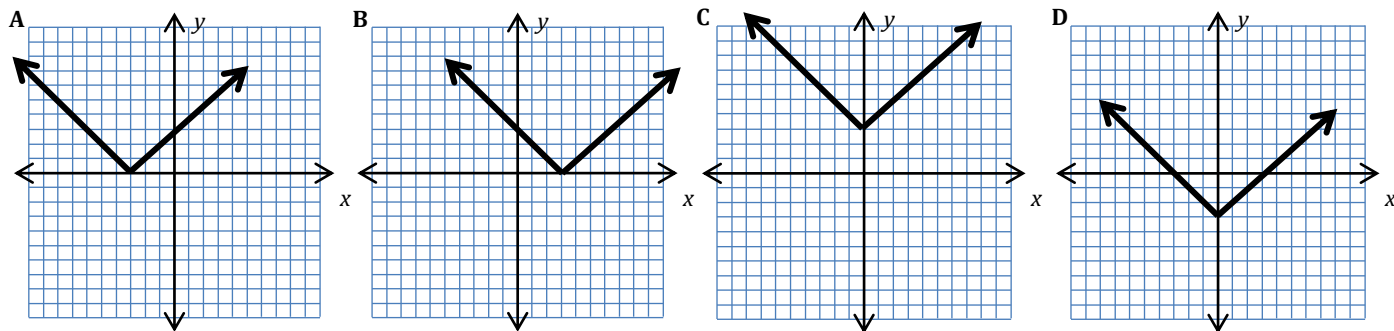
21. Solve the literal equations.

(a) Solve for r : $A = P + Prt$

(b) Solve for z : $2x - 3z + 12 = 0$

22. The following questions explore the relationship between $y = f(x)$ and $y = af(x - h) + k$, and the effect of a , h , and k in transforming the function $f(x)$

- (a) Which of the following graphs represents the function $f(x) = |x + 3|$? Which one represents $f(x) = |x| + 3$?



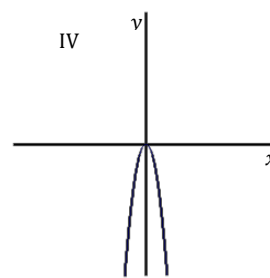
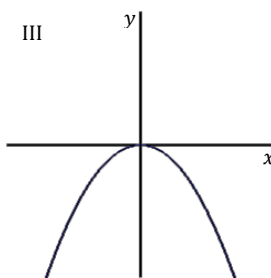
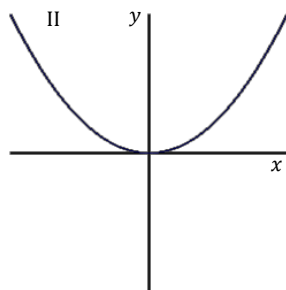
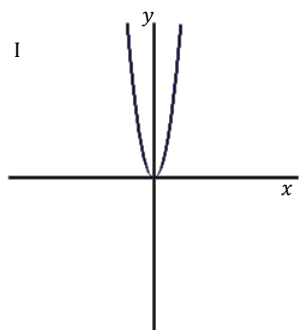
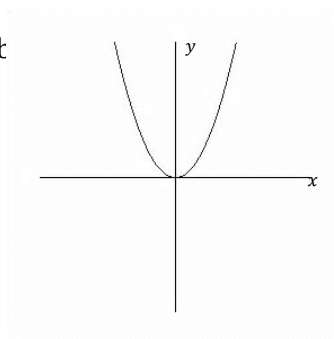
- (b) The graph to the right represents $y = x^2$. Match the equations listed below with the graphs I, II, III, and IV.

_____ $y = -\frac{1}{3}x^2$

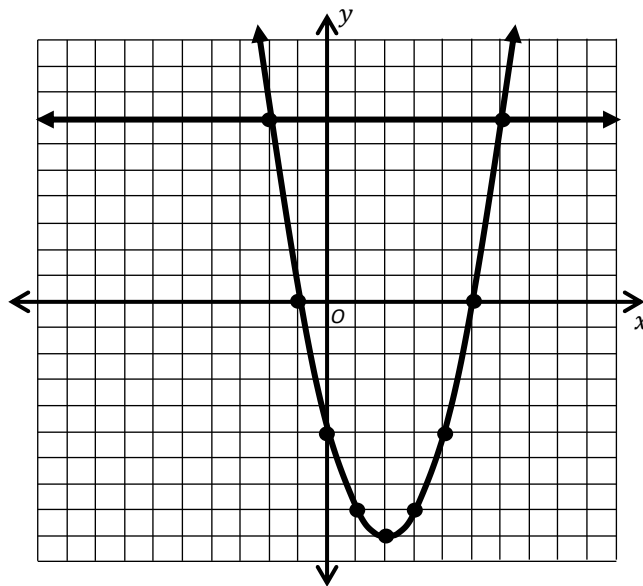
_____ $y = 4x^2$

_____ $y = -5x^2$

_____ $y = \frac{1}{2}x^2$



23. The equations $y = x^2 - 4x - 6$ and $y = 7$ are graphed on the coordinate plane below.



(a) What are the solutions to the equation $x^2 - 4x - 5 = 7$?

(b) What are the solutions to the inequality $x^2 - 4x - 5 \leq 7$?

24. Solve the following linear equations.

(a) $-4(3 - x) = 2(x - 6)$

(b) $2(4x + 6) + 8 = 6x$

(c) $3x - 2(x + 1) = 0$

(d) $6(x + 2) + 1 = 2(x + 4)$

(e) $7x = 4x - 2$

(f) $7x + 19 = 55 - 2x$

(g) $2x + 7 + x = 4(x + 2) - 5$

(h) $4(1 - x) + 3x = -2(x + 1)$

25. Find the slope of the line containing the points (6,5) and (3,9).
26. Find the x -intercept and the y -intercept of the line $3x - 4y = -48$.
27. Find an equation of the line with slope $3/5$ and y -intercept of -4 .
28. Find an equation of the line that passes through the points (1,2) and (3, -4).
29. Find an equation of the line that passes through the point $(-1,4)$ and is parallel to the line with equation $y = 3x - 1$.
30. Find an equation of the line that passes through the point $(-2, -5)$ and is perpendicular to the line with equation $y = x + 2$.
31. Compute the value of each of the following.
- (a) Evaluate $y = 3x^2 - 5$ if $x = 4$.
- (b) Evaluate $y = 4x^3 + 1$ if $x = -3$.
- (c) Evaluate $f(x) = -x^2 - 14x$ when $x = 2$.

32. Completely simplify each of the following. Simplify any radical expression (do not write as a decimal).

(a) $(-3a^2 + 4a - 7) + (2a^2 - 7a + 8)$

(b) $(39x^4 - 4x^3 + 2x^2 - x - 7) - (13x^4 + 3x^3 - 2x^2 - x + 8)$

(c) $(y + 7)(y - 3)$

(d) $(x + 12)(x - 12)$

(e) $(p - 2)(3p + 2)$

(f) $(6m - 5)^2$

(g) $(3t - 5)(3t + 5)$

(h) $(2x - 3)(3x^2 + x - 1)$

(i) $z^2 \cdot z^4$

(j) x^{15}/x^5

(k) $(11x)^2$

(l) $(s^3)^2$

(m) $\left(\frac{2y^3}{3x^2}\right)^3$

(n) $(5x^2y^4)(3x^4y^3)$

(o) $\sqrt{50}$

(p) $\sqrt{100}$

(q) $\sqrt{32}$

(r) $3\sqrt{20}$

(s) $\sqrt{14}\sqrt{7}$

(t) $\sqrt{\frac{32}{25}}$

33. Completely factor each of the following. (HINT: When factoring, first take out any common factors, then factor using the difference of squares method or some other method of factoring.)

(a) $5x^2 - 20x$

(b) $7x^3 - 21x^2$

(c) $x^2 + 16x + 64$

(d) $x^2 - 49$

(e) $x^2 - x - 72$

(f) $3x^2 - 5x - 2$

(g) $x^2 + 7x + 12$

(h) $2x^2 - x - 15$

(i) $x^2 - 8x + 16$

(j) $-6x^2 + 3x + 3$

(k) $x^2 + 10x - 24$

(l) $-2x^2 + 6x$

(m) $4x^2 - 49$

(n) $3x^2 - 11x + 6$

(o) $81x^2 - 1$

(p) $3x^2 + 8x + 5$

(q) $3x^2 + 11x + 10$

(r) $6x^3 - 3x^2 - 18x$

(s) $6x^2 + 13x + 5$

(t) $10x^2 - 7x - 12$

34. Solve each of the following by factoring.

(a) $(2x + 1)(x + 3) = 0$

(b) $x^2 - 16 = 0$

(c) $x^2 - 3x - 10 = 0$

(d) $x^2 - 6x = 0$

(e) $2x^2 - 13x + 15 = 0$

(f) $x^2 + 7x - 8 = 0$

(g) $4x^2 - 24x = 0$

35. Solve each of the following using the quadratic formula.

(a) $2x^2 - 3x - 2 = 0$

(b) $x^2 + 5x + 6 = 0$

(c) $x^2 - 4x - 1 = 0$

(d) $2x^2 - 8x + 5 = 0$

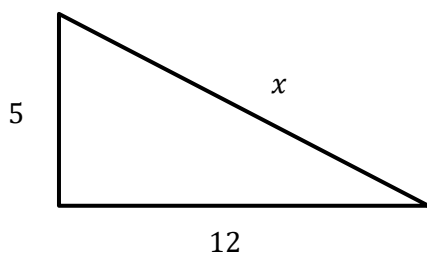
36. Solve the quadratic equation by taking a square root.

(a) $(x - 7)^2 = 81$

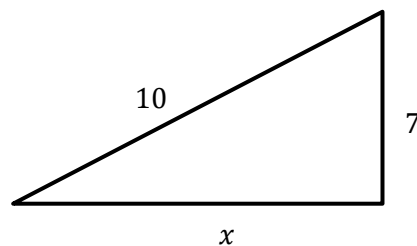
(b) $(x + 5)^2 = 64$

37. For the items below, determine the side of the right triangle marked x .

(a)



(b)



38. The lengths of the three sides of a triangle are given. Determine whether the triangle is a right triangle.

(a) Lengths are 12, 16, and 30.

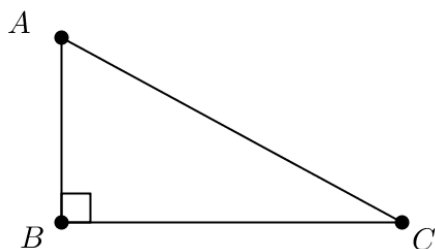
(b) Lengths are 2, 3, and 4.

39. Find the distance between the pair of points.

(a) $(7, -2)$ and $(-6, 3)$.

(b) $(-6, 3)$ and $(2, 9)$

40. Given right $\triangle ABC$, use the item bank to identify the following. Items may be used more than once.



Item Bank					
$\angle A$	$\angle B$	$\angle C$	$\overline{AB}$	$\overline{BC}$	$\overline{AC}$
$\frac{BC}{AC}$	$\frac{BC}{AB}$	$\frac{AB}{BC}$	$\frac{AB}{AC}$	$\frac{AC}{BC}$	$\frac{AC}{AB}$

(a) Leg opposite $\angle A$

(g) Cosine ratio of $\angle C$

(b) Leg opposite $\angle C$

(h) Tangent ratio of $\angle C$

(c) Sine Ratio of $\angle A$

(i) The angle whose Sine ratio is $\frac{BC}{AC}$.

(d) Cosine of ratio $\angle A$

(j) The angle whose Tangent ratio is $\frac{AB}{BC}$

(e) Tangent ratio of $\angle A$

(k) Hypotenuse

(f) Sine ratio of $\angle C$