AP Calculus
June 1, 2019

Summer Practice for AP Calculus AB and BC

In order to be successful in AP Calculus, you must have strong prerequisite skills. Although your calculus teacher may give you quick reminders about concepts that were taught in prior years, there will not be time for re-teaching these prerequisite skills. The following summer packet will help review these skills. The packet will be due at the beginning of the school year. Opportunities will be given to all students to get extra help on the concepts. A quiz (no retesting) will follow. (See the information that follows this summer practice for a fairly comprehensive list of skills which will be needed at various points throughout AP Calculus.)

IMPORTANT SUMMER PACKET DIRECTIONS!!!

- NO CALCULATOR!
- MUST BE DONE ON LINED PAPER. No work is to be done on this packet of questions.
- MUST BE DONE IN ORDER IN A VERY ORGANIZED WAY. Copy the section number and problem letter.
- COPY EACH QUESTION.
- SHOW NEAT AND COMPLETE WORK. Do not skip steps.
- If you can't do a problem, don't just skip it or write in sketchy, incomplete work. Instead, FIND HELP, REVIEW THE SKILL, and confidently complete the problem.
- THIS IS TO BE DONE INDIVIDUALLY. Do not work with a partner or group. Packets that are substantially similar will receive a zero.

If you have any questions about the course or the summer packet, please contact Mr. Kraft (AP Calculus BC) or Mr. Cangelosi (AP Calculus AB) at the e-mail addresses below. (Please note that we may not access our school e-mail accounts frequently during the summer.)

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NO CALCULATOR:

(1) Factor the following expressions, if possible:

(a) $x^2 - 11x + 28$ (b) $2x^2 - 7x - 15$ (c) $20x^2 + 39x + 18$ (d) $x^2 - 7x$ (e) $16x^3 + 6x^2$
(f) $x^2 + 81$ (g) $4x^2 - 9$ (h) $x^3 + y^3$ (i) $8x^6 - 27$

(2) Simplify, if possible:

(a) $2x + 8$ (b) $\dfrac{5x^2 + 15x}{10x^4 + 30x}$ (c) $\dfrac{8x - 3}{4x + 2}$ (d) $\dfrac{x^2 - x - 6}{x^2 - 4x + 3}$

(3) Solve for *x*. This is just a reminder that you should be showing all steps, one step at a time:

(a) $7(-2x+3)=14$

(b) $6x-10=-4x+40$

(c) $3x^2=-15x$

(d) $2x^2-17x=9$

(e) $\ln x=5$

(f) $\log_4(x+3)=2$

(g) $e^x=10$

(h) $5^{3x}-1=3$

(i) $3e^{2x}=50$

(j) $\sqrt{x^2-17}=x-1$

(k) $\sqrt{4+x}+0.5x(4+x)^{-1/2}=0$

(l) $x^2e^{-x}-2xe^{-x}=0$

(m) $2x^3-4x^2+5x+3=x^3+x^2+19x+3$

(n) $|2x-3|=7$

(o) $-\dfrac{1}{x+2}=\dfrac{1}{2}+\dfrac{1}{3}$

(p) $\dfrac{3}{x-5}+\dfrac{2}{x+1}=\dfrac{6}{x^2-4x-5}$

(q) $\dfrac{x^2-5}{2x+1}=0$

(r) $\dfrac{2\sin(x)}{3}+1=\dfrac{5}{3}$

(4) Using your knowledge of the unit circle, evaluate:

(a) $\sin\pi$ (b) $\sin 0$ (c) $\sin\dfrac{3\pi}{2}$ (d) $\sin\dfrac{\pi}{3}$ (e) $\sin\dfrac{5\pi}{4}$

(f) $\cos 0$ (g) $\cos\dfrac{\pi}{2}$ (h) $\cos\pi$ (i) $\cos\dfrac{\pi}{6}$ (j) $\cos\dfrac{5\pi}{3}$

(k) $\tan 0$ (l) $\tan\dfrac{\pi}{2}$ (m) $\tan\dfrac{3\pi}{2}$ (n) $\tan\dfrac{\pi}{4}$ (o) $\tan\dfrac{5\pi}{6}$

(p) $\csc 0$ (q) $\csc\dfrac{\pi}{6}$ (r) $\sec 0$ (s) $\sec\dfrac{\pi}{4}$ (t) $\cot\dfrac{\pi}{2}$ (u) $\cot\dfrac{7\pi}{4}$

(v) $\sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$ (w) $\cos^{-1}0$ (x) $\sec^{-1}1$

(5) Simplify the following trig expressions:

(a) $\sin x\cot x$ (b) $\tan x\csc x\cos x$ (c) $\dfrac{\sin^2 x}{\csc^2 x}$ (d) $\sec x\sin x\cos x$

(e) $\sin^2 x+\cos^2 x$ (f) $1+\cot^2 x$

(6) Simplify without any negative exponents in the final answer:

(a) $x^2 \bullet x^7$ (b) $4x^{-3} \bullet x$ (c) $(3x^5y^{-4})(5x^{-8}y^9)$

(d) $\dfrac{x^{10}}{x^4}$ (e) $\dfrac{x^{-3}}{x^7}$ (f) $\dfrac{-8x^4y^3}{4x^6y}$

(g) $(x^2)^5$ (h) $\left(\dfrac{x^6}{y^{-2}}\right)^{-1}$ (i) $(9x^4)^3$

(j) x^0y^7 (k) $\left(\dfrac{512\sqrt{x}}{x^7} - \sin x\right)^0$

(7) Rewrite in exponential form: (a) $\sqrt[3]{x}$ (b) $\sqrt[5]{x^2}$

(8) Rewrite in radical form: (a) $x^{1/2}$ (b) $x^{-3/4}$

(9) Simplify: (a) $\sqrt{64x^2y^{10}}$ (b) $\sqrt{\dfrac{16x^4}{100}}$

(10) Sketch the following equations as accurately as you can using your knowledge of the transformation rules. Draw in any asymptotes using dashed lines.

(a) $y = 4x - 2$ (b) $y = 7$ (c) $x = -3$

(d) $y = x^2$ (e) $y = x^2 + 3$

(f) $y = x^3$ (g) $y = (x+2)^3$

(h) $y = \ln(x)$ (i) $y = \ln(x) - 4$

(j) $y = \sin(x)$ (k) $y = \sin\left(x - \dfrac{\pi}{2}\right)$

(l) $y = e^x$ (m) $y = -e^x$ (n) $y = e^{-x}$

(o) $y = \cos(x)$ (p) $y = 2\cos(x)$ (q) $y = \dfrac{1}{2}\cos(x)$

(r) $y = |x|$ (s) $y = |4x|$ (t) $y = \left|\dfrac{1}{4}x\right|$

(u) $y = \begin{cases} 3x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ (v) $y = \begin{cases} 2x+3, & x > 1 \\ 5, & x = 1 \\ x^3, & x < 1 \end{cases}$

(11) Rewrite in exponential form: (a) $\log_3 9 = 2$ (b) $\log_4 \frac{1}{16} = -2$

(12) Rewrite in logarithmic form: (a) $5^x = 20$ (b) $2^{-3} = \frac{1}{8}$

(13) Evaluate:

(a) $\log_4 4$ (b) $\log_5 1$ (c) $\log \frac{1}{10}$

(d) $\ln e$ (e) $\ln 1$ (f) $\ln \frac{1}{e}$ (g) $\ln e^2$

(14) Simplify, if possible. If not possible, write "already simplified".

(a) $\ln 8 + \ln 5$ (b) $\ln 20 + \ln \frac{1}{2}$ (c) $\ln 12 - \ln 4$ (d) $\ln 500 - \ln 20$

(e) $\frac{\ln 15}{\ln 3}$ (f) $(\ln 2)(\ln 6)$ (g) $e^{\ln(x+3)}$ (h) $\ln e^{\sin x}$

(i) $e^{-2\ln x}$

(15) Rewrite without an exponent: $\ln x^3$

(16) Rewrite using an exponent: $-\ln 2$

(17) If you could use a calculator, how would you evaluate the following problems?

(a) $\log_7 40$ (b) $\log_5 0.27$

(18) Are the following expressions defined? Indicate "yes" or "no."

(a) $\frac{17}{5+3-8}$ (b) $\frac{6x}{x+3},\ x=0$ (c) $\frac{x^2}{x-5},\ x=5$

(d) $\sqrt{x-10},\ x=14$ (e) $\sqrt{x+8},\ x=-8$ (f) $\sqrt{x-1},\ x=0$

(g) $\ln 1$ (h) $\ln 0$ (i) $\ln(-3)$

(19) Indicate if the following statements are "true" or "false."

(a) The graph of $y = \frac{x^2-4}{x+2}$ is equivalent to the graph of $y = x-2$.

(b) The solutions for $x^3 = x$ are the same as those for $x^2 = 1$.

(c) The graph of $y = \sqrt{x^2}$ is equivalent to the graph of $y = x$.

(20) Given that $f(x) = x^2 + 3x$ and $g(x) = \sin x$, determine:

(a) $f(7)$ (b) $g(\pi)$ (c) $f\left(g\left(\frac{3\pi}{2}\right)\right)$ (d) $f(g(x))$

(e) $g(f(x))$ (f) $f(f(x))$ (g) $f(x+c)$ (h) $h(1)$ *where* $h(x) = f^2(x+1)$

(i) $j(\pi)$ *where* $j(x) = \sqrt{f(g(x)+2)}$

(21) Perform the following operations: (Remember, no calculator.)

(a) $936 + 879$ (b) $10,487 - 8,529$ (c) $87 \bullet (-56)$ (d) $6255 \div 3$

(e) $72.58 + 3.6$ (f) $778.457 - 64.51$ (g) $(-6.44)(-4.4)$ (h) $865.2 \div 0.06$

(i) $\frac{3}{4} + \frac{1}{8}$ (j) $\frac{25}{3} - \frac{3}{4}$ (k) $\frac{10}{15} \bullet \frac{27}{4}$ (l) $\frac{12}{5} \div \frac{18}{5}$

(m) $\frac{7}{x} + \frac{2x}{x-3}$ (n) $\frac{2x+8}{3x} \bullet \frac{12x^2}{x+4}$ (o) $\frac{x^2 - 7x + 12}{x^2 + 7x + 10} \div \frac{4x - 16}{3x + 6}$

(22) Evaluate for x = 9:

(a) $x^{3/2}$ (b) $f(x) = \begin{cases} x + 41, & x \le 8 \\ x^2 - 5, & x > 8 \end{cases}$ (c) 4^{x-6}

(23) Simplify:

(a) $(x-2)^3$ (b) $(3x+5)^2$ (c) $5x - (6 - 3x)$

(24) Describe the end behavior of the following functions:

(a) $f(x) = 4x^5 - 9x^3 - x^2 + 6$ (b) $f(x) = -11x^8 - 7x^3 + 2x$

(25) Write the inverse of the following functions:

(a) $f(x) = 18x - 5$ (b) $f(x) = 3(x-5)^2$

(26) Identify the type of discontinuity most associated with the following functions:

(a) a rational function has a denominator that can "cancel out." Example: $y = \frac{x^2 - 4}{x+2}$

(b) a rational function has a denominator that cannot "cancel out." Example: $y = \frac{x^3 - 8}{x+5}$

(c) a piecewise function. Example: $y = \begin{cases} 2x+10, & x<3 \\ -5x^2, & x \geq 3 \end{cases}$

(d) log functions and some trig functions. Example: $y = \ln x$, $y = \tan x$

(27) Using only the features of a graphing calculator, solve $-(x+4)^2 + 7 = 0$. Round correctly to 3 decimal places. Then, find the absolute maximum of $y = -(x+4)^2 + 7$.

(28) Using your graphing calculator only…

(a) What is the result when negative 4 is raised to the power of 6 ?
(b) What is the result when 10 is divided by the expression 2π ?
(c) What is the result when 2 is raised to the power of 3+2 ?

(29) True or False:

(a) Displacement is the change in position and can be either positive or negative.
(b) Velocity has direction; therefore, it can be positive or negative.
(c) If the velocity is -30 mph, the speed is 30 mph.

(30) Geometry Problems. Include correct units.

(a) If a circle has a radius of 4 in., find its diameter, circumference, and area.
(b) If a rectangle has a length of 10 cm and a width of 3 cm, find its perimeter and area.
(c) If a cube has a side length of 3 ft., find its surface area and volume.
(d) If a triangle has a base of 5 in. and a height of 4 in., find its area.
(e) Given that the side across from the 30° angle in a 30-60-90 triangle is 7 cm, what are the lengths of the other two sides?
(f) Given that the side across from a 45° angle in a 45-45-90 triangle is 2 yd., what is the length of the hypotenuse?
(g) The two legs of a right triangle are 35 ft. and 40 ft. What is the length of the hypotenuse?
(h) If a trapezoid has base lengths of 2 cm and 8 cm and the height is 5 cm, what is the area?
(i) The graph of $x^2 + y^2 = 16$ would be what shape?

The following problems are for AP Calculus BC only.

(31) AP Calculus BC only: Sequences and Series

(a) Find the sum of the infinite series, or show why the infinite sum does not exist:

$$3+\frac{9}{5}+\frac{27}{25}+\frac{81}{125}+\ldots$$

(b) Express the series in Sigma notation: 11 – 18 + 25 – 32 + 39

(c) Express the series in Sigma notation: 175 – 35 + 7 – …

(32) AP Calculus BC only: Vectors

(a) Find an ordered pair to represent u in each equation if $\overline{\mathbf{v}} = \langle -3,2 \rangle$ and $\overline{\mathbf{w}} = \langle 5,-2 \rangle$

u = w – 2v   u = 5w + 2v

(b) Write the ordered pair that represents the vector from A to B. Then find the magnitude and direction of $\overline{\mathbf{AB}}$:   A(-2, -10) B(11, -5)

(c) Find the dot product, then state if the vectors are parallel, perpendicular, or neither:

$\langle -4,-2 \rangle \bullet \langle -6,3 \rangle$

(33) AP Calculus BC only: Polars

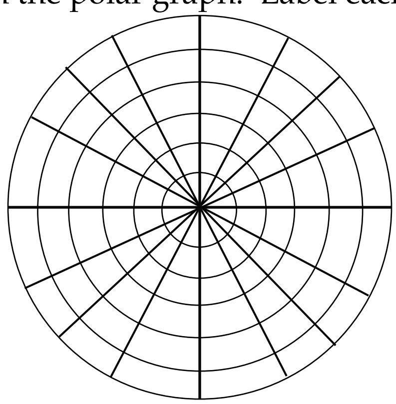
(a) Graph the following points on the polar graph. Label each point with the capital letter.

$A(-2, \frac{\pi}{3})$

$B(3, \frac{5\pi}{4})$

$C(1, -90°)$

$D(-4, -120°)$



(b) Convert the point from rectangular to polar form. SHOW ALL WORK!

$(-4, -4)$

(c) Convert the point from polar to rectangular form. SHOW ALL WORK!

Note: (x, y) must be expressed in exact (radical) form whenever possible.

$(-5, \frac{2\pi}{3})$

Prerequisite Skills for AP Calculus

What are prerequisite skills?
They are the math skills that you should already be familiar with before taking AP Calculus.
Why are prerequisite skills important?
They are the building blocks for everything we do in AP Calculus. In calculus, we study 3 big and exciting ideas: limits, derivatives, and integrals. To fully understand these concepts, you should have strong knowledge of basic math, algebra 1 and 2, pre-calculus, and to a lesser extent, geometry. The majority of errors that students make in this course are basic math and algebra mistakes. Some students will understand the new calculus skills but keep getting problems wrong because they keep making computational or algebraic errors.
Will you teach us the prerequisite skills?
The 3 big ideas of limits, derivatives, and integrals will take up most of our time. There will be little time to reteach things like adding fractions, using the unit circle, or solving exponential equations.
What are the specific prerequisite skills that I need to know?
I have attached a list of prerequisite skills. Most skills must be done without a calculator.
I've looked at the list of prerequisite skills and now I'm really worried! What should I do?
There may be nothing to be worried about. Everyone will have skills that need "refreshing." Here are some examples of when you might be concerned:

topic	I'll be fine.	I might be concerned.
adding fractions	"I know I can do this, but let me first practice a few problems."	"I've always been terrible with fractions. Math just isn't my strength."
using the unit circle	"I'll need to quickly review how to sketch in all of the values."	"I never learned it. It was just too crazy!"
solving exponential equations	"Someone please remind me of the first step."	"I have trouble solving linear and quadratic equations. Forget this!"

Where can I go for more help?
For quick questions and reminders about the prerequisite skills, you can always ask the teacher during class, before school, or at lunch. Don't be shy! If you need time-intensive re-teaching of a pre-requisite skill, relying on your calculus teacher may not be the best choice. The teacher needs to spend the majority of time helping students with *calculus* skills. Instead, you may want to consider the following resources:

- using Google, youtube, Khan Academy, etc. to find on-line tutorials on the subject
- borrowing a textbook from Algebra, Geometry, Algebra 2, or Pre-Calculus
- working with another student, a family member, or tutor

The Prerequisite Skills (This list should be fairly complete, but there may be other skills.)

- adding, subtracting, multiplying, and dividing with whole numbers (without a calculator)
- adding, subtracting, multiplying, and dividing with fractions (without a calculator)
    - includes finding common denominators:
      $\frac{4}{5}+\frac{7}{x+1} \rightarrow \frac{4(x+1)}{5(x+1)}+\frac{7(5)}{(x+1)(5)} \rightarrow \frac{4x+4+35}{5(x+1)} \rightarrow \frac{4x+39}{5x+5}$
    - includes reciprocals: $\frac{-x^2+x}{3} \div x = \frac{-x^2+x}{3} \bullet \frac{1}{x} = etc...$
    - includes splitting fractions correctly: $\frac{x+7}{2} = \frac{x}{2}+\frac{7}{2}$ *BUT* $\frac{2}{x+7} \neq \frac{2}{x}+\frac{2}{7}$
- adding, subtracting, multiplying, and dividing with decimals (without a calculator)
- adding, subtracting, multiplying, and dividing with negative numbers (without a calculator)
- evaluating functions for a given *x*-value, including piecewise functions and fractional exponents
    - example: $f(x)=2^{x-5}$; $f(9)=2^{9-5}=2^4=16$
    - example: $f(x)=\begin{cases} 3x-1, & x \le 2 \\ 5x^3 \quad , & x > 2 \end{cases}$ $f(4)=5(4)^3=5(64)=320$
    - example: $f(x)=x^{3/2}$; $f(16)=16^{3/2}=4^3=64$
- distributing
    - $-3x(2x-5)=-6x^2+15x$
- multiplying polynomials using FOIL or distribution
    - $(2x-3)(3x+1)=6x^2+2x-9x-3=6x^2-7x-3$
    - $(3x-7)^2 \neq 9x^2+49$, *but...*
    - $(3x-7)^2=(3x-7)(3x-7)=9x^2-21x-21x+49=9x^2-42x+49$
    - $(x-2)^3=(x-2)(x-2)(x-2)=(x-2)(x^2-4x+4)=x^3-4x^2+4x-2x^2+8x-8=x^3-6x^2+12x-8$
- factoring
    - polynomials: $10x^2-15x \rightarrow 5x(2x-3)$
    - sums of squares: $4x^2+9$ does not factor
    - differences of squares: $a^2-b^2=(a+b)(a-b)$; $4x^2-9=(2x+3)(2x-3)$
    - sums of cubes:
      $a^3+b^3=(a+b)(a^2-ab+b^2)$; $8x^3+1000=(2x+10)(4x^2-20x+100)$
    - differences of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$; $x^3-8=(x-2)(x^2+2x+4)$
    - trinomials using reverse FOIL: $6x^2+x-15=(2x-3)(3x+5)$

- simplifying expressions correctly
  - $8x+6 \neq 4x+3$
  - $\frac{6x+5}{3x-2} \neq \frac{{}^{2}\cancel{6}\,\cancel{x}+5}{\cancel{3}\,\cancel{x}-2} \neq \frac{2+5}{-2}$
  - $\frac{2}{8x} \neq \frac{1}{4}x$; however, $\frac{2}{8x} = \frac{1}{4x}$
  - $\frac{10}{5\pi} \neq 2\pi$; however, $\frac{10}{5\pi} = \frac{2}{\pi}$
- understanding that when you simplify, you may lose important information
  - the graph of $y=\sqrt{x^2}$ is a "V", but if you take the square root of x^2, the graph of $y=x$ is a line
  - the graph of $y=\frac{x^2-4}{x-2}$ has a point discontinuity at $x = 2$, but if you factor and simplify, the resulting $y=x+2$ would seem to be a continuous line
  - $2x^2=-8x$ has TWO solutions, $x=\{0,-4\}$, but if you divide both sides by x, you lose a solution: $2x^2=-8x \rightarrow 2x=-8 \rightarrow x=-4$
- using exponent rules and root rules correctly
  - $x^4 \bullet x^3 = x^7$
  - $\frac{x^8}{x^2}=x^6$
  - $(x^5)^2 = x^{10}$
  - $x^{-4}=\frac{1}{x^4}$
  - $(4x^2)^3 = 4^3(x^2)^3 = 64x^6$
  - $\left(\frac{2}{x^7}\right)^3 = \frac{2^3}{(x^7)^3} = \frac{8}{x^{21}}$
  - $\sqrt{4x^2}=2x$
  - $\sqrt{\frac{4}{x^2}}=\frac{2}{x}$
  - $\sqrt{4+x^2} \neq 2+x$
- rewriting in exponential form or radical form
  - $\sqrt[4]{x^3}=x^{\frac{3}{4}}$
  - $x^{\frac{1}{2}}=\sqrt{x}$

- recognizing that certain functions are defined only at particular x values
  - example: $f(x)=\sqrt{x-7}$ is defined only when $x \geq 7$
  - example: $f(x)=\dfrac{4}{x+17}$ is defined except when $x=-17$
  - example: $f(x)=\ln(x)$ is defined only when $x>0$
  - example: $f(x)=\tan(x)+5$ is defined except when $x=\left\{-\dfrac{3\pi}{2},-\dfrac{\pi}{2},\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{2},\ldots\right\}$
- using the unit circle (from memory) to find trig values (especially the first quadrant, but other quadrants as well). Note: We will always be working in radians, not degrees:
  - examples: $\sin\dfrac{\pi}{4},\ \sec\dfrac{\pi}{3},\ \tan\pi,\ \cos\dfrac{5\pi}{4}$
- using the trig identities
  - the reciprocal of sine is cosecant
  - the reciprocal of cosine is secant
  - the reciprocal of tangent is cotangent
  - $\dfrac{\sin x}{\cos x}=\tan x$
  - $\sin^2 x+\cos^2 x=1$, from which you can derive: $1+\cot^2 x=\csc^2 x$, $\tan^2 x+1=\sec^2 x$
- solving linear equations
  - $4(3x-5)=-16 \rightarrow 3x-5=-4 \rightarrow 3x=1 \rightarrow x=\dfrac{1}{3}$
- solving quadratic, cubic, and other polynomial equations
  - $7x^2-3=25 \rightarrow 7x^2=28 \rightarrow x^2=4 \rightarrow x=\pm 2$
- solving exponential equations by taking the natural log of both sides
  - $3(2)^x=18 \rightarrow 2^x=6 \rightarrow \ln 2^x=\ln 6 \rightarrow x\ln 2=\ln 6 \rightarrow x=\dfrac{\ln 6}{\ln 2}$
- solving logarithmic equations by rewriting in exponential form
  - $\ln(3x)+1=5 \rightarrow \ln(3x)=4 \rightarrow e^4=3x \rightarrow \dfrac{e^4}{3}=x$
- solving trigonometric equations
  - $4\sin x=\pi \rightarrow \sin x=\dfrac{\pi}{4} \rightarrow x=\sin^{-1}\left(\dfrac{\pi}{4}\right) \rightarrow x=\dfrac{\sqrt{2}}{2}$
- solving rational equations
  - $\dfrac{x-3}{7}=\dfrac{4}{x} \rightarrow x^2-3x=28 \rightarrow x^2-3x-28=0 \rightarrow (x-7)(x+4)=0 \rightarrow x=\{7,-4\}$

- solving absolute value equations by creating a $\pm$
  - $|x-3|=7 \rightarrow x-3=\pm 7 \rightarrow x=\{10,-4\}$
- solving radical equations
  - $\sqrt{3x-2}=4 \rightarrow 3x-2=16 \rightarrow 3x=18 \rightarrow x=6$
- rewriting exponential equations in log form and vice versa
  - $4^x=9 \rightarrow \log_4 9=x$
  - $\ln y=3x \rightarrow e^{3x}=y$
- graphing linear equations using slope and y-intercept
- graphing and citing characteristics (domain, range, intercepts, asymptotes, etc.) of the basic functions (without a calculator): constant,

  $e^x,\ \ln x,\ x^2,\ x^3,\ \sin x,\ \cos x,\ \tan x,\ \frac{1}{x},\ |x|,\ \sqrt{x}$
- knowing all transformations rules, especially translating left, right, up, and down; reflecting over the *x*-axis and *y*-axis, and dilating with respect to the *x*-axis.
- graphing any basic function using transformation rules (without a calculator)
  - examples: $f(x)=e^{-x}$, $f(x)=4x^2-3$, $f(x)=|x|+6$
- graphing piecewise functions
- writing the equations of asymptotes
  - vertical asymptotes are in $x=$ form, such as $x=3$
  - horizontal asymptotes are in $y=$ form, such as $y=3$
- knowing the end behavior rules for polynomial functions
  - example: $y=-4x^3+7x-8$, as $x\rightarrow -\infty,\ y\rightarrow \infty$, as $x\rightarrow \infty,\ y\rightarrow -\infty$
- writing the inverse of a function
  - example: $y=4(x+3) \rightarrow x=4(y+3) \rightarrow \frac{x}{4}=y+3 \rightarrow \frac{x}{4}-3=y$
- simplifying and evaluating composite functions
  - Given $f(x)=x^3$ and $g(x)=\sin x$, then $f(g(x))=(\sin x)^3$ *or* $\sin^3 x$
- identifying the types of discontinuity: point (removable), infinite, or jump
- using key features of your <u>graphing calculator</u>
  - graph any function
  - solve any equation using the "intersect" or "zero" feature
  - find the *x*-intercepts of any function using the "zero" feature
  - find the maximum or minimum of any function
  - create a table of values
  - zoom in and out, or change the "window" of a graph
  - use the keys for *e* (approximately 2.718) and π (approximately 3.14)
  - press "ans" to use your previous answer
  - press "entry" to repeat an earlier entry

- using parentheses correctly on your graphing calculator – especially with older calculators
    - 2^{7x} is entered as 2^(7x)
    - negative 3 raised to the power of 2 is entered as $(-3)^2$
    - $\frac{4}{x+3}$ is entered as 4/(x+3)
    - given $4 = 2\pi x$, you would solve for x by entering 4/(2π)
- knowing that "displacement" is the change in position (positive or negative)
- knowing the difference between speed and velocity – velocity has direction
    - an elevator might have a velocity of –10 mph because it is moving downwards, but its speed is 10 mph
- memorizing basic geometric formulas
    - the area of a circle is $A = \pi r^2$
    - the circumference of a circle is $C = 2\pi r$
    - the area of a rectangle is $A = bh$; the perimeter is the sum of the lengths of the sides
    - the volume of a cube is $V = l^3$
    - the surface area of a cube is $S = 6l^2$
    - the area of a triangle is $A = \frac{1}{2}bh$
    - the area of a trapezoid is $A = \frac{b_1 + b_2}{2} \bullet h$
- knowing the ratio of the side lengths of special right triangles
    - the ratio of side lengths for a 30-60-90 triangle is $1, \sqrt{3}, 2$
    - the ratio of side lengths for a 45-45-90 triangle is $1, 1, \sqrt{2}$
- knowing the relationship between the diameter and radius of a circle
    - $d = 2r$
- using the Pythagorean Theorem to solve problems involving right triangles.
    - $x^2 + y^2 = h^2$, where h is the hypotenuse
- recognizing the equation of a circle
    - $x^2 + y^2 = r^2$ $\quad y = \pm\sqrt{r^2 - x^2}$ $\quad y = \sqrt{r^2 - x^2}$ (semi-circle)
- remembering to distribute subtraction signs
    - $3x - (5x - 2) = 3x - 5x + 2 = -2x + 2$

- simplifying exponential and logarithmic expressions
  - $e^{\ln x} = x$
  - $\ln e^x = x$
  - $\ln a + \ln b = \ln(ab)$
  - $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$
- using the logarithm power rule
  - $\ln x^a = a \ln x$ *Ex.* $\ln x^3 = 3\ln x$
- using the change of base formula
  - $\log_b a = \frac{\log a}{\log b}$ *ex.* $\log_4 20 = \frac{\log 20}{\log 4}$
- rounding decimals correctly to 3 decimal places
  - $4.1524388 \approx 4.152$
  - $8.8735919 \approx 8.874$
  - $7.399763 \approx 7.400$
- using correct units
  - units of length: cm, in.
  - units of area: cm^2, $in.^2$
  - units of volume: cm^3, $in.^3$

The following skills are for AP Calculus BC only.

- Sequences and Series
  - Given a series in Sigma (summation) notation, expand that series.
  - Given a series in expanded format, express that series in Sigma (summation) notation.
  - Identify a Geometric Series, and its common ratio.
  - Find the sum of an infinite Geometric series.

- Vectors
  - Must have working knowledge of vectors (magnitude, direction, unit vectors).
  - perform vector operations – addition, scalar multiplication, dot product.

- Parametrics
  - Must have working knowledge of parametric equations.

- Polars
    - Plot polar coordinates.
    - Convert points and equations from rectangular to polar, and from polar to rectangular.
    - Recognize polar functions and their characteristics (Circle/Oval, Cardioid/Limaçon, Rose).