

# *IB Standard Level Summer 2019 Prerequisite Skills Packet*

The topics and problems in this packet are considered prerequisite skills from Geometry, Algebra 2, and Pre-Calculus. These skills are required topics encompassed within the IB Standard Level Mathematics Curriculum. It is assumed you are already familiar and comfortable with these topics as they will be used throughout the course as you learn new concepts.

**This packet is due the first day of school. It will count as a large homework grade.**

If the topics are unfamiliar to you, you need to work independently over the summer to catch up. Here are tips if you need help:

- Look things up online (do a google search of the topic)
- Khan Academy and Purple math are great website with tutorials and explanations
- Consult your old Geometry, Algebra 2, and Pre-Calculus notes

All work should be complete and correct. All work must be shown (when necessary) to earn full credit.

During IB SL Math, you will have use of the ***IB Formula Booklet***. Therefore, you may look up any formulas you may need to complete these problems (such as the law of cosines or the area of a sector).

Questions #39-42 are IB problems from past exams so you can see right away that topics from previous courses are included on the IB Math SL Exam.

1. Let  $f$  and  $g$  be functions that are inverses of each other.

Complete the following statements.

- a. If the point  $(a, b)$  is on the graph of  $f$ , then the point \_\_\_\_\_ is on the graph of  $g$ .
- b. If  $f(3) = 7$ , then  $g(7) =$  \_\_\_\_\_.
- c. The graphs of  $f$  and  $g$  are symmetric with respect to the line \_\_\_\_\_.
- d. The range of  $f$  is the same as the \_\_\_\_\_ of  $g$ .
- e. The domain of  $f$  is the same as the \_\_\_\_\_ of  $g$ .

2. Each function below is a transformation of the function  $f(x) = e^x$ . After each given transformation, write the function rule.

- a. The graph of function  $g$  is the graph of  $f(x) = e^x$  translated one unit to the right.  
 $g(x) =$  \_\_\_\_\_
- b. The asymptote of the graph of function  $h$  has the equation  $y = -4$ .  
 $h(x) =$  \_\_\_\_\_
- c. The graph of function  $p$  is the graph of  $f(x) = e^x$  reflected across the  $x$ -axis.  
 $p(x) =$  \_\_\_\_\_
- d. The graph of function  $s$  is the graph of  $f(x) = e^x$  reflected across the  $y$ -axis.  
 $s(x) =$  \_\_\_\_\_
- e. The graph of function  $w$  is the graph of  $f(x) = e^x$  dilated by a factor of 2 with respect to the  $x$ -axis.  $w(x) =$  \_\_\_\_\_

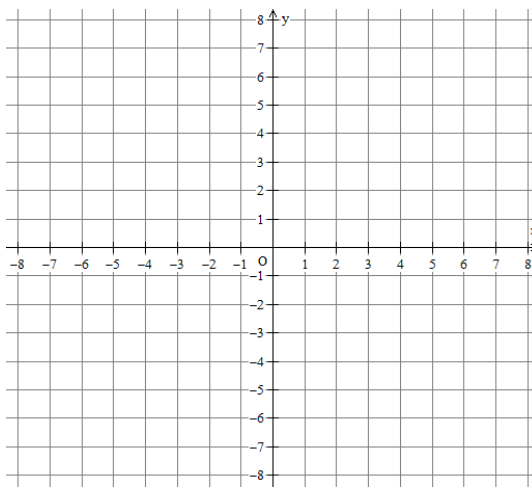
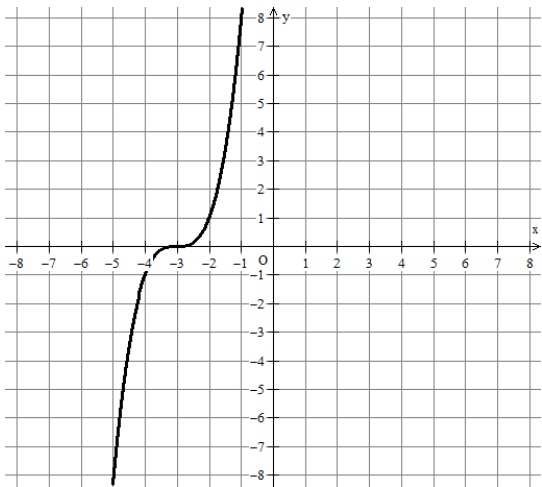
3. An angle  $\theta$  in standard position passes through a point  $P(x, y)$  in the coordinate plane.

Complete the table below.

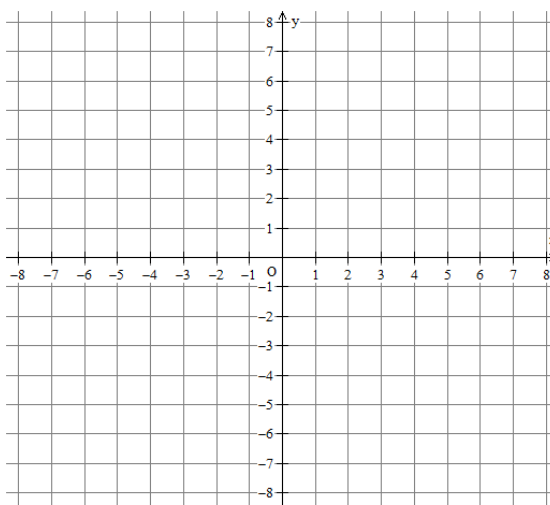
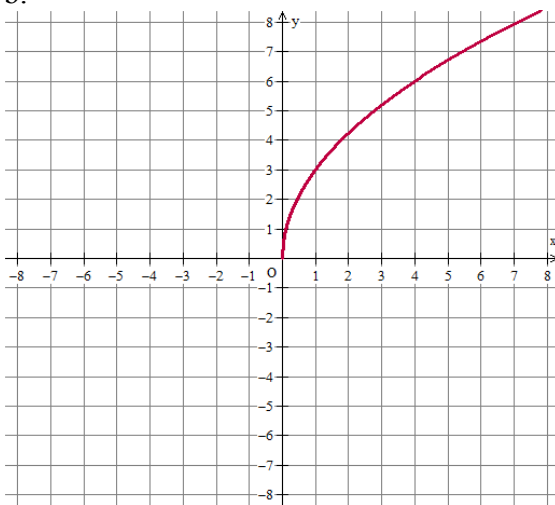
| Values of $x$ and $y$            | Quadrant of the Angle $\theta$ | Sign of $\sin \theta$ | Sign of $\cos \theta$ |
|----------------------------------|--------------------------------|-----------------------|-----------------------|
| $x$ is positive, $y$ is positive |                                |                       |                       |
| $x$ is positive, $y$ is negative |                                |                       |                       |
| $x$ is negative, $y$ is positive |                                |                       |                       |
| $x$ is negative, $y$ is negative |                                |                       |                       |

4. For each graph below, sketch the inverse function on the graph to its right.

a.

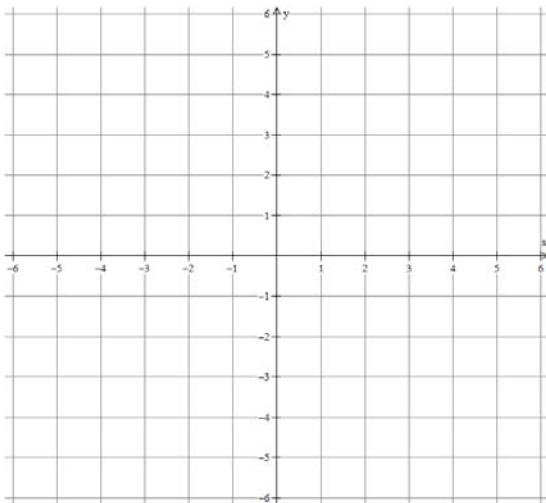


b.

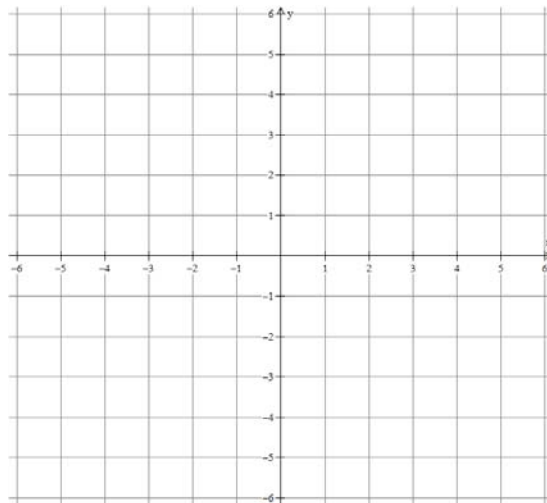


5. Graph the following.

a.  $y = 3 + \frac{1}{x-1}$



b.  $y = -2 + \frac{1}{x-3}$

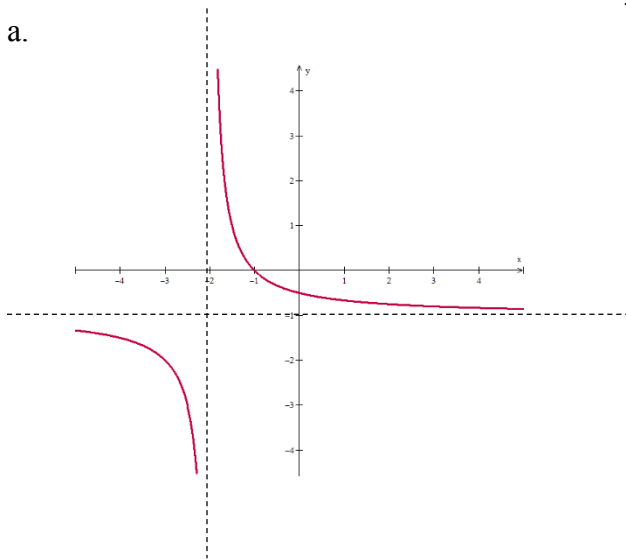


6. Let  $f(x) = x^2 + 2x - 7$ ,  $g(x) = 2x^2 + 9$ , and  $h(x) = 5x + 3$ .

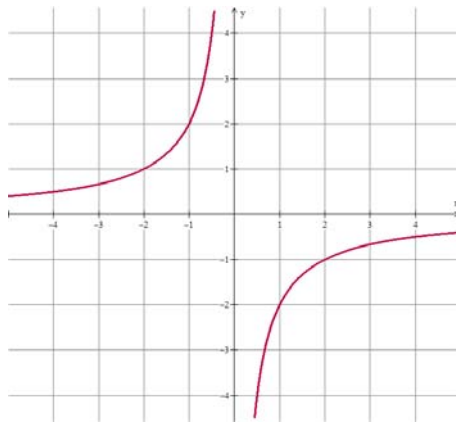
- Write  $(f + g)(x)$  as a polynomial in standard form.
- Write  $(f - g)(x)$  as a polynomial in standard form.
- What is the degree of the product  $(f \cdot g)(x)$ ?
- What is the degree of the product  $(g \cdot h)(x)$ ?
- Write  $(f \cdot h)(x)$  as a polynomial in standard form.
- Write  $(f \cdot g)(x)$  as polynomial in standard form.

7. The graphs below are transformations of  $y = \frac{1}{x}$ . Write the equation for each graph.

a.



b.



8. An angle  $\theta$  is in quadrant 3 with  $\sin \theta = -\frac{5}{13}$ . What is the value of  $\cos \theta$ ?

9. Let  $f(x) = \cos x$ . The four graphs described below are transformation of function  $f$ . Write the function equation for each.

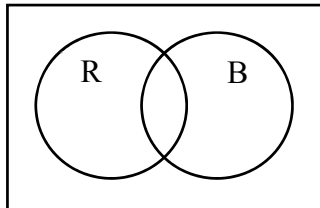
- The graph of function  $g$  is the result of the graph of function  $f$  being dilated vertically by a factor of three.  $g(x) = \underline{\hspace{2cm}}$
- The graph of function  $h$  is the result of the graph of function  $f$  being translated to the left four units and up six units.  $h(x) = \underline{\hspace{2cm}}$
- The graph of function  $k$  is the result of the graph of function  $f$  being dilated horizontally by a factor of 7.  $k(x) = \underline{\hspace{2cm}}$ .
- The graph of function  $p$  is the graph of function  $f$  being dilated horizontally and vertically by a factor of  $\frac{1}{2}$ .  $p(x) = \underline{\hspace{2cm}}$

10. In the Venn Diagrams below, the events shown are

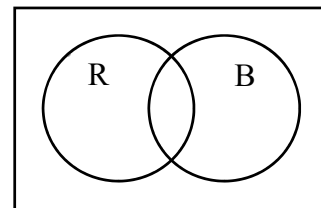
$R$ : A person who likes to run and  $B$ : A person who likes to bike.

In each situation below, shade in the appropriate areas.

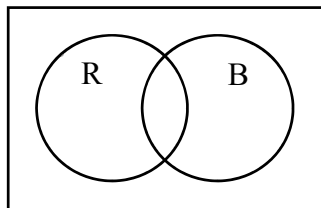
- a) People who like to ride and like to bike.



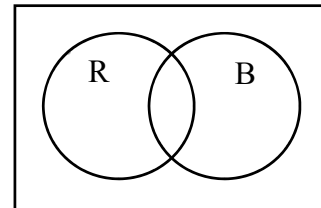
- b) People who like to ride or like to bike.



- c) People who do not like to ride.



- d) People who like to ride, but do not like to bike.



11. One-hundred people (male and female) were asked which of two sports they preferred (hockey or basketball). The results are shown in the two-way table below.

|            | Hockey (H) | Basketball (B) | Total |
|------------|------------|----------------|-------|
| Male (M)   | 14         | 56             | 70    |
| Female (F) | 6          | 24             | 30    |
| Total      | 20         | 80             | 100   |

One person is selected at random from this group. Determine the following. Prove answers in the form of a fraction.

- The probability that the person likes basketball.
- The probability that the person is female.
- The probability that the person prefers hockey and is male.
- The probability that the person is female or prefers hockey.
- Given that the person selected is male, the probability that the person prefers hockey.
- Given that the person selected prefers basketball, the probability that the person is female.

12. The life span of a certain insect is normally distributed has a mean life span of 14 days and a standard deviation of 2 days.

- What percentage of the population has a life span between 12 and 16 days?
- What percentage of the population has a life span greater than 18 days?
- What percentage of the population has a life span between 14 and 20 days?
- What percentage of the population has a life span less than 12 days?

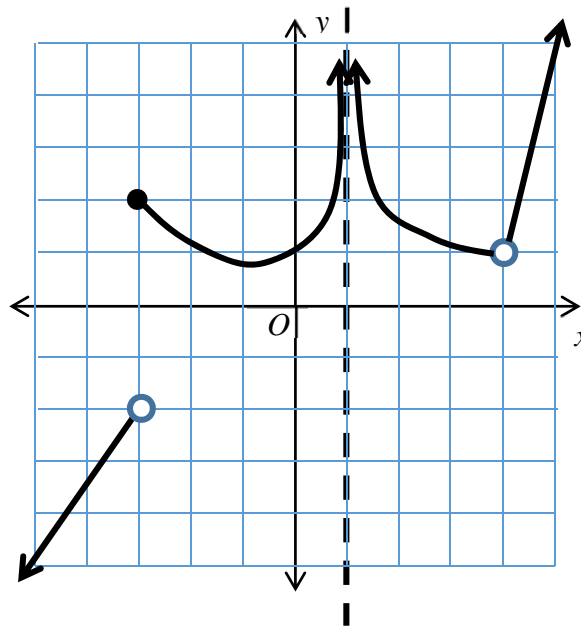
13. Which of the following describes the right-end behavior of the function  $f(x) = \frac{3}{x^2}$ ?

- |          |  |          |   |
|----------|--|----------|---|
| <b>A</b> | $\lim_{x \rightarrow \infty} f(x) = -\infty$ | <b>B</b> | $\lim_{x \rightarrow \infty} f(x) = 0$      |
| <b>C</b> | $\lim_{x \rightarrow \infty} f(x) = 3$       | <b>D</b> | $\lim_{x \rightarrow \infty} f(x) = \infty$ |

14. Look at the graph of the piece-wise function below.

Which type of discontinuity does the graph have at the following  $x$ -values?

- a.  $x = -3$
- b.  $x = 1$
- c.  $x = 4$



15. Given the points  $A(2, -1)$  and  $B(0, 4)$ , what is  $\overline{AB}$  in component form?

16. Look at the vectors  $\vec{u}$  and  $\vec{v}$  below.



a) Represent graphically the vector  $\vec{u} + \vec{v}$  using the head-to-tail and parallelogram methods.

b) Represent the vector  $\vec{u} - \vec{v}$  graphically.

17. For items a through c, use the vectors  $\vec{a} = \langle 2, -3 \rangle$ ,  $\vec{b} = \langle 6, 5 \rangle$ , and  $\vec{c} = \langle r, -9 \rangle$ .

a) Find the measure of the angle between  $\vec{a}$  and  $\vec{b}$  to the nearest tenth of a degree.

b) Find the value of  $r$  such that the vectors  $\vec{a}$  and  $\vec{c}$  are parallel.

c) Find the value of  $r$  such that the vectors  $\vec{b}$  and  $\vec{c}$  are perpendicular.

18. How many different pizzas can be made using 3 toppings from a list of 8 available toppings?

19. How many different ways are there to arrange 5 photographs on a shelf?

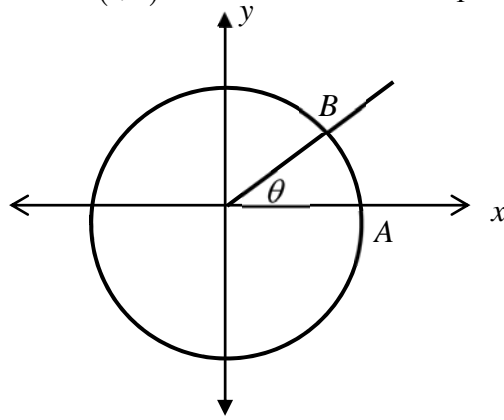
20. Expand the binomial:  $(x-2)^4$

21. What is the coefficient of  $x^3$  in the expansion of  $(x+2)^5$ ?

22. What is the coefficient of the  $x^9y$  term in the expansion of  $(x^3+5y)^4$ ?



23. On the unit circle below the coordinates of point  $A$  are  $(1,0)$  and the coordinates of point  $B$  are  $(0.8,0.6)$ . Find the value of the following.



a.  $\sin \theta$

b.  $\cos \theta$

c.  $\tan \theta$

24. What is the value of  $b$  such that  $y = \cos(bx)$  has a period of  $\frac{\pi}{3}$ ?

25. Determine the exact value of the following.

a.  $\sin\left(\frac{\pi}{6}\right)$

b.  $\cos\left(\frac{5\pi}{4}\right)$

c.  $\cos\left(\frac{5\pi}{6}\right)$

d.  $\sin\left(\frac{3\pi}{2}\right)$

e.  $\cos(\pi)$

f.  $\tan\left(\frac{\pi}{2}\right)$

g.  $\sin\left(\frac{\pi}{4}\right)$

h.  $\cos\left(-\frac{4\pi}{3}\right)$

26. Determine the exact value of the following.

a.  $\text{Sin}^{-1}\left(\frac{1}{2}\right)$

b.  $\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

c.  $\text{Sin}^{-1}(-1)$

d.  $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

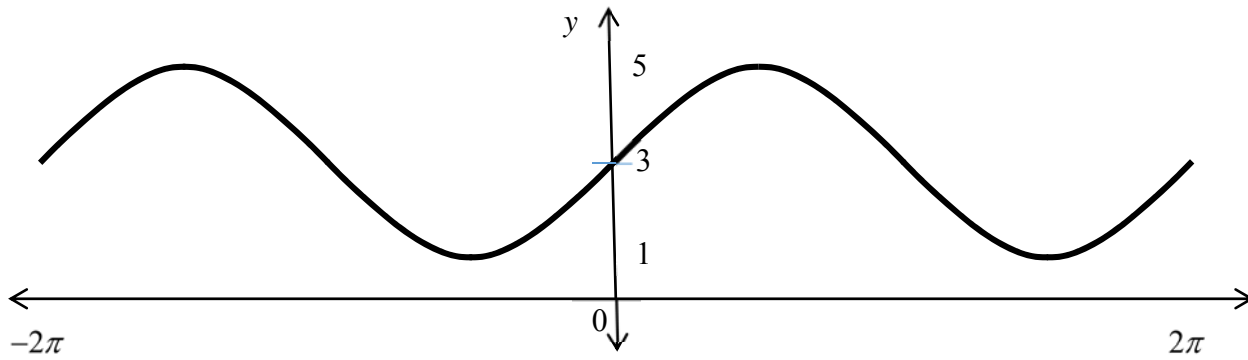
e.  $\text{Cos}^{-1}\left(\frac{1}{2}\right)$

f.  $\text{Cos}^{-1}(0)$

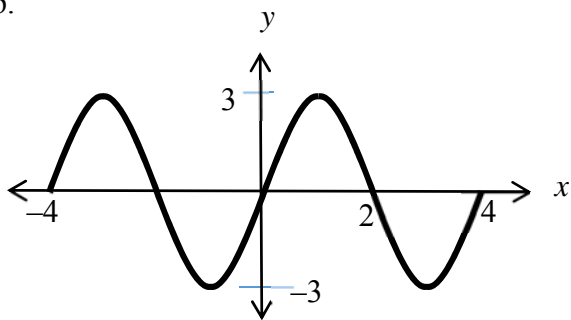
27. Determine the equation that best describes a sine curve with amplitude 3, period of 6, and a phase shift of  $\frac{\pi}{2}$  to the right.

28. Write a sinusoidal equation for each of the following graphs.

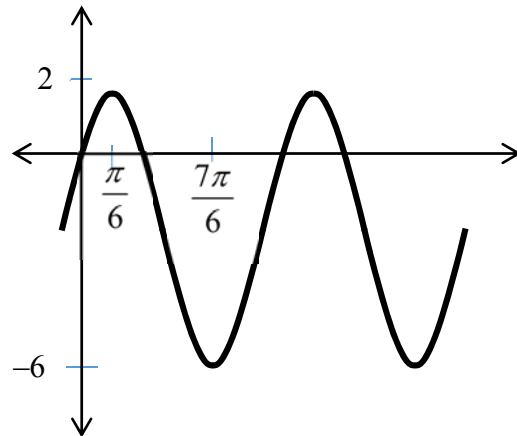
a.



b.



c.



29. Which of the following is equivalent to  $3 \ln x + \ln z - \ln y$ ?

**A**  $3 \ln \left( \frac{xz}{y} \right)$

**B**  $\ln \left( \frac{xz}{y} \right)^3$

**C**  $\ln \left( \frac{x^3 z}{y} \right)$

**D**  $\ln \left( \frac{(xz)^3}{y} \right)$

30. Which of the following is equivalent to  $\frac{\log 20}{\log 2}$ ?

**A** 1

**B**  $\log 18$

**C**  $\log_2 20$

**D**  $\log_{20} 2$

31. For the function  $g(x) = -\ln(x+2) - 5$ ,
- What are the domain and range?
  - What are the equations of any asymptotes?
  - Describe the transformations of the graph of  $f(x) = \ln x$  that results in the graph of  $g(x)$ ?

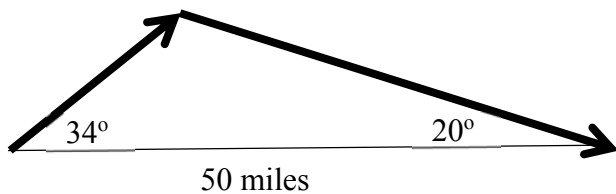
32. Which of the following is equivalent to  $\log_5\left(\frac{5}{x^3}\right)$ ?

- |          |                 |          |                 |
|----------|-----------------|----------|-----------------|
| <b>A</b> | $5 - 3\log_5 x$ | <b>B</b> | $1 - 3\log_5 x$ |
| <b>C</b> | $-3\log_5 x$    | <b>D</b> | $2\log_5 x$     |

33. Which of the following is equivalent to  $\log\left(\frac{y^2}{\sqrt{x}}\right)$ ?

- |          |  |          |                                    |
|----------|--|----------|------------------------------------|
| <b>A</b> | $2\log y - 2\log x$                        | <b>B</b> | $2\log y - \frac{1}{2}\log x$      |
| <b>C</b> | $\log(2y) - \log\left(\frac{1}{2}x\right)$ | <b>D</b> | $\frac{\log(y^2)}{\log(\sqrt{x})}$ |

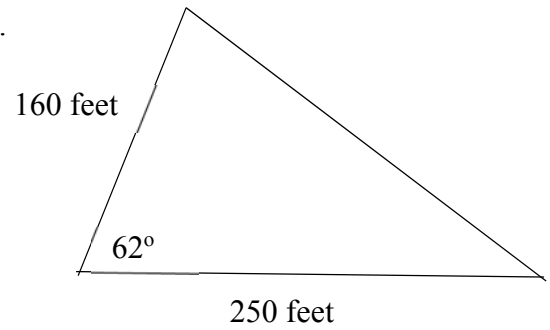
34. An airplane needs to take a detour around a group of thunderstorms, as shown in the figure below. How much farther does the plane have to travel due to the detour? Your answer should be correct to three places after the decimal point.



35. Determine the area of triangle  $ABC$  if  $a = 4$ ,  $b = 10$ , and  $m\angle C = 30^\circ$ .

36. A real estate appraiser wishes to find the value of the lot below.

a. Determine the length of the third side of the lot.



b. Find the area of the lot. Your answer should be correct to three places after the decimal point.

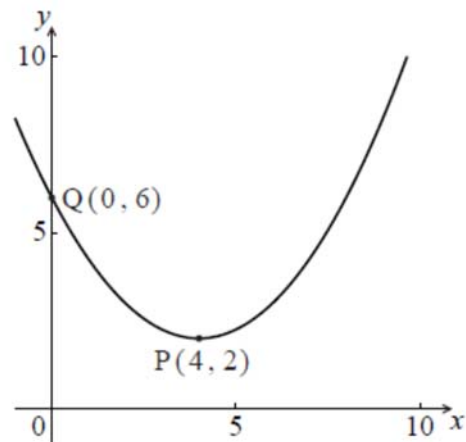
37. In  $\triangle ABC$ ,  $a = 9$ ,  $b = 12$ ,  $c = 16$ . What is the measure of  $\angle B$ ? Your answer should be correct to the nearest tenth of a degree.

38. From a point 200 feet from its base, the angle of elevation from the ground to the top of a lighthouse is 55 degrees. How tall is the lighthouse? Your answer should be correct to three places after the decimal point.

## Sample IB Style Questions:

39. Let  $f$  be a quadratic function. Part of the graph of  $f$  is shown below. The vertex is at  $P(4, 2)$  and the  $y$ -intercept is at  $Q(0, 6)$

- a) Write down the equation for the axis of symmetry.



The function  $f$  can be written in the form:  $f(x) = a(x - h)^2 + k$

- b) Write down the values of  $h$  and  $k$ .

- c) Find  $a$ .

40. The first three terms of an infinite geometric sequence are 32, 16, and 8.

- a) Write down the value of  $r$ .

- b) Find  $u_6$ .

- c) Find the sum to infinity of this sequence.

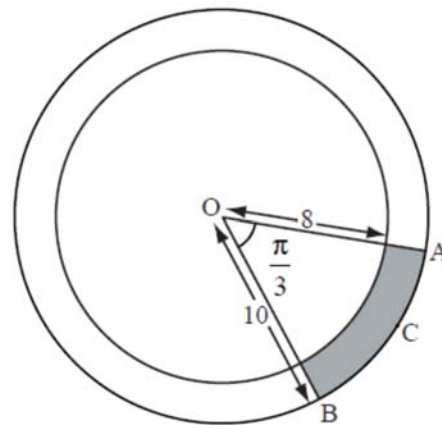
41. Use trigonometry identities to complete the problem below.

a) Show that  $4 - \cos 2\theta + 5 \sin \theta = 2 \sin^2 \theta + 5 \sin \theta + 3$ .

b) Hence, solve the equation  $4 - \cos 2\theta + 5 \sin \theta = 0$  for  $0 \leq \theta \leq 2\pi$ .

42. The diagram shows two concentric circles with centre O. The radius of the small circle is 8cm and the radius of the larger circle is 10cm. Points A, B, and C are on the circumference of the larger circle such that  $\angle AOB$  is  $\frac{\pi}{3}$  radians.

a) Find the length of the arc ACB.



*diagram  
not to scale*

b) Find the area of the shaded region.