

Calculus Summer Review Packet 2019

Your name: _____

The problems in this packet are designed to help you review topics that are important to your success in calculus. The skills and knowledge required for these problems are considered prerequisites. We will not reteach these concepts.

If the topics are unfamiliar to you, you need to do work on your own over the summer to catch up. Here are some tips if you are in need of help:

- Topics to enter into an Internet search engine are **bolded** at the start of each section of problems.
- Kahn Academy has excellent video tutorials.
- Purple math is another site that is very useful

In general, you must know how to do all these problems WITHOUT a calculator. One exception is Section 13 which is meant to show you useful calculator tricks that may be unfamiliar to you.

- **This packet will be collected and graded on DAY 1 of school.**
- **Work should be complete and correct.**
- **The packet will be graded for both completion and accuracy.**
- **Your first test grade for the quarter will be on this material.**

Section 1 **Factor polynomials** and simplify. Show the work that leads to your answer.

1. $\frac{x-3}{x^2+x-12}$

2. $\frac{x^3+8}{x+2}$

3. $\frac{6-x}{x^2-36}$

Section 2 **Reduce complex fractions** and simplify each expression in order to obtain a single fraction.

4. $\frac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

5. $\frac{5}{x+h} - \frac{5}{h}$

6. $\frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

Section 3 **Trig Identities**

7. $\sin^2 x + \cos^2 x =$ _____ 8. $\tan^2 x + 1 =$ _____ 9. $\cot^2 x + 1 =$ _____

10. $\sin 2x =$ _____ 11. $\cos 2x =$ _____

Section 4 **Simplifying radicals, fractions, logarithms and exponents.** True or false. If **false**, put write a correct right-hand side. If the expression does not simplify, rewrite the same expression on the left.

12. $\sqrt{4x^2 + 25} = 2x + 5$

13. $\sqrt[3]{\frac{27x^4}{8}} = \frac{3x^{4/3}}{2}$

14. $\frac{x^2}{2x-5} = \frac{x^2}{2x} - \frac{x^2}{5} = \frac{x}{2} - \frac{x^2}{5}$

15. If $\ln y = 4 + \ln 6$, then $y = e^4 + e^{\ln 6}$

Section 5 **Apply rules of exponents** to simplify expressions.
Do **NOT** leave negative exponents in an answer.

16. $x^{2/3}(x^2 - x + 3x^{1/2})$

17. $\frac{\sqrt[5]{x^2}}{x^2}$

18. $e^{4 \ln x}$

19. $e^{5 + \ln x}$

20. $(5a^{2/3})^3$

21. $-3x^{-3}$

22. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

23. $27^{2/3}$

Section 6 Solving algebraic equations. Topics: factoring, quadratic formula, trigonometry, absolute value, inequalities, exponentials, and logarithms. Solve for x.

24. $3y + 5xz = 0$

25. $7y^2 + 4xz - 6y - x = 8$

26. $\frac{x^2 - 2x - 8}{x + 2} = 0$

27. $x^2 - 2x - 30 = 5$

28. $(x - 3)^2 = 25$

29. $2x^2 + 7x = 3$

30. $16x^2 = 2x$

31. $x^2 \leq 1$

32. $\sin 2x = \sin x, 0 \leq x \leq 2\pi$

34. $\ln(2x + 5) = \ln 3 + \ln(x - 1)$

34. $\log_3 x + \log_3 4 = 2$

35. $\cos x = \frac{1}{2}, 0 \leq x \leq 2\pi$

36. $x^2 + 2x - 15 < 0$

37. $\left| \frac{x^2 - 2}{5} \right| < 1$

38. $\cos^2 x = \cos x, 0 \leq x \leq 2\pi$

39. $2\sin x + \sqrt{3} = 0, 0 \leq x \leq 2\pi$

Section 7 Miscellaneous. Follow the directions for each problem.

40. **Evaluate and simplify composite functions.** If $f(x) = 4x^2 - 2x + 3$, evaluate $f(x + h)$

41. **Expand and simplify functions in sigma notation:** $\sum_{n=1}^4 \frac{n^2}{3}$

42. An **even function** is symmetric about the _____.

 An **odd function** is symmetric about the _____.

Section 8 **Write the equations of the lines.**

43. slope of -2, containing the point (-3, 1)
44. containing the points (4,-2) and (-3,0)
45. slope of 0, passing through the point (-5,1)
46. undefined slope, passing through the point (2,7)
47. perpendicular to the line $2y - 3x = 8$, passing through (-1,3)

Section 9 **Find the domain of the following rational functions.**

48. $f(x) = \frac{3x-2}{4x+1}$

49. $f(x) = \frac{x^2 - 5x - 6}{x^2 - 3x - 18}$

50. $f(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

51. $f(x) = \log(2x - 12)$

Section 10 Find the equation(s) of the **horizontal and vertical asymptotes** for each rational function.

52. $y = \frac{2x}{x-3}$

53. $y = \frac{x+4}{x^2-1}$

Section 11 **Unit Circle Values.** Determine the exact value of each expression **without a calculator**. Be mindful of restrictions on Capitalized Arc Trig functions.

54. $\sin \frac{2\pi}{3}$

55. $\cos \frac{7\pi}{6}$

56. $\tan 270^\circ$

57. $\cot \frac{3\pi}{4}$

58. $\sec 135^\circ$

59. $\csc \frac{7\pi}{4}$

60. $\text{Arcsin}\left(-\frac{1}{2}\right)$

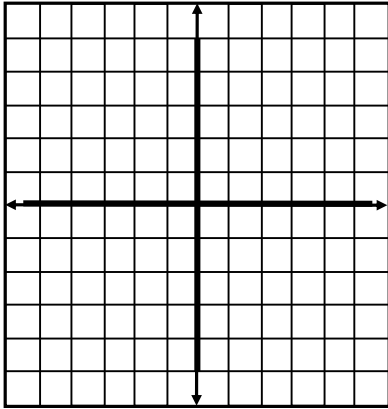
61. $\text{Arc tan}(-\sqrt{3})$

62. $\text{Arccos}\left(\frac{\sqrt{2}}{2}\right)$

Section 12

Graphs. The following are graphs you will need to know in calculus without a calculator. Sketch the graph of each function and state its domain and range. Determine and label an appropriate scale for each graph. (Check on Desmos graphing calculator if you are unsure.)

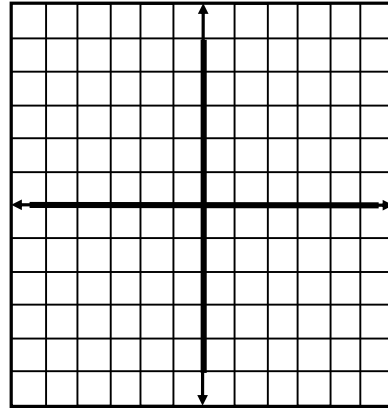
63. $y = \sin x$



Domain _____

Range _____

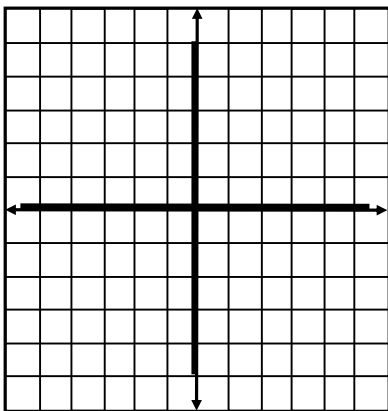
64. $y = \cos x$



Domain _____

Range _____

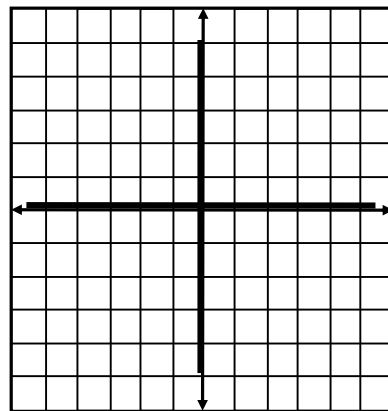
65. $y = \sqrt{x}$



Domain _____

Range _____

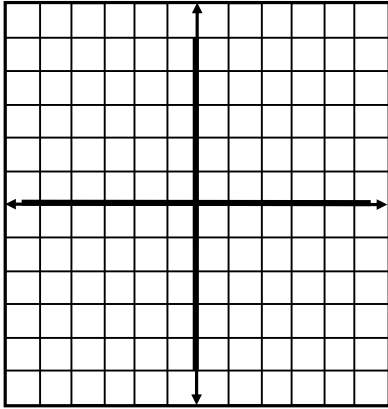
66. $y = e^x$



Domain _____

Range _____

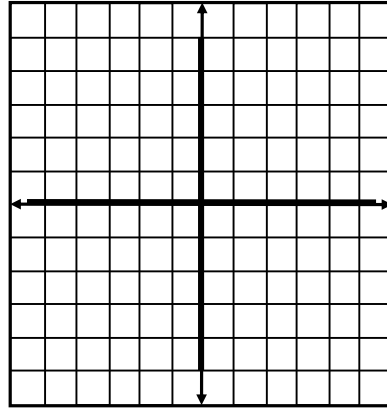
67. $y = \ln x$



Domain _____

Range _____

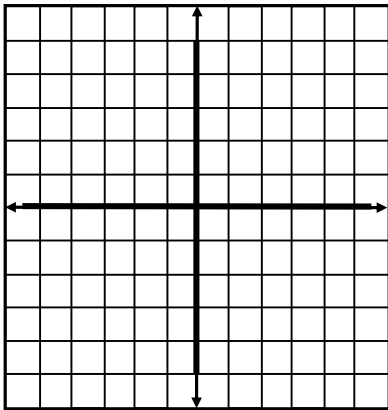
68. $y = \frac{1}{x}$



Domain _____

Range _____

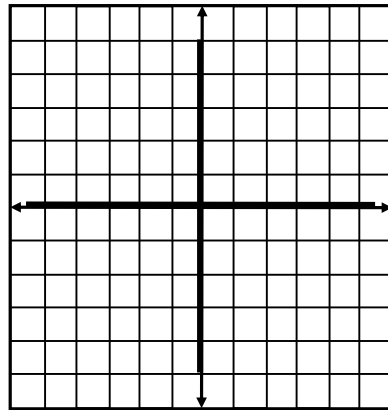
69. $y = \text{Arc tan } x$



Domain _____

Range _____

70. $y = \begin{cases} 3x-1, & x < -1 \\ x^2, & -1 \leq x < 2 \\ 4, & 2 < x \leq 5 \end{cases}$



Domain _____

Range _____

Section 13 How to use the graphing calculator.

The following are the uses of the calculator in AP Calculus:

- 1) plot the graph of a function within an arbitrary viewing window.
- 2) find the zeros of functions (solve equations numerically).
- 3) numerically calculate the derivative of a function.
- 4) numerically calculate the value of a definite integral.

If a calculation is given as a decimal approximation, it should be correct to three places after the decimal point unless otherwise indicated. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology; for example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

1. *Solving for zeroes, storing values, and evaluating several values in the same equation.*

Given $f(x) = 2x^3 - 4x + 1$, find the value(s) of x such that $f(x) = 0$. Store each value in your calculator and evaluate $g(x) = \frac{1}{2}x^4 - 2x^2 + x$ for each zero of $f(x)$.

Steps:

1. Type the equation for $f(x)$ into Y_1 .
2. Zoom 6 (standard 10 by 10 window)
3. You will see three zeroes. Hit '2nd' 'TRACE' and choose '2: zero'.
4. To calculate the one furthest to the left (which appears to occur between -2 and -1), you will be asked to give a 'LEFT BOUND'. Either scroll using the left and right scroll buttons OR just type in an x -value to the left of the zero but reasonably close (and definitely on the viewing window). Then you will be asked for a 'RIGHT BOUND'. Do the same but be careful not to choose a # to the right of the next zero. You will be asked to 'GUESS'. Just hit 'ENTER'. You should get 'X=-1.525687'.
5. Hit '2nd' 'MODE' to return to the home screen. Type 'X', 'STO →', 'ALPHA', 'A'. Your value is now stored.
6. Repeat steps 2-5 for the next two zeroes. However, in step 5, store the second zero to 'B' and the third to 'C'. (The 2nd zero is .25865202 and the 3rd is 1.2670351)

There are three ways to calculate $g(x)$ at each of the values you just found.

1st method:

You're done with $f(x)$ so go back to Y_1 and type in $g(x)$.

1. Type 'VARS', scroll over to 'Y-VARS' and choose '1:Function'. Hit '1' or 'ENTER', the type '(A)' and you should get -3.471986531.
2. Do the same for B and C. You should get .1270881481 and -.6551016167, respectively.

Acceptable answers: 1) (-1.525, -3.471) OR (-1.526, -3.472) OR (-1.525, -3.472) OR (-1.526, -3.471)
2) (.258, .127) OR (.259, .127)
3) (1.267, -.655)

2nd method:

1. Hit 'ZOOM 6'.
 2. Hit '2nd', 'TRACE' and choose '1:VALUE'
 3. You will be prompted to enter an x-value. Type in 'A'. You will get the x-and y-coordinates which are your answers.
 4. Repeat step 2 and 3 for B and C.
- **If one of your x-values is outside of the viewing window, you will get an error. For instance, $x=11$ will give you an error. Simply change your window to go beyond 11 (setting the Xmax to 12 would work).

3rd method:

1. Hit '2nd', 'WINDOW' and change the 'Indpnt' to 'Ask'.

```
TABLE SETUP
TblStart=0
ΔTbl=1
Indent: Auto [ON]
Depend: AUTO Ask
```

2. Hit '2nd', 'Graph' and enter A, B, and C.

X	Y ₁
-1.526	-3.472
.25865	.12709
1.267	-.6551

X=

2. Solving equations

If you need to solve an equation like: $5x^4 - 2x = \cos x$ on the interval $[0, 3]$, set the equation equal to zero.

1. You can enter this into $Y_1 = 5x^4 - 2x - \cos(x)$
2. Follow the steps for 'solving for zeroes' from the 1st example. You should get $x = .826$

3. Using parenthesis

If you need to divide 20 by 3π , do NOT type in $20/3\pi$. This will use the order of operations and divide 20 by 3 THEN multiply by π . Rather, type in $20/(3\pi)$.

Now you try:

71. Given $f(x) = 2x^3 - 6x + 2$, find the value(s) of x such that $f(x) = 0$. Store each value in your calculator and evaluate $g(x) = \frac{1}{2}x^4 - 3x^2 + 2x$ for each zero of $f(x)$. List your answer(s) correct to three decimal places.
72. Solve for x : $x^3 - 3x = \ln x$. List your answer(s) correct to three decimal places.
73. Evaluate: $\frac{15}{2\pi}$. List your answer correct to three decimal places.

Congratulations! You have finished the calculus summer packet. Please use the space below if you would like to make some comments to your calculus teacher concerning the packet or this upcoming class.