Office of the Superintendent of Schools MONTGOMERY COUNTY PUBLIC SCHOOLS Rockville, Maryland

June 14, 2012

MEMORANDUM

To: Members of the Board of Education

From: Joshua P. Starr, Superintendent of Schools

Subject: Status Update on the Recommendations of the Math Work Group

The purpose of this memorandum is to update the Board of Education (Board) on changes to the mathematics program that align with recommendations of the K–12 Mathematics Work Group Report presented to the Board on November 9, 2010. Convened in January 2009, the work group was established to address factors such as the scope of the curriculum, pacing of instruction, professional development, and the emphasis on acceleration. Additionally, despite a strong record of mathematics achievement for all students—with increases in performance for all student groups—the gap between Black or African American and Hispanic/Latino students, and their White or Asian counterparts continued as illustrated in Attachment A.

Recognizing that strong schools are the pathway to our nation's long-term economic success, Maryland and 44 other states adopted the internationally driven Common Core State Standards (CCSS) in 2010. The CCSS are a set of standards in mathematics and English/language arts designed through a state-led initiative to prepare students to be college and/or career ready by the end of Grade 12. Subsequently, the Board adopted the CCSS, which have been incorporated into Montgomery County Public Schools' (MCPS) Grades K–12 curriculum frameworks in mathematics and English/language arts. The implementation of the CCSS in Maryland will include new assessments developed by the Partnership for Assessment of Readiness for College and Careers (PARCC) tentatively scheduled for statewide administration in school year 2014–2015.

The K-12 Mathematics Work Group Report recommended aligning mathematics with the CCSS to ensure a focused, coherent, and rigorous curriculum. The CCSS articulates what students need to know and be able to do, by focusing on the development of deep mathematical proficiency through *mathematical practices*.

The work group report proposed a vision for mathematics classrooms in MCPS that promotes deep mathematical understanding. In this environment—

- students are fluent and resourceful problem solvers working together;
- the curriculum offers students multiple opportunities to learn challenging mathematics;
- there are ambitious expectations for all students, including those who are exceptionally talented in mathematics:
- highly effective teachers have the resources and support to expertly engage students with the mathematics curriculum;
- technology is used to support and engage students as an essential component of the teaching and learning environment;
- the learning needs and diverse backgrounds of all students are supported through differentiated instruction; and
- students confidently engage in complex mathematical tasks chosen carefully by teachers.

The K-12 Mathematics Work Group Report recommended actions in five areas—written curriculum, implemented curriculum (instructional practices), assessed curriculum, targets and acceleration, and teacher preparation and development—that taken together, promote the realization of our vision for mathematics classrooms in MCPS.

Implementing the Vision

During the past three years, and concurrent with the adoption of the CCSS, MCPS developed Curriculum 2.0 for kindergarten–Grade 3 using the CCSS in mathematics, English/language arts, and writing. Implementation of Curriculum 2.0 provides a window into the mathematics classroom envisioned by the mathematics work group and highlights implications of transitioning to the CCSS content and mathematical practices. Implications include increasing teachers' content knowledge, changing instructional practices, designing and implementing new assessments, providing acceleration and remediation, setting appropriate targets, and providing comprehensive professional development to employees throughout the system including teachers, administrators, and support staff. Significant work is ongoing to ensure that Curriculum 2.0 is aligned with the recommendations in the K–12 Mathematics Work Group Report while meeting the requirements of the CCSS.

Written Curriculum—Guiding the recommendations for the written curriculum, the mathematics work group envisioned that the curriculum should be rigorous, coherent, comprehensive, and aligned with the CCSS. Currently, written curriculum has been developed for K–3 and is under development for Grades 4, 5, and 6. The resulting written curriculum developed from the CCSS in mathematics is substantially more challenging than the 2001 MCPS curriculum based on the Maryland State Curriculum.

MCPS 2001 Curriculum Framework/ Maryland State Curriculum	Curriculum 2.0/ Internationally Driven CCSS
Perceived as a collection of skills and	Carefully designed progression of knowledge
processes.	that builds deep conceptual understanding of
	mathematics.
Design led to a classroom emphasis on	Design requires focused instruction. Focus is
teaching isolated procedures.	defined as the balance among concepts,
	procedure, and application.
"Spiral" curriculum—Concepts introduced,	Concept introduced and mastered in one grade
reintroduced, and mastered over several grade	level, then deepened with a new concept in the
levels.	next grade.
Grade-level standards not sufficiently	Many standards are "pulled down" from upper
challenging for some students.	grades.
Language of standards was general and not	Fewer, higher, clearer standards provide better
specific—resulting in teaching to test.	direction for teaching and learning.
Curriculum developed independently of all	Elementary curriculum integrated with other
other subjects.	content areas and thinking and academic skills.

Mathematics in Curriculum 2.0 is more focused, with fewer topics taught at each grade level, allowing more time to develop mastery across all five strands of mathematical proficiency—Understanding, Computing, Applying, Reasoning, and Engaging. The focus of the CCSS allows students to construct their understanding of mathematics over time, dig deeper into concepts and procedures, and make meaningful connections. Curriculum 2.0 fosters students understanding of concepts and procedures to develop a deep and lasting knowledge of mathematics.

Implemented Curriculum—The mathematics work group articulated that instructional strategies, technology, and school structures are all important components of effective mathematics teaching and learning. Moreover, the work group found that teacher competency is the key to successful curriculum implementation. The mathematical practices that support student attainment of the CCSS describe content expertise that all teachers must elicit from students to develop mathematical competence and demonstrate their understanding. Great teachers use these practices in their classroom every day to engage students in learning mathematics on a deep conceptual level. Dr. Francis (Skip) Fennell, Professor of Education at McDaniel College and member of the writing team for the CCSS states, "Conceptual understanding is no longer optional." Conceptual understanding requires teachers to see the connections among mathematical proficiency, mathematical practices, and the essential deep understandings. MCPS has designed curriculum resources and professional development that fosters teachers' conceptual understanding (Attachment B). As a result, the mathematical practices are integrated into the design of Curriculum 2.0.

Assessed Curriculum—The paradigm presented in the K-12 Mathematics Work Group Report requires assessment for learning that is used to drive day-to-day instruction (formative) and assessment of learning that is used to measure student understanding at a point in time

(summative). It also requires that teachers use multiple assessment formats and modes, and offer multiple opportunities for students to demonstrate what they know and can do.

Curriculum 2.0 embraces this vision of ongoing formative assessment as the most effective way to build a strong foundation of mathematical understanding. Curriculum 2.0 includes mathematics formative assessment items organized by marking periods and grouped by measurement topics. These formative assessment items also contain examples that show how students may represent their understanding and can be used to measure the depth expected in all five strands of mathematical proficiency. Measures of Academic Progress–Primary Grades (MAP-P) and Measures of Academic Progress–Mathematics (MAP-M) provide opportunities for benchmark assessment *for* learning for students in Grades K–2 and Grades 3–8, respectively. These assessment data will be used to guide instructional planning. Additionally, benchmark assessments provide consistent data for monitoring mathematics in classrooms, schools, and the district.

MCPS is closely monitoring developments at the state and national level as assessments are created to measure the CCSS. Current information from the Maryland State Department of Education indicates that some PARCC test items will be field tested in 2012–2013 and 2013–2014, and may replace the Maryland School Assessment in 2014–2015. As benchmark assessments are implemented and new state measures are put in place, professional development will be required on the effective use of assessment data to plan instruction that builds students' deep mathematical understanding.

Targets and Acceleration—The mathematics work group envisioned that the MCPS mathematics program would be challenging and rigorous for all students and would be taught to mastery. The work group recommended that acceleration be based on the needs of the learner, be supported by data, and be flexible as the student moves through a course or content. The work group found that efforts to increase students' access to higher-level mathematics beginning in the elementary grades was accomplished through the use of acceleration strategies that included skipping units, grade levels, or courses. As a result, not all students were adequately prepared for success in some courses. Additionally, the work group recommended that district targets be aligned with mastery of mathematical proficiency as defined by the CCSS.

As the Curriculum 2.0 team analyzed CCSS at each grade level; received feedback from principals, teachers, and parents; and reviewed student performance, they determined that the CCSS alone would not sufficiently challenge every child. MCPS leadership researched best practices in mathematics acceleration and consulted with authors of the CCSS to develop a model of acceleration and enrichment based on learning progressions, or a careful sequencing of the building blocks that make up deep student understanding of key concepts. By mapping out these building blocks, MCPS created a model that allows teachers to easily determine the next logical experience for a student through acceleration, enrichment, or further instructional support. Since learning progressions are at the foundation of CCSS, the MCPS enrichment and acceleration model shows promise for challenging each child while avoiding gaps in their

understanding that may occur with grade-level acceleration and content skipping. MCPS is committed to challenging every child with this additional acceleration and enrichment model, while recognizing that some students who demonstrate deep mathematical understanding across all five strands of proficiency will need to advance a grade level in mathematics to receive sufficient challenge.

The learning progressions, combined with the Universal Design for Learning architecture of Curriculum 2.0, also will serve students who are struggling in mathematics. By moving down the learning progression in the opposite direction, teachers are able to determine the building blocks a struggling student may be missing in order to reach grade-level proficiency. Close study of this model will be necessary as teachers and students work to implement Curriculum 2.0 over the next few years.

Teacher Preparation and Development—The mathematics work group found that high quality mathematics professional development is job-embedded, data-driven, research based, differentiated, sustained over time, and balanced between content and pedagogy. The professional development vision for Curriculum 2.0 includes the development of key messages for all staff members and a focus on job-embedded collaborative professional development supported by building capacity of instructional leadership and individual staff members. Core team training and job-alike professional development have focused on building the capacity of instructional leaders. Based on feedback from principals and teachers, core team training was expanded to include Grade 3 teachers along with team leaders. Substitute time was allocated for teachers to engage in collaborative instructional planning and curriculum study. In professional learning communities, teachers will collaboratively study short videos, interactive presentations, graphics, and written information to understand the curriculum goals, content, and expectations of Curriculum 2.0.

Challenges

Implementation of Curriculum 2.0 content and *mathematical practices* requires that teachers instruct students in ways that move beyond the techniques they experienced in their own education and perhaps beyond what they currently use in their classroom practice. Our implementation of the CCSS over the past two years has revealed the following challenges that will need to be addressed as we continue to implement mathematics aligned with the CCSS:

- Building the capacity of teachers to develop conceptual understanding in their students across all five areas of mathematical proficiency.
- Building the capacity of instructional leaders, including principals, to promote and monitor strong mathematical teaching practices that promote deep understanding in all students.
- Communicating with stakeholders, including parents, the reason for this significant shift in mathematics teaching and learning and its benefits for students.

• Communicating with stakeholders, including parents, how the vision for mathematics teaching and learning supports and challenges all students and leads to equity in instruction and student achievement.

Next Steps

During the next two years, we will continue to develop and implement mathematics curriculum, instruction, and assessments in anticipation of the 2014–2015 administration of the new state assessments of the CCSS and provide professional development for teachers, instructional leaders, and other support staff members in our schools and offices. Additional actions required to effectively implement our mathematics vision include the following:

- Determine secondary course progressions that build on the elementary curriculum, assessment, and instruction.
- Prepare secondary teachers and instructional leaders, including principals, for implementing mathematics curriculum aligned with the CCSS.
- Review existing resources and develop a budget for full implementation of Pre-K-12 mathematics including resources for professional development and instructional materials.
- Collaborate with the Office of Human Resources and Development regarding hiring of effective mathematics teachers.
- Provide ongoing support and information to schools, offices, and the community to facilitate effective change management during the implementation.
- Continue to monitor the implementation of the CCSS and evaluate the impact on student achievement.

At the table for today's discussion are Mr. Erick J. Lang, associate superintendent, Office of Curriculum and Instructional Programs; Ms. Theresa A. Cepaitis, director, Elementary Integrated Curriculum; Mr. Martin M. Creel, director, Department of Enriched and Innovative Programs; Mr. Edward C. Nolan, supervisor, Pre-K–12 Mathematics; Ms. Rebecca A. Jones, principal, Westbrook Elementary School; Mrs. Tamisha L. Sampson, principal, Sargent Shriver Elementary School; and Mrs. Carole A. Working, principal, Quince Orchard High School.

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Attachments

Montgomery County Public Schools Mathematics Data

Below are data points included in the Montgomery County Public School strategic plan, *Our Call to Action: Pursuit of Excellence*. Systemwide data are collected and included in the *Annual Report on Our Call to Action*. Additionally, several of these data points also are keys in the Seven Keys to College Readiness.

Strategic Plan Data Point and Key 3: Advanced Math in Grade 5—The proficiency of Grade 5 students in Math 6 or higher-level mathematics became a data point in the district's strategic plan, *Our Call to Action: Pursuit of Excellence* in 2006 at which time, systemwide results were collected and reported in the *Annual Report on Our Call to Action*. Since the baseline year of 2001, proficiency rates for students have risen 17.1 percentage points from 31.9 percent in 2006 to 49.0 percent in 2010. African American and Hispanic students have steadily increased achievement, rising 16.6 and 13.7 percentage points, respectively.

Grade 5 Students Proficient in Math 6 or Higher by Racial/Ethnic Group

Grade 5 Student		% Successful	ly Completed	
Racial/Ethnic Group	2001	2008	2009	2010
All Students	31.9	43.1	48.8	49.0
Asian American Students	52.1	64.0	70.0	70.1
African American Students	15.2	25.1	29.0	31.8
White Students	43.3	56.8	64.1	63.3
Hispanic Students	13.4	22.8	27.3	27.1

Strategic Plan Data Point and Key 4: Algebra 1 by Grade 8—Since the baseline year of 2001, Grade 8 algebra or higher-level mathematics completion by all students has increased by 24.7 percentage points from 43.1 percent in 2001 to 67.8 percent in 2010. The rate of increase was 4.6 percentage points greater for African American (29.3 percent) and 7.7 percentage points greater for Hispanic students (32.4 percent).

Grade 8 Algebra or Higher-Level Mathematics Completion by Racial/Ethnic Group

Grade 8 Student		% Successful	ly Completed	•
Racial/Ethnic Group	2001	2008	2009	2010
All Students	43.1	59.6	65.5	67.8
Asian American Students	60.6	78.8	84.6	85.3
African American Students	21.2	38.4	46.6	50.5
White Students	55.5	74.7	80.1	82.0
Hispanic Students	16.4	38.8	45.8	48.8

Data Sources:

- 1) K-12 Mathematics Work Group Report (Fall 2010)
- 2) Successful Completion of Algebra 2 with a C or Higher by Grade 11 (October 6, 2011, memorandum to high school principals)

Strategic Plan Data Point: Grade 9 Algebra or Higher-level Mathematics Completion—Since the baseline year of 2001, Grade 9 algebra or higher-level mathematics completion rates have climbed 10 percentage points from 71.5 to 81.5 percentage points. The achievement gap is narrowing, with gains of 23.5 percentage points for both African American and Hispanic students.

Grade 9 Algebra or Higher-Level Mathematics Completion by Racial/Ethnic Group

Grade 9 Student		% Successful	ly Completed	
Racial/Ethnic Group	2001	2008	2009	2010
All Students	71.5	77.0	78.0	81.5
Asian American Students	89.0	88.8	89.5	91.2
African American Students	49.4	65.3	66.6	72.9
White Students	84.3	88.2	89.2	91.8
Hispanic Students	44.2	62.1	63.5	67.7

Strategic Plan Data Point: Grade 10 Geometry Completion—Since the baseline year of 2004, Grade 10 geometry completion rates have risen 9.3 percentage points, from 70.2 percent in 2004 to 79.5 percent in 2010. African American and Hispanic students' completion rates increased 20.2 and 18.5 percentage points, respectively, doubling the rate of increase for all students.

Grade 10 Geometry Completion by Racial/Ethnic Group

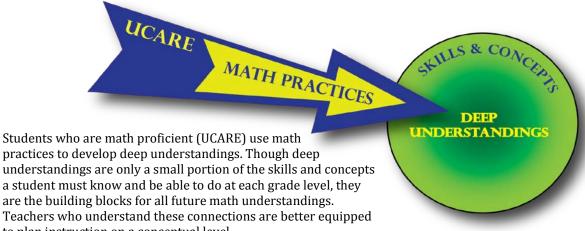
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Grade 10 Student		% Successful	ly Completed	
Racial/Ethnic Group	2004	2008	2009	2010
All Students	70.2	73.8	77.4	79.5
Asian American Students	85.1	87.9	88.2	92.1
African American Students	47.8	55.4	63.2	68.0
White Students	84.8	88.9	91.1	90.8
Hispanic Students	44.0	52.0	59.6	62.5

Strategic Plan Data Point and Key 5: Algebra 2 by Grade 11 —Since the baseline year of 2010, the Grade 11 Algebra 2 successful completion rate for all racial/ethnic groups increased at least 4.0 percentage points.

Grade 11 Algebra 2 Completion by Racial/Ethnic Group

Grade 11 Student	% Successfully Completed		
Racial/Ethnic Group	2010	2011	
All Students	54.0	59.8	
Asian	73.4	78.7	
Black or African American	34.2	39.4	
White	69.6	74.7	
Hispanic/Latino	32.6	37.9	
Two or More Races	61.3	65.3	

The Connections That Lead To Deep Understandings in Math



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Deep Understandings

Kindergarten

- Understand the relationship between numbers and quantities.
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Grade 1

- Understand and apply properties of operations and the relationship between addition and subtraction.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Grade 2

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Grade 3

- Understand properties of multiplication and the relationship between multiplication and division.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.
- Develop understanding of fractions as numbers.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Grade 4

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.
- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.
- Geometric measurement: understand concepts of angle and measure angles.

Grade 5

- Understand the place value system.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.