Sail into Summer with Math!

For Students Entering Algebra 1

This summer math booklet was developed to provide students in kindergarten through the eighth grade an opportunity to review grade level math objectives and to improve math performance.

Student Responsibilities

Students will be able to improve their own math performance by:

- Completing the summer math booklet
- Reviewing math skills throughout the summer**.**

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Student Signature	Grade	Date

Parent Responsibilities

Parents will be able to promote student success in math by:

- Supporting the math goal of the cluster of schools,
- Monitoring student completion of the summer math booklet,
- Encouraging student use of math concepts in summer activities**.**

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Parent Signature	Date

Algebra 1 Summer Mathematics Packet

Table of Contents

Fraction Operations

Hints/Guide:

When adding and subtracting fractions, we need to be sure that each fraction has the same denominator, then add or subtract the numerators together. For example:

$$\frac{1}{8}+\frac{3}{4}=\frac{1}{8}+\frac{6}{8}=\frac{1+6}{8}=\frac{7}{8}$$

That was easy because it was easy to see what the new denominator should be, but what about if it was not so apparent? For example: $\frac{7}{12}+\frac{8}{15}=$

For this example, we must find the Lowest Common Denominator (LCM) for the two denominators 12 and 15.

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, . . .
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, . . .
The LCM of 12 and 15 is 60

So, $\frac{7}{12}+\frac{8}{15}=\frac{35}{60}+\frac{32}{60}=\frac{35+32}{60}=\frac{67}{60}=1\frac{7}{60}$.

Note: Be sure that answers are always in lowest terms

To multiply fractions, we multiply the numerators together and denominators together, and then simplify the product. To divide fractions, we find the reciprocal of the second fraction (flip the numerator and the denominator) and then multiply the two together. For example:

$$\frac{2}{3}\bullet\frac{1}{4}=\frac{2}{12}=\frac{1}{6} \text{ and } \frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\bullet\frac{4}{3}=\frac{8}{9}$$

Exercises: Perform the indicated operation No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $\frac{6}{7}+\frac{2}{3}=$
2. $\frac{8}{9}+\frac{3}{4}=$
3. $\frac{9}{11}-\frac{2}{5}=$
4. $\frac{5}{7}-\frac{5}{9}=$
5. $\frac{6}{11}\bullet\frac{2}{3}=$
6. $\frac{7}{9}\bullet\frac{3}{5}=$
7. $\frac{6}{7}\div\frac{1}{5}=$
8. $\frac{7}{11}\div\frac{3}{5}=$
9. $\left[\frac{2}{3}-\frac{5}{9}\right]\div\left[\frac{4}{7}+\frac{1}{6}\right]=$
10. $\frac{3}{4}+\frac{4}{5}\left[\frac{5}{9}+\frac{9}{11}\right]=$
11. $\left[\frac{3}{4}+\frac{4}{5}\right]\left[\frac{5}{9}+\frac{9}{11}\right]=$

Decimal Operations

Hints/Guide:

When adding and subtracting decimals, the key is to line up the decimals above each other, add zeroes so all of the numbers have the same place value length, then use the same rules as adding and subtracting whole numbers. The answer will have a decimal point in line with the problem. For example:

$$34.5 + 6.72 + 9.045 = \begin{array}{r} 34.5 \\ 6.72 \\ \underline{9.045} \\ 50.265 \end{array}$$

To multiply decimals, the rules are the same as with multiplying whole numbers, until the product is determined and the decimal point must be located. The decimal point is placed the same number of digits in from the right side of the product as the number of decimal place values in the numbers being multiplied. For example,

8.54 · 17.2, since 854 · 172 is 146888, then we count the number of decimal places in the factors (3) and move in from the right three places, so the final product is 146.888

To divide decimals by a whole number, the division process is the same as for whole numbers, but the decimal points are lined up in the dividend and the quotient. For example, to divide 51.06 by 3, the process is the same as if the problem were 5,106 divided by 3, with the decimal point from the quotient moving up into the quotient to create the final answer of 17.02

$$3\overset{\displaystyle 17.02}{\overline{)51.06}}$$

Exercises: Perform the indicated operation — No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. 15.709 + 2.34 + 105.06 =
2. 64.308 + 164.18 + 1005.7 =
3. 87.4 – 56.09 =
4. 500.908 – 4.72 =
5. 6108.09 – 2004.704 =
6. 9055.3 – 242.007 =
7. $\begin{array}{r} 63 \\ \underline{\times\ .04} \end{array}$
8. $\begin{array}{r} .87 \\ \underline{\times\ .23} \end{array}$
9. $\begin{array}{r} 8.904 \\ \underline{\times\ \ 2.1} \end{array}$
10. $\begin{array}{r} 4.2 \\ \underline{\times\ .602} \end{array}$
11. $35\overline{)70.35}$
12. $14\overline{)50.512}$
13. $23\overline{)74.888}$

Add and Subtract Mixed Numbers

Hints/Guide:

When adding mixed numbers, we can add the whole numbers and the fractions separately, then simplify the answer. For example:

$$4\frac{1}{3}+2\frac{3}{4}=4\frac{8}{24}+2\frac{18}{24}=6\frac{26}{24}=6+1\frac{2}{24}=7\frac{2}{24}=7\frac{1}{12}$$

When subtracting mixed numbers, we subtract the whole numbers and the fractions separately, then simplify the answer. For example:

$$7\frac{3}{4}-2\frac{15}{24}=7\frac{18}{24}-2\frac{15}{24}=5\frac{3}{24}=5\frac{1}{8}$$

$$5\frac{1}{4}-3\frac{3}{8}=5\frac{2}{8}-3\frac{3}{8}=4\frac{10}{8}-3\frac{3}{8}=1\frac{5}{8}$$ Note: regrouping needed in order to subtract

Exercises: Solve in lowest terms. No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $3\frac{1}{2}+5\frac{3}{5}=$

2. $6\frac{17}{25}+8\frac{4}{7}=$

3. $6\frac{2}{3}+9\frac{7}{9}=$

4. $8\frac{3}{10}-6\frac{7}{9}=$

5. $9\frac{7}{15}-2\frac{7}{12}=$

6. $12\frac{8}{9}-7\frac{3}{4}=$

Multiply and Divide Mixed Numbers

Hints/Guide:

To multiply mixed numbers, we can first convert the mixed numbers into improper fractions. This is done my multiplying the denominator by the whole number part of the mixed number and them adding the numerator to this product. This sum is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the mixed number. For example: $3\frac{2}{5}$ leads to 3 · 5 + 2 = 17, so $3\frac{2}{5} = \frac{17}{5}$.

Once the mixed numbers are converted into improper fractions, we multiply and simplify just as with regular fractions. For example: $5\frac{1}{5} \bullet 3\frac{1}{2} = \frac{26}{2} \bullet \frac{7}{2} = \frac{182}{10} = 18\frac{2}{10} = 18\frac{1}{5}$

To divide mixed numbers, we must convert to improper fractions then multiply by the reciprocal of the second fraction and simplify. For example: $2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \bullet \frac{3}{10} = \frac{15}{20} = \frac{3}{4}$

Exercises: Solve in lowest terms. No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $6\frac{2}{3} \bullet 7\frac{3}{7} =$

2. $3\frac{1}{3} \bullet 6\frac{4}{5} =$

3. $7\frac{1}{8} \bullet 6 =$

4. $4\frac{1}{4} \div \frac{5}{7} =$

5. $3\frac{2}{3} \div 4\frac{3}{7} =$

6. $\frac{3}{4} \div 2\frac{3}{11} =$

7. $6\frac{1}{5} \div 8\frac{2}{5} =$

8. $8\frac{2}{7} \div 7\frac{8}{9} =$

9. $6\frac{4}{7} \div 3\frac{3}{5} =$

Laws of Exponents

Hints/Guide:

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

Adding powers	$a^m a^n = a^{m+n}$
Multiplying powers	$(a^m)^n = a^{mn}$
Subtracting powers	$\frac{a^m}{a^n} = a^{m-n}$
Negative powers	$a^{-n} = \frac{1}{a^n}$
To the zero power	$a^0 = 1$

Here are some examples of problems simplified using the above powers:

$4^3 \bullet 5^5 = 4^8$ $\quad (4^3)^3 = 4^9$ $\quad 4^5 \div 4^3 = 4^2$ $\quad 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ $\quad 4^0 = 1$

Exercises: Simplify the following problems using exponents (Do not multiply out).

1. $5^2 5^4 =$
2. $7^{-3} 7^5 =$
3. $(12^4)^3 =$
4. $(6^5)^2 =$
5. $5^9 \div 5^4 =$
6. $10^3 \div 10^{-5} =$
7. $7^{-3} =$
8. $3^{-4} =$
9. $124^0 =$
10. $-9^0 =$
11. $(3^5 \bullet 3^2)^3 =$
12. $5^3 \bullet 5^4 \div 5^7 =$

Integers I

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add its additive inverse. For example, 6 – 11 = 6 + -11 = -5

Exercises: Solve the following problems.

1. (-4) + (-5) =
2. –9 – (-2) =
3. 6 - (-9) =
4. (-6) – 7 =
5. 7 – (-9) =
6. 15 – 24 =
7. (-5) + (-8) =
8. –15 + 8 – 8 =
9. 14 + (-4) – 8 =
10. 14.5 – 29 =
11. –7 – 6.85 =
12. –8.4 – (-19.5) =
13. 29 - 16 + (-5) =
14. –15 + 8 – (-19.7) =
15. 45.6 – (-13.5) + (-14) =
16. –15.98 – 6.98 – 9 =
17. –7.24 + (-6.28) – 7.3 =
18. 29.45 – 56.009 – 78.2 =
19. 17.002 + (-7) – (-5.23) =
20. 45.9 – (-9.2) + 5 =

Integers II

Hints/Guide:

The rules for multiplying integers are:

Positive · Positive = Positive Negative · Negative = Positive
Positive · Negative = Negative Negative · Positive = Negative

The rules for dividing integers are the same as multiplying integers

Exercises: Solve the following problems.

1. $4 \cdot (-3) \cdot 6 =$

2. $5\ (-12) \cdot (-4) =$

3. $(4)(-2)(-3) =$

4. $\frac{(-5)(-6)}{-2} =$

5. $\frac{6(-4)}{8} =$

6. $\frac{-56}{2^3} =$

7. $6\ (-5 - (-6)) =$

8. $8\ (-4 - 6) =$

9. $-6\ (9 - 11) =$

10. $\frac{-14}{2} + 7 =$

11. $8 - \frac{-15}{-3} =$

12. $-3 + \frac{-12 \bullet (-5)}{4} =$

13. $\frac{-6-(-8)}{-2} =$

14. $-7 + \frac{4+(-6)}{-2} =$

15. $45 - 14\ (5 - (-3)) =$

16. $(-4 + 7)\ (-16 + 3) =$

17. $16 - (-13)\ (-7 + 5) =$

18. $\frac{4+(-6)-5-3}{-6+4} =$

19. $(-2)^3\ (-5-(-6)) =$

20. $13\ (-9 + 17) + 24 =$

Solving Equations I

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In one-step equations, we merely undo the operation - addition is the opposite of subtraction and multiplication is the opposite of division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side.

Examples:

1. $x + 5 = 6$
   $\underline{-5 \quad -5}$
   $x = 1$
   Check: $1 + 5 = 6$
   $6 = 6$

2. $t - 6 = 7$
   $\underline{+6 \quad +6}$
   $t = 13$
   Check: $13 - 6 = 7$
   $7 = 7$

3. $\frac{4x}{4} = \frac{16}{4}$
   $x = 4$
   Check: $4(4) = 16$
   $16 = 16$

4. $6 \bullet \frac{r}{6} = 12 \bullet 6$
   $r = 72$
   Check: $72 \div 6 = 12$
   $12 = 12$

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $x + 8 = -13$
2. $t - (-9) = 4$
3. $-4t = -12$
4. $\frac{r}{4} = 24$
5. $y - 4 = -3$
6. $h + 8 = -5$
7. $\frac{p}{8} = -16$
8. $-5k = 20$
9. $-9 - p = 17$

Solving Equations II

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In two-step equations, we must undo addition and subtraction first, then multiplication and division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side. Examples:

1. 4x - 6 = -14

   + 6 + 6

   $\frac{4x}{4} = \frac{-8}{4}$

   x = -2

   Solve: 4 (-2) - 6 = -14

   -8 - 6 = -14

   -14 = -14

2. $\frac{x}{-6} - 4 = -8$

   + 4 + 4

   $-6 \bullet \frac{x}{-6} = -4 \bullet -6$

   x = 24

   Solve: (24/-6) - 4 = -8

   -4 - 4 = -8

   -8 = -8

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. -4t - 6 = 22
2. $\frac{m}{-5} + 6 = -4$
3. -4r + 5 = -25
4. $\frac{x}{-3} + (-7) = 6$
5. 5g + (-3) = -12
6. $\frac{y}{-2} + (-4) = 8$

Solving Equations III

Hints/Guide:

When solving equations that include basic mathematical operations, we must simplify the mathematics first, then solve the equations. For example:

$$5(4-3)+7x=4(9-6)$$
$$5(1)+7x=4(3)$$
$$5+7x=12$$
$$-5 \qquad -5$$
$$\frac{7x}{7}=\frac{7}{7}$$
$$x=1$$

Check:
$$5(4-3)+7(1)=4(9-6)$$
$$5+7=4(3)$$
$$12=12$$

Exercises: Solve the following equations using the rules listed on the previous pages:
SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4x + 8 - 6 = 2(9 - 2)$

2. $\frac{t}{5} - 7 + 31 = 8(6 - 4)$

3. $5(t - 4) = 9(7 - 3)$

4. $9 - 5(4 - 3) = -16 + \frac{x}{3}$

5. $6t - 9 - 3t = 8(7 - 4)$

6. $7(6 - (-8)) = \frac{t}{-4} + 2$

7. $7(3 - 6) = 6(4 + t)$

8. $4r + 5r - 6r = 15 + 6$

9. $3(5 + x) = 5(7 - (-2))$

Equations - Variables on Each Side

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division. Remember the golden rule of equation solving. Examples:

$$\begin{array}{rcl} 8x - 6 &=& 4x + 5 \\ -4x & & -4x \\ 4x - 6 &=& 5 \\ +6 & & +6 \\ \frac{4x}{4} &=& \frac{11}{4} \\ x &=& 2\frac{3}{4} \end{array}$$

$$\begin{array}{rcl} 5 - 6t &=& 24 + 4t \\ +6t & & +6t \\ 5 &=& 24 + 10t \\ -24 & & -24 \\ \frac{-19}{10} &=& \frac{10t}{10} \\ -1\frac{9}{10} &=& t \end{array}$$

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4r - 7 = 8r + 13$
2. $14 + 3t = 5t - 12$
3. $4x + 5 = 3x - 3$
4. $6y + 5 = 4y - 13$
5. $5x - 8 = 6 - 2x$
6. $7p - 8 = -4p + 6$

Inequalities

Hints/Guide:

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality as many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

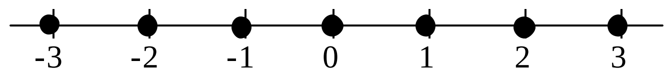
<u>Inequality</u> - a statement containing one of the following symbols:

$<$ is less than　　$>$ is greater than　　$\leq$ is less than or equal to

$\geq$ is greater than or equal to　　_ is not equal to

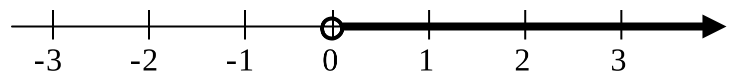
Examples:

1. Integers between -4 and 4.



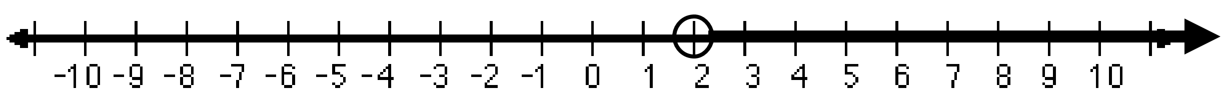
2. All numbers between -4 and 4.

3. The positive numbers.



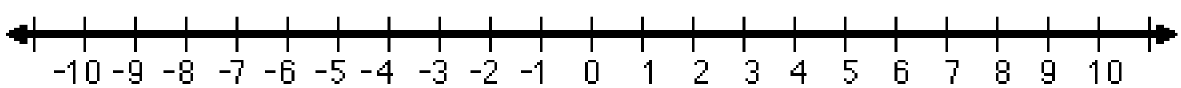
So, to solve the inequality -4x < -8 becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore x > 2 is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:

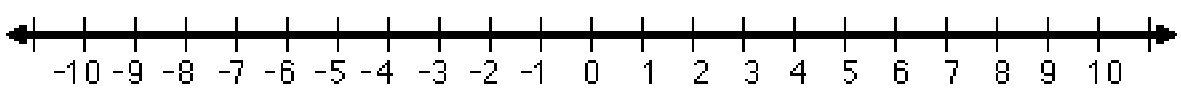


Exercises: Solve the following problems:　　　　No Calculators!

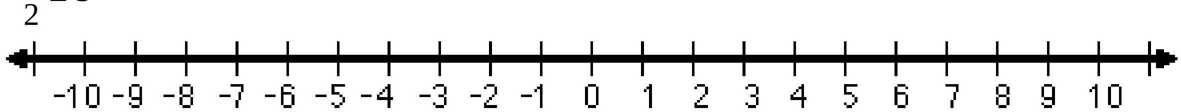
1. 4x > 9



2. -5t $\geq$ -15



3. $\frac{x}{2} \geq 3$



4. $\frac{x}{-4} > 2$