

Framework for mathematics

The framework for MYP mathematics outlines five branches of mathematical study.

- **Number**
- **Algebra**
- **Geometry and trigonometry**
- **Statistics and probability**
- **Discrete mathematics**

Schools can use the framework for mathematics as a tool for curriculum mapping when designing and planning their mathematics courses. Schools are not expected to address all the branches of the framework in each year of the programme nor are they asked to cover every concept and skill suggested in the framework. However, it is important that over the five years (or complete duration) of the programme, students experience learning in all five branches of the framework for mathematics.

In this section, concepts and skills for each of the five branches of the framework for mathematics are suggested. The concepts and skills are **examples** of what students may expect to study at the two levels: standard mathematics and extended mathematics.

Concepts and skills

Number

The ability to work with numbers is an essential skill in mathematics. Students are expected to have an understanding of number concepts and to develop the skills of calculation and estimation. Students should understand that the use of numbers to express patterns and to describe real-life situations goes back to humankind's earliest beginnings, and that mathematics has multicultural roots.

Concepts	Skills
Standard and extended mathematics	
Forms of numbers: integers, fractions, decimals, exponents, standard form (scientific notation) and surds/radicals Number systems: set of positive integers and zero (\mathbb{N}), integers (\mathbb{Z}), rationals (\mathbb{Q}), irrationals (\mathbb{Q}') and real numbers (\mathbb{R})	Ordering numbers Transformation between different forms of numbers Simplification of numerical expressions in the number systems and forms of number Recognizing and classifying numbers in different number systems
The four number operations	Using the four number operations (addition, subtraction, multiplication and division) with integers, decimals and simple fractions
Prime numbers and factors including greatest common divisor and least common multiple	Representing a number as the product of its prime factors and using this representation to find the greatest common divisor and least common multiple
Number lines	Expressing the solution set of a linear inequality on the number line
Estimation	Using different forms of rounding, decimal approximation and significant figures Using appropriate forms of rounding to estimate results
Units of measurement	Converting between different units of measurement and between different currencies
Ratio, percentage, direct and inverse proportion	Dividing a quantity in a given ratio Finding a constant of proportionality, setting up equations and graphing direct and inverse relationships
Number sequences	Predicting the next term in the number sequence (linear, quadratic, triangular, Fibonacci)
Extended mathematics only	
Fractional exponents	Using the rules of indices to simplify numerical expressions involving radicals and exponents
Absolute and percentage error in estimations	Calculating the absolute and percentage error

Algebra

Algebra is an abstraction of the concepts first used when dealing with number and is essential for further learning in mathematics. Algebra uses letters and symbols to represent number, quantity and operations, and employs variables to solve mathematical problems.

Students who wish to continue studying mathematics beyond the MYP will require knowledge of concepts and skills in algebra. Teachers should, where appropriate, assist students' understanding of algebra by using real-life contexts for the application of algebraic knowledge and skills in problem-solving situations.

Concepts	Skills
Standard and extended mathematics	
Addition, subtraction, multiplication and division of algebraic terms	Expanding and simplifying algebraic expressions
Factorization of linear and quadratic expressions	Factorizing algebraic expressions
Substitution	Using substitution to evaluate expressions
Rearranging algebraic expressions	Changing the subject of the formula
Algebraic fractions	Solving equations involving algebraic fractions
Integer exponents (including negative number exponents)	Using the laws of exponents
Patterns and sequences	Finding and justifying or proving general rules/formulae for sequences
Functions: types of functions—linear, quadratic domain and range	Solving the linear function $f(x) = mx + c$, its graph, gradient and y-intercept Graphing different types of functions and understanding their characteristics Determining the range, given the domain
Graphs	Sketching and interpreting graphs
Equations: linear simultaneous quadratic	Solving equations algebraically and using graphs
Inequalities	Solving and graphing linear inequalities
Extended mathematics only	
Logarithms and exponents: fractional exponents logarithms with different base numbers (including natural logarithms)	Using the laws of logarithms
Functions and graphs: types of functions: trigonometric, exponential, logarithmic, reciprocal function ($f(x) = \frac{1}{x}$ and their transformations), the square root function inverse and composite function	Graphing different types of functions and understanding their characteristics Determining inverse and composite functions and their graphs
Equations involving the functions above	Solving equations algebraically and using graphs
Inequalities	Solving non-linear inequalities Linear programming
Arithmetic and geometric series	Developing and justifying or proving general rules/formulae for sequences Finding the sum of the series Finding unknowns (ratio, term, and so on)
Matrices	Performing basic operations with matrices Using matrices (solving equations, transformations, growth models, and so on)

Geometry and trigonometry

The study of geometry and trigonometry enhances students' spatial awareness and provides them with the tools for analysing, measuring and transforming geometric quantities in two and three dimensions.

Concepts	Skills
Standard and extended mathematics	
Geometrical elements and their classification	Naming and classifying different geometrical elements (point, line, angle, regular and irregular planar figures, solids)
Distance	Measuring distance (between two points, between a line and a point)
Angle properties	Solving problems using the properties of: angles in different figures or positions acute, right and obtuse angles in triangles angles in intersecting and parallel lines angles in regular and irregular polygons angles in circles
Triangle properties	Solving problems involving triangles by using: Pythagoras' theorem and its converse properties of similar triangles properties of congruent triangles
Perimeter/area/volume	Finding the perimeter (circumference), area and volume of regular and irregular two-dimensional (2D) and three-dimensional (3D) shapes
The Cartesian plane	Identifying the different components of the Cartesian plane: axes, origin, coordinates (x, y) and points Understanding and using the Cartesian plane, plot graphs and measuring distances between points
Trigonometric ratios in right-angled triangles	Relating angles and sides of right-angled triangles using sines, cosines and tangents Solving problems in right-angled triangles using trigonometric ratios
Constructions	Using geometry tools to make basic constructions and using these in solving problems
Simple isometric transformation	Transforming a figure by rotation, reflection, translation and enlarging
Loci	Using locus concept to solve problems in two dimensions
Extended mathematics only	
Vectors and vectors spaces	Adding, subtracting and scalar multiplication of vectors
Similarity and congruence theorems	Justifying or proving theorems for congruence, similarity, shape and angles
Trigonometric ratios for angles bigger than 90°	Justifying or proving simple trigonometric identities to simplify and solve equations where $0^\circ \leq \theta \leq 360^\circ$
Sine and cosine rules	Using the sine and cosine rules to solve problems

Statistics and probability

This branch of mathematics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses and predict the occurrence of events.

Through the study of statistics, students should develop skills associated with the collection, organization and analysis of data, enabling them to present information clearly and to discover patterns. Students will also develop critical-thinking skills, enabling them to differentiate between what happens in theory (probability) and what is observed (statistics).

Students should understand both the power and limitations of statistics, becoming aware of their legitimate use in supporting and questioning hypotheses, but also recognizing how statistics can be used to mislead as well as to counter opinions and propaganda.

Students should use these skills in their investigations and are encouraged to use information and communication technology whenever appropriate.

Concepts	Skills
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Standard and extended mathematics	
Graphical analysis and representation (pie charts, histograms, line graphs)	Constructing and interpreting graphs
Population sampling	Selecting samples and making inferences about populations
Measures of central tendency/location (mean, mode, median, quartile, percentile)	Calculating the mean, median and mode and choosing the best measure of central tendency
Measures of dispersion (range, inter-quartile range)	Calculating quartiles and percentiles and discussing their uses
Probability of an event Probability of exclusive and combined events Probability of successive trials	Calculating probabilities of simple events Calculating probabilities of mutually exclusive events and combined events Using tree diagrams to determine the probability of repeated events
Extended mathematics only	
Normal distribution and standard deviation	Making inferences about normal distributed data given the mean and the standard deviation
Linear regression Correlation	Drawing the line of best fit
Conditional probability	Calculating conditional probability

Discrete mathematics

Discrete mathematics is a relatively new branch of mathematics that has its roots in abstract algebra and has adopted the language and notation of graph theory. Discrete mathematics contributes to the understanding of systems and formal structures. Its techniques have become increasingly important for people to analyze and solve problems in technology, science, engineering, business and other complex systems.

Students should develop logical-thinking skills and be able to articulate their understanding through the use of Venn diagrams, structure diagrams and flow charts. Discrete mathematics provides new approaches to learning (ATL) in the MYP.

Students should be aware of the real-world applications of discrete mathematics, which may include road or rail networks, computer networks, communications networks, optimal routes, time- and project-management techniques, and critical path analysis.

Concepts	Skills
Standard and extended mathematics	
Sets Venn diagrams	Performing operations Drawing and interpreting Venn diagrams Using Venn diagrams to solve problems in real-life contexts
Logic	Expressing ideas in two-value systems (Boolean algebra) Applying truth tables to determine the truth for complex statements
Networks (including trees)	Locating paths and tours Analyzing networks to find complete paths, shortest distance paths Solving problems involving optimal solutions Devising and describing procedures for performing complete calculations Using networks and flowcharts to solve problems in real-life contexts
Algorithms	Analyzing and using well-defined procedures for solving complex problems
Extended mathematics only	
Topology	Classifying and describing topological objects and simplifying knots
Directed networks	Performing critical path analyses
Codes and ciphers	Encoding and decoding information using translations and modular arithmetic Generating and translating bar codes Encoding and decoding with RSA codes that involve public and private keys