MATH/SCIENCE/COMPUTER SCIENCE MAGNET PROGRAM
Takoma Park Middle School
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Dear Magnet Geometry Student,

We hope that you are enjoying your summer vacation. During the weeks that remain, notice that you live in a fabulous world full of mathematics. You will discover geometry in living organisms as well as in works of art and architecture- lines, planes, angles, polygons & polyhedra, circles & spheres. Where do you see geometry around you?

Enclosed you will find a set of summer problems. Please complete this sheet in accordance with the directions and bring your solutions the first day of class. You may discuss the problems with other students. Also, answers are included for the Algebra Review. Please correct this section before bringing the packet to class. The summer problems are not mandatory, but we strongly encourage you to complete them. They are a great way to review key algebra skills, sharpen your problem-solving skills, and preview topics in geometry.

It’s a good idea to get your supplies ready before school starts. Required materials for Magnet Geometry include a three ring binder, loose leaf paper, pencils, pens, a ruler, a protractor, and a compass. If you wish to purchase a good quality compass, bring $2.50 the first week of school, and we will have some available. You will also need a calculator with trigonometric functions (sin, cos, tan). If you do not currently have a graphing calculator, you do not need to purchase one for this course; a scientific calculator is fine.

If you have questions before school starts, you are welcome to email us. Come prepared to work hard, think creatively, ask questions, and learn a lot. We look forward to meeting all of you in August. An exciting world of geometry awaits!

Sincerely,

Sarah Manchester
Magnet Geometry Teacher
Use the following key to check your answers in the algebra review portion of the problems.

1. Group – closure, associative, identity, inverse
   Abelian group – all group properties, plus commutative

2a. All natural number values of $n$
   b. All prime values of $n$
   c. When $n$ is not prime, the inverse property doesn’t hold. The elements of $\mathbb{Z}_n$ which have a common factor (other than 1) with $n$ do not have multiplicative inverses.

3. Reflexive, symmetric, transitive
   a. Transitive
   b. Reflexive, symmetric
   c. Reflexive, transitive

4a. $15\sqrt{3} - 14\sqrt{5}$
   b. $29 - 12\sqrt{5}$
   c. $\frac{\sqrt{6}}{4}$
   d. $7 - 2\sqrt{5}$

5a. $x = 8$ or $x = -3$
   b. $y = \frac{2}{3}$ or $y = 5$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = \frac{3 \pm 3\sqrt{17}}{4}$

7a. $PQ = \sqrt{181}$
   b. $(1, -3.5)$

8a. $x - 3y = 7$
   b. $y = \frac{-16}{3}x + 5$

9a. $(2, 13)$
   b. $(3.5, -0.1)$
MAGNET GEOMETRY SUMMER PROBLEMS

Complete the following problems on separate paper, showing all steps. After completing the Algebra Review, check your answers using the key provided on the back of the letter. Please bring your solutions on the first day of class for correction and discussion.

A. Algebra Review

The problems in this section will help you review some algebraic concepts and skills necessary for success in geometry. You may refer to your algebra notes or other resources for guidance. Make sure to show all steps on problems #4-9.

1. What properties must hold in order for a system \((S,*)\) to be a group? an Abelian group?

2a. For which values of \(n\) is \((\mathbb{Z}_n,+)\) an Abelian group?
   
b. For which values of \(n\) is \((\mathbb{Z}_n\setminus\{0\},\cdot)\) an Abelian group?
   
c. Explain why \((\mathbb{Z}_n\setminus\{0\},\cdot)\) is not a group for certain values of \(n\).

3. List the properties of an equivalence relation, then tell which properties hold for each system.
   
a. \((\mathbb{R}, >)\)  
b. \(\{\text{Takoma students}\}, \text{‘has a class with’} \)  
c. \(\{\text{sets}\}, \text{‘is a subset of’} \)

4. Simplify each expression, putting your answer in simplest radical form.
   
a. \(\sqrt{180} - 4\sqrt{125} + 3\sqrt{75}\)  
b. \((2\sqrt{5} - 3)^2\)  
c. \(\sqrt{\frac{3}{8}}\)  
d. \(\frac{14 - \sqrt{80}}{2}\)

5. Solve each quadratic equation by factoring.
   
a. \(x^2 - 24 = 5x\)  
b. \(3y^2 - 17y + 10 = 0\)

6. State the quadratic formula, then use it to solve the following equation: \(2a^2 - 3a = 18\)

7. The coordinates of points \(P\) and \(Q\) are \((6, -8)\) and \((-4, 1)\).
   
a. Calculate the length of \(PQ\).  
b. Find the midpoint of \(PQ\).

8. Find the equation of each line using the given information. Put your answer in the form specified.
   
a. perpendicular to \(y = -3x + 1\), has an \(x\)-intercept of 7 \(\quad\) (general form)  
b. contains the point \((3, -11)\), has a \(y\)-intercept of 5 \(\quad\) (slope-intercept form)

9. Solve each system of equations using the method specified. Express your answer as an ordered pair.
   
a. \(y = 7x - 1\) \(\quad\) (substitution)  
b. \(3x + 5y = 10\) \(\quad\) (linear combination)  
   \(y = 2x + 9\)  
   \(7x - 15y = 26\)
B. Looking Ahead to Geometry

The problems in this section require you to apply some concepts and skills that will be important in your study of geometry. Remember to show your solutions completely and clearly. These problems will also be discussed on the first day of class.

1. What is the largest number of regions into which you can divide a circle with one line? Two lines? Three lines? Four lines? Make a chart showing the maximum number of regions formed when \( n \) lines intersect a circle, for \( n = 1 \ldots 6 \). Describe a pattern in the chart.

2. Find at least three ways to arrange six congruent toothpicks to obtain four congruent triangles.

3. Consider the 26 capital letters of the English alphabet written as simply as possible (no serifs, fancy fonts, etc.) Which of these 26 letters have a vertical line of symmetry? a horizontal line of symmetry? rotational symmetry? List the letters that have each property.

4. Where on earth could a man leave his house, then walk three miles north, three miles west, three miles south, and find himself back home, at the same point where he started? (Hint: This is a famous problem; there is one well-known answer, but that is not the only answer.)

5. Define a Pythagorean triple and list at least three examples.

6. Draw a regular pentagon and its five diagonals. How many triangles are formed? Show or describe how you counted the triangles. (Hint: Don’t forget to count overlapping triangles.)

7. Traditional constructions are performed using only a compass and straightedge. In 6th grade, you learned some basic constructions, such as copying an angle and bisecting a segment. What are the three ancient impossible construction problems of Euclidean geometry? In your own words, describe each problem. (You’ll need to do a little research for this one.)

8. When a clock shows the time to be 12:15, how many degrees are in the acute angle between the minute hand and the hour hand?

9. Sketch the graph of the equation \( |x| + |y| = 5 \). What type of figure is formed? What is its perimeter and area?

10. Imagine you start saving pennies during summer vacation. On the first day, you drop one penny into the piggy bank, two pennies the second day, three pennies the third day, and so on. How much money do you have at the end of summer? Vacation this year lasts 75 days.

11. Your friend is also saving pennies. She puts one penny into her piggy bank the first day, two pennies the second day, four the third day, and so on, each day doubling the number of pennies added. How much money would she have at the end of vacation, if she could keep saving at this rate? (This would be hard to do, even if Bill Gates were your dad!)

12. Geometry Jill baked a rectangular cake for her brothers, John and Jerry. She planned to cut the cake in half and send an equal amount home with each brother. However, when she took the cake from the fridge, she saw that her son, Geometry Joe, had cut out a rectangular chunk and eaten it. How can she make a single straight cut through the cake that will guarantee each brother an equal amount? Jill has some string on hand, but no measuring tools. Also, she may not cut the cake through the middle, parallel to the cake’s top surface. Each brother should receive an equal amount of the frosted top. (This one is tricky. You may not solve it successfully, but make an attempt.)