The semester A examination for Precalculus consists of two parts. Part 1 is selected response on which a calculator will not be allowed. Part 2 is short answer on which a calculator will be allowed.

Pages with the symbol indicate that a student should be prepared to complete questions like these with or without a calculator.

The formulas below are provided in the examination booklet.

**Trigonometric Identities:**

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\csc^2\theta = 1 + \cot^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\tan\left(\dfrac{\theta}{2}\right) = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

**Law of Cosines and Sines:**

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

**Area of a Triangle:**

Area $\triangle = \dfrac{1}{2}ab\sin C$

Area $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{1}{2}(a+b+c)$

**Length of Arc:**

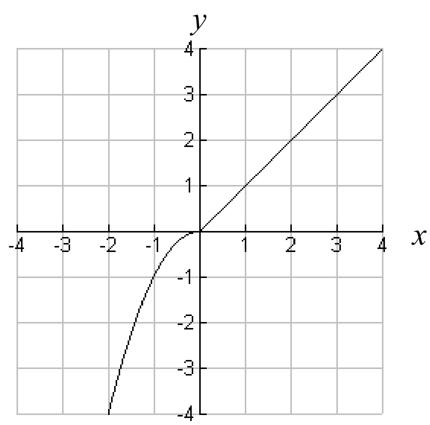
$s = r\theta$, $\theta$ in radians

$s = \dfrac{\theta}{360}(2\pi r)$, $\theta$ in degrees

**PART 1 NO CALCULATOR SECTION**

1. Sketch the graph of the piece-wise function $f(x)=\begin{cases} x^2\ , & \text{if } x<0 \\ \sqrt{x}+1, & \text{if } x\geq 0 \end{cases}$

2. Look at the graph of the piecewise function below.




Which of the following functions is represented by the graph?

**A** $f(x)=\begin{cases} x^2, & \text{if } x\leq 0 \\ x, & \text{if } x>0 \end{cases}$

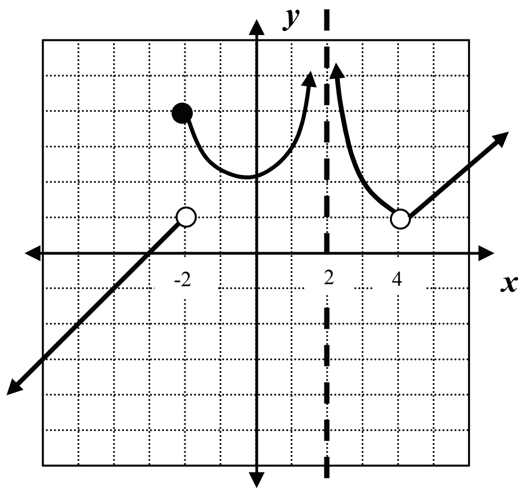
**B** $f(x)=\begin{cases} -x^2, & \text{if } x\leq 0 \\ x, & \text{if } x>0 \end{cases}$

**C** $f(x)=\begin{cases} x, & \text{if } x\leq 0 \\ -x^2, & \text{if } x>0 \end{cases}$

**D** $f(x)=\begin{cases} -x, & \text{if } x\leq 0 \\ x^2, & \text{if } x>0 \end{cases}$



3. Look at the graph of the piecewise function below.




What type of discontinuity does the graph have at the following $x$ values?

a. $x = -2$

b. $x = 2$

c. $x = 4$

4. Which of the following is true about the function $f(x) = \dfrac{x+4}{x-3}$?

**A** The function is continuous for all real numbers.

**B** The function is discontinuous at $x = 3$ only.

**C** The function is discontinuous at $x = -4$ only.

**D** The function is discontinuous at $x = 3$ and $x = -4$.

5. Determine whether each function below is even, odd, or neither even nor odd.

a. $g(x) = \sin x + x^3$

b. $h(x) = x^2 - 4$

c. $r(x) = \cos x + 4x$

6. If $f(x)=x^{\frac{2}{3}}$, which of the following statements is NOT true?

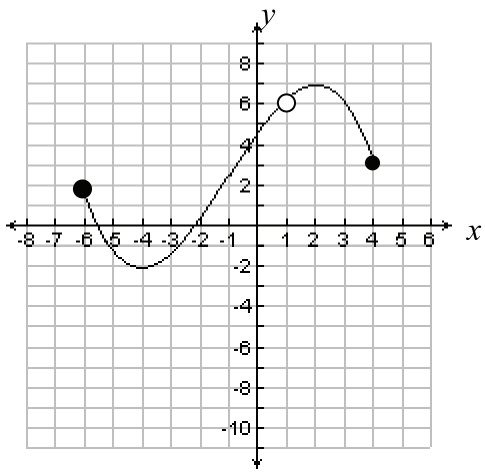
   **A** The graph of $f(x)$ is symmetric with respect to the $y$-axis.

   **B** $f(x)$ is an even function.

   **C** The range of $f(x)$ is all real numbers.

   **D** As $x \to \infty,\ f(x) \to \infty$

7. Look at the graph of the function below.




   a. What is the domain of this function? ________________________

   b. What is the range of this function? __________________________

8. For each function below, find a formula for $f^{-1}(x)$ and state any restrictions on the domain.

   a. $f(x)=\sqrt{x+2}$

   b. $f(x)=x^3+4$

9. True or false.

   a. The function $g(x)=5f(x)-2$ represents a vertical stretch of the graph of $f(x)$ by a factor of 5, followed by a vertical translation down 2 units.

   b. The function $g(x)=7f\left(\frac{x}{4}\right)$ represents a vertical and horizontal shrinking of the graph of $f(x)$.

10. Match the transformations that would create the graph of $g(x)$ from the graph of $f(x)$.

_______ $g(x) = 3f(x)$ **A** Stretch the graph of $f(x)$ horizontally.

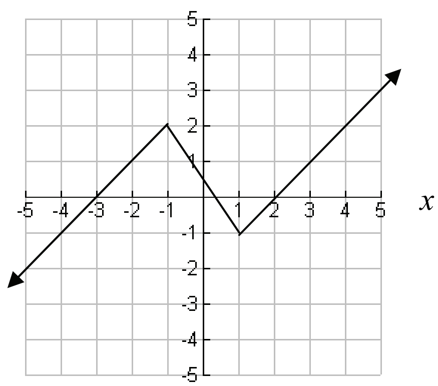
_______ $g(x) = f(3x)$ **B** Stretch the graph of $f(x)$ vertically.

_______ $g(x) = f\left(\frac{1}{3}x\right)$ **C** Shrink the graph of $f(x)$ horizontally.

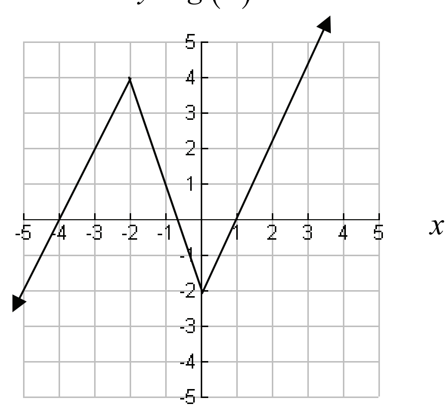
_______ $g(x) = \frac{1}{3}f(x)$ **D** Shrink the graph of $f(x)$ vertically.

For items 11 and 12, use the graphs of $f(x)$ and $g(x)$ below.

$y = f(x)$




$y = g(x)$




11. Which of the following represents the relationship between $f(x)$ and $g(x)$?

**A** $g(x) = 2f(x+1)$

**B** $g(x) = \frac{1}{2}f(x-1)$

**C** $g(x) = f(2x) - 1$

**D** $g(x) = f\left(\frac{1}{2}x\right) + 1$

12. Sketch the graph of $y = |f(x)|$.

13. Write the definitions of the six circular functions in terms of $x, y,$ and $r$.

14. If $\sin\theta = -\dfrac{4}{5}$ with $\cos\theta > 0$ , what are the values of the other five trigonometric functions?

15. For each of the following, write the quadrant in which the terminal side of $\theta$ lies.

    a. $\sin\theta > 0, \tan\theta < 0$

    b. $\cos\theta < 0, \tan\theta > 0$

    c. $\sec\theta < 0, \csc\theta < 0$

16. Convert to radian measure. Leave your answer in terms of $\pi$.

    a. $40^o$

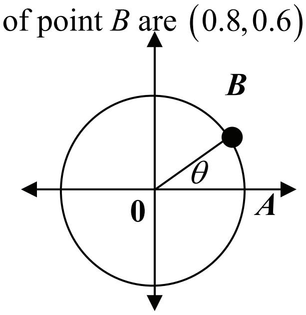
    b. $165^o$

17. The coordinates of point $A$ are $(1,0)$ and the coordinates of point $B$ are $(0.8, 0.6)$ as shown below. Find the value of the following.

    a. $\sin\theta$

    b. $\cos\theta$

    c. $\tan\theta$



18. Determine the exact value of the following.

| | | | | | |
|---|---|---|---|---|---|
| a. | $\sin\left(\frac{\pi}{6}\right)$ | b. | $\cos\left(\frac{5\pi}{4}\right)$ | c. | $\tan\left(\frac{5\pi}{3}\right)$ |
| d. | $\sin\left(\frac{3\pi}{2}\right)$ | e. | $\cos(\pi)$ | f. | $\tan\left(\frac{\pi}{2}\right)$ |
| g. | $\tan\left(-\frac{7\pi}{4}\right)$ | h. | $\cos\left(-\frac{4\pi}{3}\right)$ | i. | $\sin\left(-\frac{11\pi}{6}\right)$ |
| j | $\sin\left(\frac{\pi}{4}\right)$ | k. | $\cos\left(\frac{5\pi}{6}\right)$ | l. | $\tan\left(\frac{7\pi}{6}\right)$ |
| m. | $\sec\left(-\frac{5\pi}{4}\right)$ | n. | $\cot\left(\frac{5\pi}{6}\right)$ | o. | $\csc\left(\frac{4\pi}{3}\right)$ |

19. Sketch the graphs of the six circular functions on the interval $-2\pi \le x \le 2\pi$

20. Which of the following functions does NOT have a period of $2\pi$ ?

**A** $f(x) = \sin x$

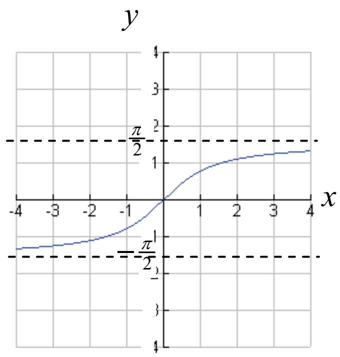
**B** $f(x) = \tan x$

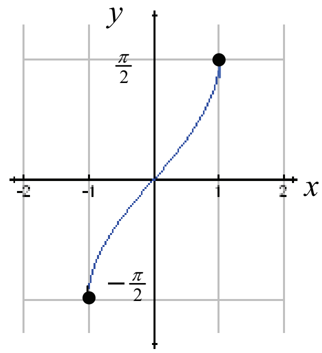
**C** $f(x) = \sec x$

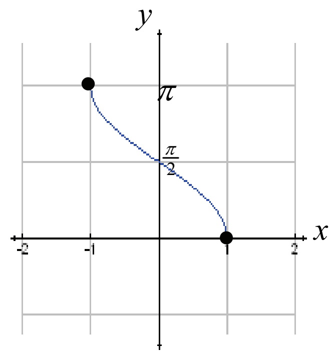
**D** $f(x) = \csc x$

21. a. Write an equation for each inverse function graph (i-iii).

i. ________________ ii. ________________ iii. ________________




b. Use the following intervals to complete the table below.

$[-1,1]$ $(-\infty,\infty)$ $[0,\pi]$ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

| | $\text{Sin}^{-1}x$ | $\text{Cos}^{-1}x$ | $\text{Tan}^{-1}x$ |
|---|---|---|---|
| Domain | | | |
| Range | | | |

22. Determine the exact value of the following.

a. $\text{Sin}^{-1}\left(\frac{1}{2}\right)$

b. $\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

c. $\text{Tan}^{-1}\left(\sqrt{3}\right)$

d. $\text{Sin}^{-1}(-1)$

e. $\text{Cos}^{-1}(0)$

f. $\text{Tan}^{-1}(-1)$

23. Determine the exact value of the following.

a. $\cos\left(\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

b. $\sin\left(\text{Tan}^{-1}(-1)\right)$

c. $\tan\left(\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

24. Determine the exact value of the following.

a. $\sin\left(\text{Csc}^{-1}\left(\frac{8}{5}\right)\right)$

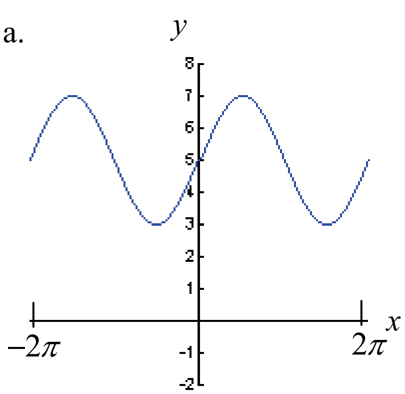
b. $\tan\left(\text{Sin}^{-1}\left(\frac{12}{13}\right)\right)$

c. $\text{Cos}^{-1}\left(\cos\left(\frac{11\pi}{6}\right)\right)$

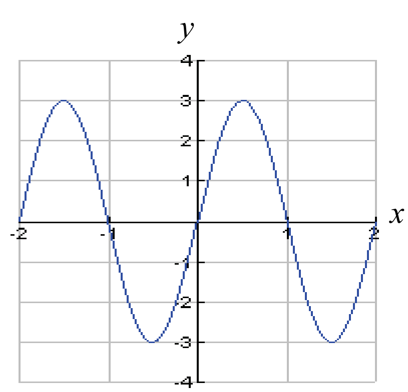
d. $\sin\left(\text{Tan}^{-1}\left(-\frac{4}{3}\right)\right)$

25. Write a sinusoidal equation for the following graphs

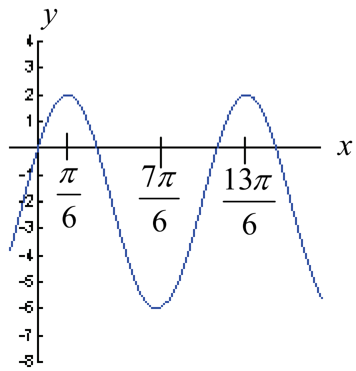
a.




b.



c.




26. Determine the equation that best describes a sine curve with amplitude 3, period of 6, and a phase shift of $\frac{\pi}{2}$ to the right.

27. State the amplitude, period, the phase shift and vertical translation of the sinusoid relative to the basic function $f(x) = \sin x$ or $f(x) = \cos x$. Sketch the graph, marking the *x*- and *y*-axes appropriately.

a. $f(x) = 2\sin\left(3\left(x - \frac{\pi}{6}\right)\right) + 5$

b. $f(x) = -5\cos(\pi(x+1))$

c. $f(x) = 5\sin(4x - \pi) - 2$.

28. Simplify the following expressions and evaluate.

a. $\sin\frac{5\pi}{8}\cos\frac{3\pi}{8} - \cos\frac{5\pi}{8}\sin\frac{3\pi}{8}$

b. $\cos\frac{5\pi}{6}\cos\frac{\pi}{6} - \sin\frac{5\pi}{6}\sin\frac{\pi}{6}$

29. If $\sin A = \frac{5}{13}$ and $\cos A < 0$, determine the value of the following.

a. $\sin 2A$

b. $\cos 2A$

30. Prove the following identities.

a. $\sin\theta\cot\theta = \cos\theta$

b. $(\sin x + \cos x)^2 = 1 + \sin 2x$

c. $\frac{\csc x}{1+\cot^2 x} = \sin x$

d. $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \sec\theta\csc\theta$

e. $\sin(x+y) + \sin(x-y) = 2\sin x\cos y$

f. $\sin^2\theta + \sin^2\theta\tan^2\theta = \tan^2\theta$

31. Solve the following equations on the interval $0^o \le \theta < 360^o$.

a. $2\sin\theta = -\sqrt{2}$

b. $3\cos\theta + 4 = 5\cos\theta + 5$

32. Solve the following equations on the interval $0 \le x < 2\pi$

a. $\tan x + 1 = 0$

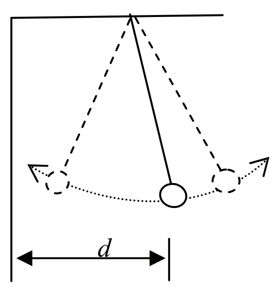
b. $2\sin^2 x - 3\sin x + 1 = 0$

**PART 2 CALCULATOR SECTION**
A calculator may be used on items 33 through 48. Make sure that your calculator is in the appropriate mode (radian or degree) for each item. Round to the number of decimal places specified in each item.

33. Complete the following chart using $s = r\theta$.

| Radius | Angle(radians) | Arc Length |
|---|---|---|
| 6 inches | $\frac{\pi}{4}$ | |
| | $\frac{5\pi}{6}$ | $15\pi$ feet |
| 10 meters | | 30 meters |

34. A ball on a string is swinging from the ceiling, as shown in the figure below.




Let $d$ represent the distance that the ball is from the wall at time $t$.
Assume that $d$ varies sinusoidally with time.

When $t = 0$ seconds, the ball is farthest from the wall, $d = 160$ cm.
When $t = 3$ seconds, the ball is closest to the wall, $d = 20$ cm.
When $t = 6$ seconds, the ball is again farthest from the wall, $d = 160$ cm.

a. Sketch a graph of $d$ as a function of time.

b. Write a trigonometric function for $d$ as a function of time.

c. What is the distance of the ball from the wall at $t = 5$ seconds?

d. What is the value of $t$ the first time the ball is 40 cm from the wall? Your answer should be correct to three places after the decimal point.

35. Sara is riding a Ferris wheel. Her sister, Kari, starts a stopwatch and records some data. Let $h$ represent Sara's height above the ground at time $t$. Kari notices that Sara is at the highest point, 80 feet above the ground, when $t = 3$ seconds. When $t = 7$ seconds Sara is at the lowest point, 20 feet above the ground. Assume that the height $h$ varies sinusoidally with time $t$.

   a. Write a trigonometric equation for the height $h$ of Sara above the ground as a function of time $t$.

   b. What will the height of Sara be above the ground at $t = 11.5$ seconds? Your answer should be correct to three places after the decimal point.

   c. Determine the first two times, $t > 0$, when the height of Sara above the ground is 70 feet. Your answer should be correct to three places after the decimal point.

36. At Ocean Tide Dock the first low tide of the day occurs at midnight, when the depth of the water is 2 meters, and the first high tide occurs at 6:30 A.M. with a depth of 8 meters. Assume that the depth of the water varies sinusoidally with time.

   a. Sketch and label a graph showing the depth ($d$) of the water at the dock as a function of the number of hours after midnight ($t$).

   b. Determine a trigonometric model that represents the graph.

   c. Suppose a tanker requiring at least 3 meters of water depth is planning to dock after midnight. Determine the earliest possible time that the tanker can dock.

37. Solve for $\theta$, where $0^o \le \theta < 360^o$. Your answer should be correct to the nearest tenth of a degree.

   a. $3\cos\theta + 9 = 7$

   b. $3\sin^2\theta + 7\sin\theta + 2 = 0$

38. How many triangles $ABC$ are possible if $\angle A = 20^\circ,\ b = 40$, and $a = 10$?

39. Given $\triangle ABC$, where $\angle A = 41^o$, $\angle B = 58^o$, and $c = 19.7$ cm, determine the measure of side $b$. Your answer should be correct to three places after the decimal point.

40. In $\Delta ABC$, $a = 9, b = 12, c = 16$. What is the measure of $\angle B$? Your answer should be correct to the nearest tenth of a degree.

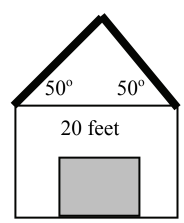
41. Determine the remaining sides and angles of a triangle with $\angle A = 58^{o}$, side $a = 11.4$ and side $b = 12.8$. Your answers (sides and angles) should correct to the nearest tenth.

42. From a point 200 feet from its base, the angle of elevation from the ground to the top of a lighthouse is 55 degrees. How tall is the lighthouse? Your answer should be correct to three places after the decimal point.

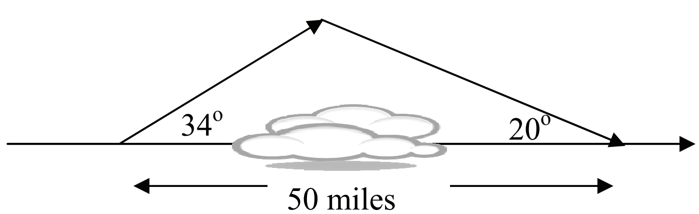
43. A truck is traveling down a mountain. A sign says that the degree of incline is 7 degrees. After the truck has traveled one mile (5280 feet), how many feet in elevation has the truck fallen? Your answer should be correct to three places after the decimal point.

44. A person is walking towards a mountain. At one point, the angle of elevation from the ground to the top of the mountain is 37 degrees. After walking another 1000 feet, the angle of elevation from the ground to the top of the mountain is 40 degrees. How high is the mountain? Your answer should be correct to three places after the decimal point.

45. The owner of the garage shown below plans to install a trim along the roof. The lengths required are in bold. How many feet of trim should be purchased? Your answer should be correct to three places after the decimal point.




46. An airplane needs to take a detour around a group of thunderstorms, as shown in the figure below. How much farther does the plane have to travel due to the detour? Your answer should be correct to three places after the decimal point.

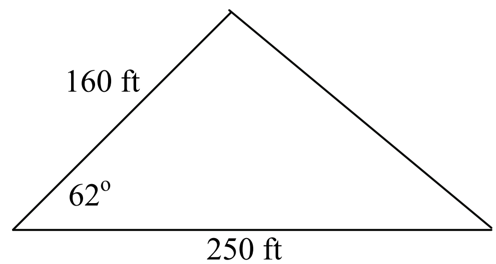


47. Determine the area of each triangle. Your answers (sides and angles) should be correct to the nearest tenth.

a. $\Delta ABC: a = 4, b = 10, m\angle C = 30^o$

b. $\Delta ABC: a = 17, b = 13, c = 18$

48. A real estate appraiser wishes to find the value of the lot below.




a. Find the area of the lot. Your answer should be correct to three places after the decimal point.

b. An acre is 43560 square feet. If land is valued at $56,000 per acre, how much is the land worth? Your answer should be correct to the nearest dollar.