The semester A examination for Algebra 2 will consist of two parts. Part 1 will be selected response. Part 2 will be short answer.

- Students may use a calculator.
- If a calculator is used to find points on a graph, the appropriate calculator function (i.e. zero, intersect, minimum or maximum) should be used. The trace function should not be used.
- Decimal approximations must be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

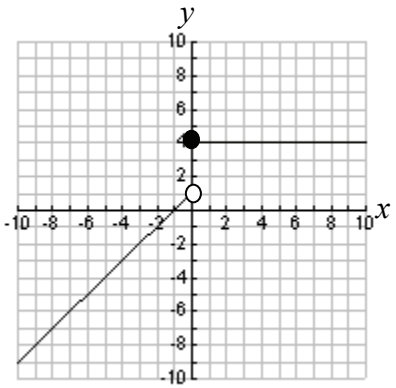
The formulas below will be provided in the examination booklet.

Quadratic Formula: If $ax^2+bx+c=0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Factoring: $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$

1. Which of the following is the graph of $f(x)=\begin{cases}x+1, & \text{if } x<0\\ 4, & \text{if } x\geq 0\end{cases}$?

A



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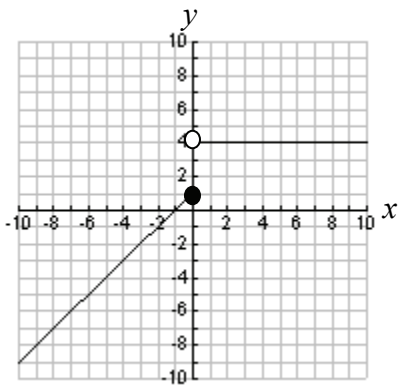

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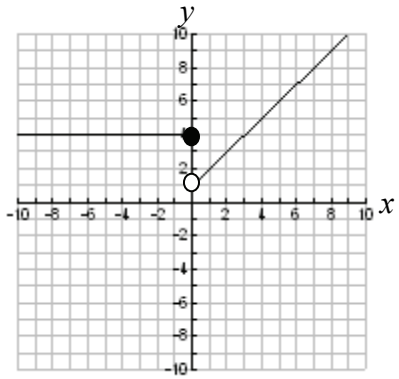

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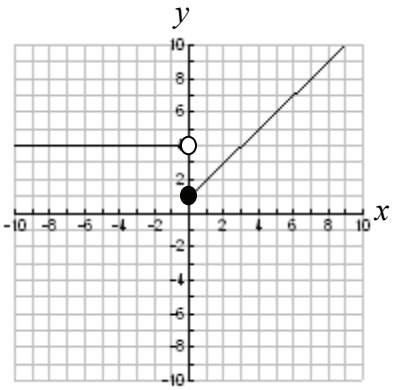
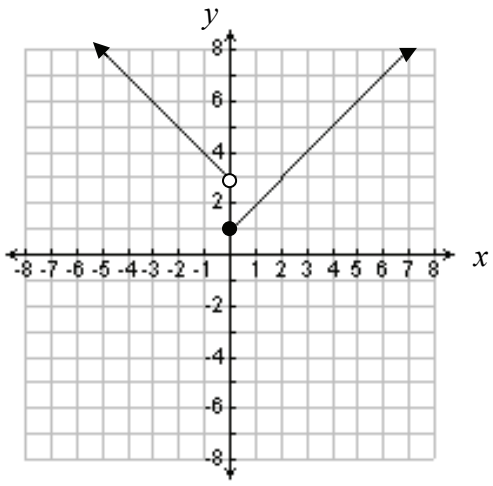


2. Write the function represented by the graph below.



$f(x) = \begin{cases} \\ \\ \end{cases}$

3. When ordering items from a catalog, the buyer has the option of having items gift-wrapped. The shipping charge for every order is \$12. In addition, if less than 50 items are wrapped, the charge is \$5.00 per item wrapped. If 50 or more items are wrapped, the charge is \$3.00 per item wrapped.

   a. Write a piece-wise function for the total charge of gift-wrapping x items, including the shipping charge.

   $f(x) = \begin{cases} \\ \\ \end{cases}$

   b. If 35 items are gift wrapped, what is the total charge including shipping?

4. Let $g(x) = 3|x+2| - 4$

   a. Sketch the graph of $g(x)$.

   b. What is the domain of $g(x)$?

   c. What is the range of $g(x)$?

   d. What is the vertex of the graph of $g(x)$?

   e. What is equation of the axis of symmetry of the graph of $g(x)$?

   f. What is the minimum value of $g(x)$?

   g. If $f(x) = |x|$, describe the transformations of $f(x)$ that were used to create $g(x)$.

   h. Is $g(x)$ continuous?

For items 5 through 12, use the following functions.

$f(x) = x - 3$ $g(x) = 2x - 8$ $h(x) = x^2 - 2$

Evaluate.

5. $f(g(3))$

6. $h(f(-7))$

Perform the following operations.

7. $f(x) + g(x)$

8. $f(x) - g(x)$

9. $f(x) \cdot g(x)$

10. $\dfrac{f(x)}{g(x)}$

11. $g(h(x))$

12. $h(f(x))$

For items 13 through 16, state whether the function is one to one.

13. $f(x) = 2x - 5$

14. $f(x) = -x^2$

15. $f(x) = |x|$

16. $f(x) = x^3 - 9x$

17. Verify algebraically that $f(x) = 7x - 6$ and $g(x) = \dfrac{x+6}{7}$ are inverse functions.

18. If $f(x)=\frac{1}{3}x+5$, which of the following represents the inverse function, $f^{-1}(x)$?

A $f^{-1}(x)=-\frac{1}{3}x-5$

B $f^{-1}(x)=3x-\frac{1}{5}$

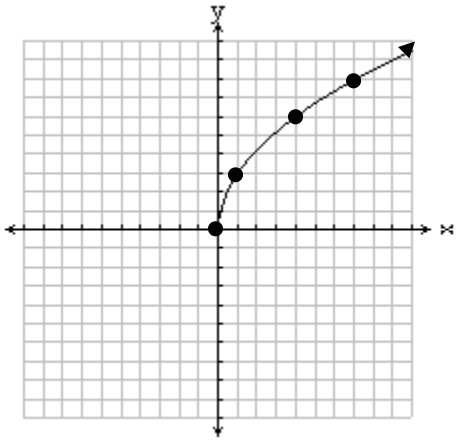
C $f^{-1}(x)=3x-5$

D $f^{-1}(x)=3x-15$

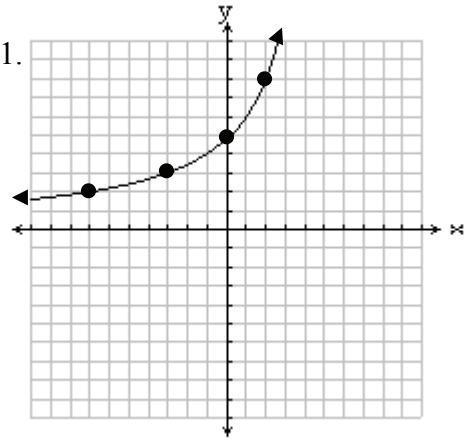
19. If $g(x)=9x-10$, determine the inverse function, $g^{-1}(x)$.

For items 20 and 21, for the graph of each function below, sketch the inverse function.

20.



21.

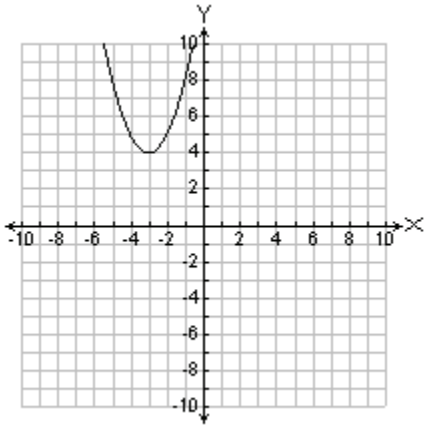


For items 22 through 24, let $f(x)=x^2$. Describe the transformations of $f(x)$ that will produce the graphs of the following functions.

22. $g(x)=4x^2+1$

23. $h(x)=\frac{1}{2}(x+5)^2+9$

24. the graph below



For items 25 through 30, use the following matrices:

$$A=\begin{bmatrix}4 & -2\\ 5 & -3\end{bmatrix} \quad B=\begin{bmatrix}7 & 1 & 9\\ 8 & 5 & 3\end{bmatrix} \quad C=\begin{bmatrix}4 & x\\ 10 & 15\end{bmatrix} \quad D=\begin{bmatrix}1 & 5 & 6 & 3 & 1\\ 9 & 7 & 3 & 4 & 0\\ 4 & 2 & 3 & 1 & 8\end{bmatrix}$$

25. What is the determinant of matrix A?

26. Write an expression for the value of the determinant of matrix C.

27. What are the dimensions of the product matrix BD?

A 2×5

B 5×2

C 5×3

D 3×5

28. If $A\cdot B=\begin{bmatrix}12 & y & 30\\ 11 & -10 & 36\end{bmatrix}$, what is the value of y?

29. For what value of x will matrix C not have an inverse matrix?

30. Write the matrix that represents A^{-1}, the inverse of matrix A.

31. Barry's Burgers offers three different types of burgers at three different prices. The types are the Hamburger, the Cheeseburger, and the BarryBurger.
One time, 3 Hamburgers, 5 Cheeseburgers, and 6 BarryBurgers cost \$25.24.
Another time, 2 Hamburgers, 7 Cheeseburgers and 5 BarryBurgers cost \$25.68.
The last time, 4 Hamburgers, 4 Cheeseburgers, and 7 BarryBurgers cost \$26.59.

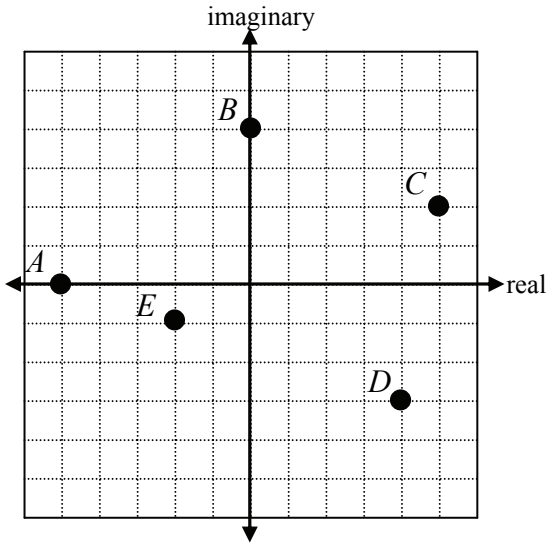
    a. Write a system of equations that represents the situation. Be sure to define the variables.

    b. Represent the system in matrix form.

    c. Determine the cost of each type of burger.

32. An astronaut on the moon throws an object. The height (in meters) of the object is given by $h = -0.8x^2 + 9.6x + 2$, where x is in seconds. The domain of this function is $0 \le x \le 10$. What is the maximum height that the object reaches?

33. Write the complex number represented by points *A*–*E* on the graph below. Each square represents one unit.



34. Identify each of the following as real, pure imaginary, and/or complex.

a. $\sqrt{3}$
b. $\sqrt{-9}$
c. $5 + 2i$

For items 35 through 40, perform the indicated operations.

35. $(3 + 2i) + (5 - 7i)$

36. $(5 - 6i)(1 - i)$

37. $(8 - 2i)(8 + 2i)$

38. $(2 - 7i)^2$

39. $\dfrac{7}{2i}$

40. $\dfrac{2 + 7i}{3i}$

For items 41 and 42, fill in the blank with the number that completes the square.

41. $x^2 - 6x + \_\_\_\_$

42. $x^2 + 10x + \_\_\_\_$

For items 43 and 44, solve for all values of x.

43. $x^2 - 3x + 11 = 0$

44. $3x^2 + 5x + 3 = 0$

For item 45a through 45c, grid in and bubble your answers in the grids below.

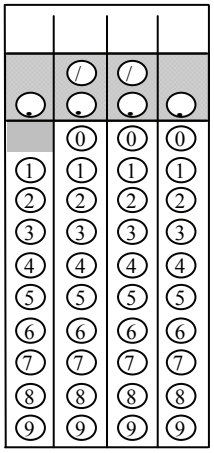
45. An arrow is shot from a height of 32 feet with an initial velocity of 56 ft/sec. The equation of the height $h(t)$ (in feet) of the arrow at time t (in seconds) is $h(t) = -16t^2 + 56t + 32$, where $0 \le t \le 4$.

    a. When will the arrow hit the ground?

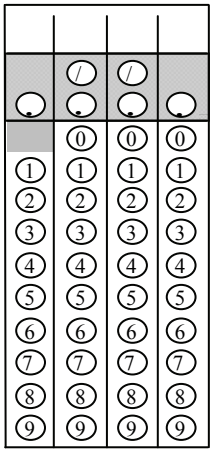
    b. How high will the arrow be at $t = 1.5$ seconds?

    c. What is the maximum height the arrow will reach?

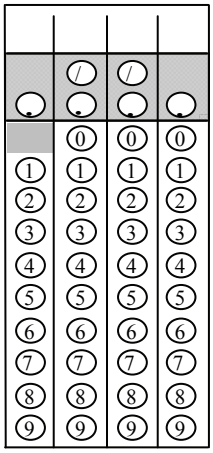
45a



45b



45c



46. If $f(x) = x^2$, on what interval of x-values is $f(x)$ decreasing?

47. What are the right- and left-end behaviors of the function $f(x) = x^3 - 4x - 8$?

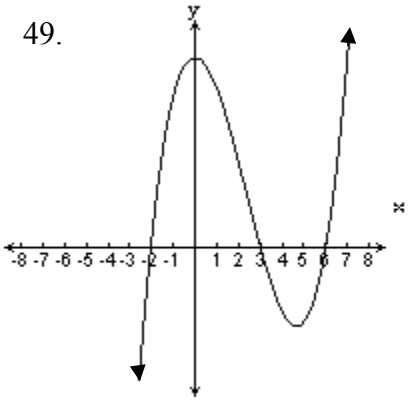
As $x \to \infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____

48. What are the right- and left-end behaviors of the function $f(x) = -x^4 - 4x^3 - 4$?

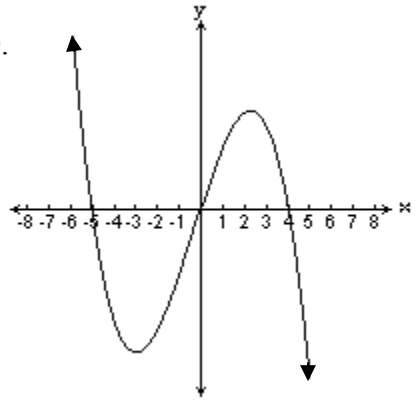
As $x \to \infty$, $f(x) \to$ _____ As $x \to -\infty$, $f(x) \to$ _____

For items 49 and 50, write an equation in factored form for the graphs shown below.

49.



50.



For items 51 and 52, factor.

51. $x^3 - 125$

52. $x^3 + 64$

For items 53 and 54, find <u>all</u> zeros of each function. Show how you determined the zeros.

53. $f(x) = x^3 - 27$

54. $g(x) = x^3 + 1000$

55. Determine if each expression below a factor of $x^3 + 6x^2 - x - 30$. Write yes or no for each expression.

a. $x - 2$ b. $x + 6$ c. $x - 5$

d. $x + 2$ e. $x + 3$ f. $x + 5$

56. Given $f(x) = x^3 + 3x^2 - x - 3$.

a. Find the zeros of the function and sketch a graph.

b. Complete: As $x \to \infty$, $f(x) \to$ ____

c. Complete: As $x \to -\infty$, $f(x) \to$ ____

57. Let $f(x) = 4x^5 - 7x^3 + 5x^2 + 15$. Determine if each number below is a <u>possible</u> rational root (zero) according to the rational root theorem. Write yes or no for each number.

a. 5 b. $-\frac{1}{4}$ c. $\frac{3}{2}$ d. 4 e. $\frac{1}{3}$

58. Using the rational root (zero) theorem, list the possible rational zeros of $f(x) = 5x^4 - 7x^2 + 4$.

59. Let $f(x) = x^4 - x - 4$.

a. How many zeros does $f(x)$ have?

b. Based on its graph, how many <u>real number</u> zeros does $f(x)$ have?

60. Let $f(x) = x^5 - 3x^2 - 4$.

a. How many zeros does $f(x)$ have?

A 0

B 1

C 4

D 5

b. Based on its graph, how many <u>real number</u> zeros does $f(x)$ have?

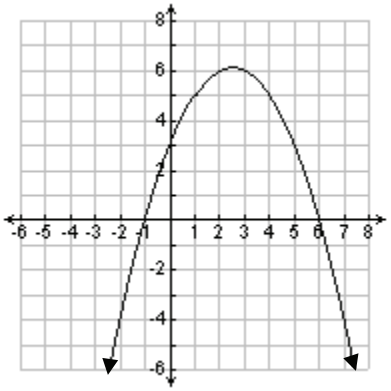
A 0

B 1

C 4

D 5

61. Look at the graph of $f(x) = -x^2 + 5x + 6$ below.



Use the graph to solve the inequality $-x^2 + 5x + 6 \le 0$.

62. The solutions of $x^3 - x^2 - 50x - 48 = 0$ are $x = -6, -1, \text{and } 8$.
Write the polynomial $x^3 - x^2 - 50x - 48$ in factored form.

63. The solutions of $x^3 - 6x^2 - 19x + 84 = 0$ are $x = -4, 3, \text{and } 7$.
Write the polynomial $x^3 - 6x^2 - 19x + 84$ in factored form.

64. Complete the chart. Round values to three decimal places.

Function	Values of any local maximums	Values of any local minimums	Interval(s) where the function is increasing	Interval(s) where the function is decreasing
$f(x) = \frac{x^3}{3} + 2x^2 + x + 3$				
$g(x) = x^4 - 5x^2 + 4$				

65. Sam throws a ball up from the top of a building. The table below shows the height of the ball above the ground.

Time (sec) t	Height (ft) $f(t)$
0	400
1	434
2	436
3	406

a. Write a quadratic function that bests models the data.

b. How high is the ball after 4 seconds?

c. When does the ball hit the ground?

For items 66 and 67 below,

a. determine the degree of the polynomial that models the data.

b. use the regression feature of your calculator to write a function that models the data.

66.

x	1	2	3	4	5
$f(x)$	5	14	37	80	149

67.

x	0	1	2	3	4
$f(x)$	3	6	11	18	27