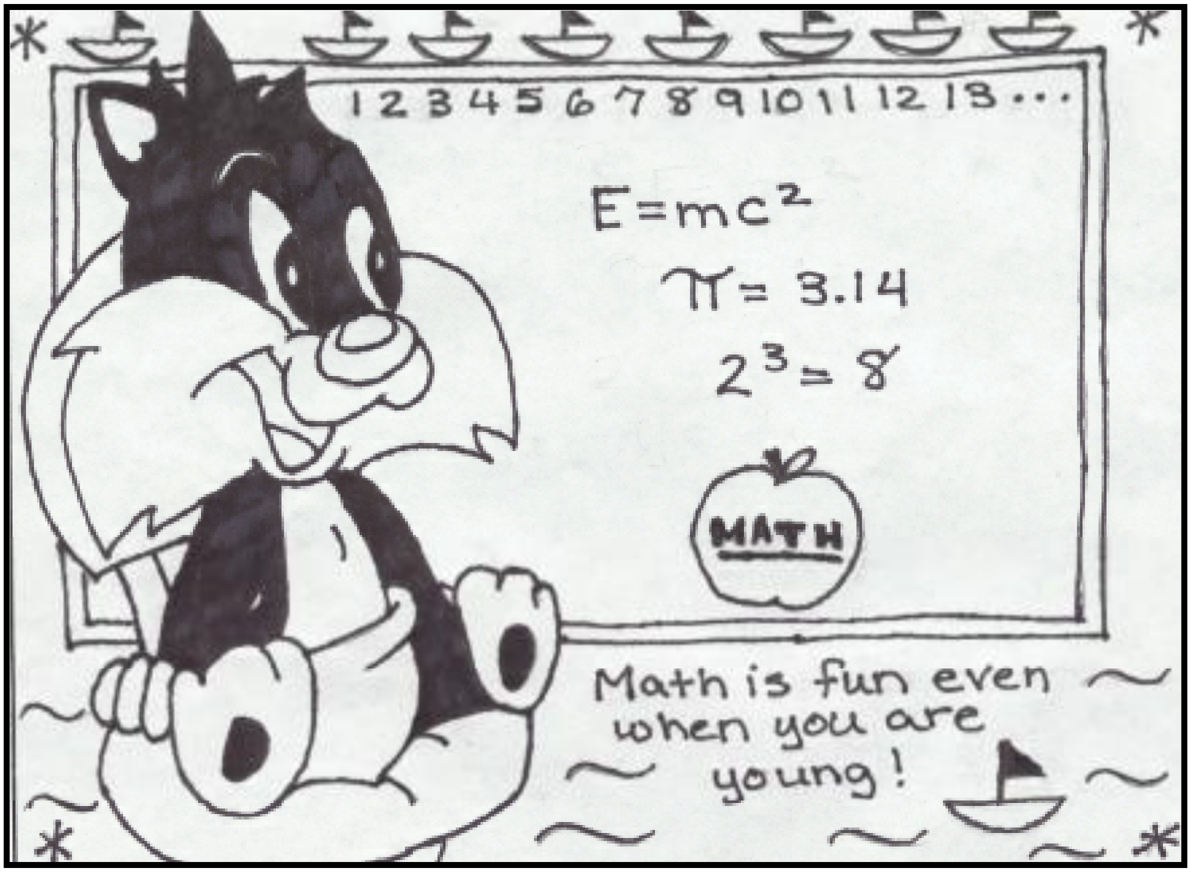
Sail into Summer with Math!



For Students Entering Honors Geometry

This summer math booklet was developed to provide students in kindergarten through the eighth grade an opportunity to review grade level math objectives and to improve math performance.

Honors Geometry Summer Packet

Table of Contents

Fraction Operations

Exercises: Perform the indicated operation No Calculators!
SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. $\frac{6}{7}+\frac{2}{3}=$

2. $\frac{8}{9}+\frac{3}{4}=$

3. $\frac{9}{11}-\frac{2}{5}=$

4. $\frac{5}{7}-\frac{5}{9}=$

5. $\frac{6}{11}\bullet\frac{2}{3}=$

6. $\frac{7}{9}\bullet\frac{3}{5}=$

7. $\frac{6}{7}\div\frac{1}{5}=$

8. $\frac{7}{11}\div\frac{3}{5}=$

9. $3\frac{1}{2}+5\frac{3}{5}=$

10. $6\frac{17}{25}+8\frac{4}{7}=$

11. $6\frac{2}{3}+9\frac{7}{9}=$

12. $8\frac{3}{10}-6\frac{7}{9}=$

13. $9\frac{7}{15}-2\frac{7}{12}=$

14. $12\frac{8}{9}-7\frac{3}{4}=$

15. $6\frac{2}{3}\bullet7\frac{3}{7}=$

16. $3\frac{1}{3}\bullet6\frac{4}{5}=$

17. $7\frac{1}{8}\bullet6=$

18. $4\frac{1}{4}\div\frac{5}{7}=$

19. $3\frac{2}{3}\div4\frac{3}{7}=$

20. $\frac{3}{4}\div2\frac{3}{11}=$

21. $6\frac{1}{5}\div8\frac{2}{5}=$

Order of Operations

Evaluate each expression. SHOW ALL WORK. No calculators.

1. $8 + 14 - 2 + 4 \times 2^3$

2. $30 - 3^3 + 8 \times 3 \div 12$

3. $3[16 - (3 + 7) \div 5]$

4. $16 \div 4(2) \times 9$

5. $[1 + 3(9 + 12)] - 4^3$

6. $18 \times 3 \div 3^3$

7. $8^2 - 1 \times 3 - 5$

8. $85 - (4 \times 2)^2 - 3 \times 7$

9. $20 - (3^5 \div 27) \times 2$

10. $10 + 5^3 - 25$

11. $6 \div (17 - 11) \times 14$

12. $(9 + 7)^2 \div 4 \times (2 \times 3)$

Proportions

Solve each proportion for the missing term. SHOW YOUR WORK. No Calculators.

1. $\frac{4}{x} = \frac{2}{10}$

2. $\frac{1}{y} = \frac{3}{15}$

3. $\frac{6}{5} = \frac{x}{15}$

4. $\frac{20}{28} = \frac{n}{21}$

5. $\frac{6}{8} = \frac{7}{a}$

6. $\frac{16}{7} = \frac{9}{n}$

7. $\frac{1}{0.19} = \frac{12}{n}$

8. $\frac{2}{0.21} = \frac{8}{n}$

9. Seth earns $152 in 4 days. At that rate, how many days will it take to earn $532?

10. Lanette drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles?

11. A blueprint for a house states 2.5 inches equals 10 feet. If the length of the wall is 12 feet, how long is the wall in the blueprint?

Squares, Square Roots, and the Laws of Exponents

Hints/Guide:

Exponents are a way to represent repeated multiplication, so that 3^4 means 3 multiplied four times, or $3 \cdot 3 \cdot 3 \cdot 3$, which equals 81. In this example, 3 is the base and 4 is the power.

Roots are the base numbers that correspond to a given power, so the square (referring to the power of 2) root of 81 is 9 because $9 \cdot 9 = 81$ and the fourth root of 81 is 3 because $3 \cdot 3 \cdot 3 \cdot 3$ is 81.

$\sqrt[n]{x}$, where n is the root index and x is the radicand

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

	Adding powers	$a^m a^n = a^{m+n}$
	Multiplying powers	$(a^m)^n = a^{mn}$
	Subtracting powers	$\dfrac{a^m}{a^n} = a^{m-n}$
	Negative powers	$a^{-n} = \dfrac{1}{a^n}$
	To the zero power	$a^0 = 1$

Exercises: Evaluate:

1. $(8-4)^2 =$
2. $(4-2)^2 (5-8)^3 =$
3. $5(8-3)^2 =$
4. $\sqrt{25-16} =$
5. $\sqrt{5(9 \bullet 125)} =$
6. $\sqrt{(8-4)(1+3)} =$

Simplify the following problems using exponents (Do not multiply out):

7. $5^2\ 5^4 =$
8. $(12^4)^3 =$
9. $5^9 \div 5^4 =$
10. $10^3 \div 10^{-5} =$
11. $7^{-3} =$
12. $3^{-4} =$
13. $(3^3 \bullet 3^2)^3 =$
14. $5^3 \cdot 5^4 \div 5^7 =$

Solving Equations I

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In two-step equations, we must undo addition and subtraction first, then multiplication and division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side. Examples:

1. 4x - 6 = -14

   + 6 + 6

   $\frac{4x}{4} = \frac{-8}{4}$

   x = -2

   Solve: 4 (-2) - 6 = -14

   -8 - 6 = -14

   -14 = -14

2. $\frac{x}{-6} - 4 = -8$

   + 4 + 4

   $-6 \bullet \frac{x}{-6} = -4 \bullet -6$

   x = 24

   Solve: (24/-6) - 4 = -8

   -4 - 4 = -8

   -8 = -8

When solving equations that include basic mathematical operations, we must simplify the mathematics first, then solve the equations. For example:

5 (4 - 3) + 7x = 4 (9 - 6)
5 (1) + 7x = 4 (3)
5 + 7x = 12
- 5 - 5
$\frac{7x}{7} = \frac{7}{7}$
x = 1

Check: 5 (4 - 3) + 7 (1) = 4 (9 - 6)
5 + 7 = 4 (3)
12 = 12

Exercises: Solve the following equations using the rules listed on the previous pages:
SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. -4t + 3t - 8 = 24
2. $\frac{m}{-5} + 6 = 4$
3. -4r + 5 - 6r = -32
4. $\frac{x}{-3} + (-7) = 6$
5. 6g + (-3) = -12
6. $\frac{y}{-2} + (-4) = 8$
7. $9 - 5(4-3) = -16 + \frac{x}{3}$
8. 6t - 14 - 3t = 8 (7 – (-2))
9. $7(6-(-8)) = \frac{t}{-4} + 2$
10. 7 (3 - 6) = 6 (4 + t)
11. 4r + 5r - 8r = 13 + 6
12. 3(7 + x) = 5(7 - (-4))
13. Explain in words how to solve a two step equation, similar to any of the equations in problems 2 through 6 above.

Solving Equations II

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division. Remember the golden rule of equation solving. Examples:

$$\begin{array}{rcl}
8x - 6 &=& 4x + 5 \\
\underline{\;\; - 4x} & & \underline{- 4x} \\
4x - 6 &=& 5 \\
\underline{\;\; + 6} & & \underline{+ 6} \\
\frac{4x}{4} &=& \frac{11}{4} \\
x &=& 2\frac{3}{4}
\end{array}
\qquad\qquad
\begin{array}{rcl}
5 - 6t &=& 24 + 4t \\
\underline{+ 6t} & & \underline{+ 6t} \\
5 &=& 24 + 10t \\
\underline{- 24} & & \underline{- 24} \\
\frac{-19}{10} &=& \frac{10t}{10} \\
-1\frac{9}{10} &=& t
\end{array}$$

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4r - 7 = 6r + 16 - 3r$
2. $13 + 3t = 5t - 9$
3. $-3x + 5 = 3x - 3$
4. $6y + 5 = 6y - 15$
5. $5x - 8 = 6 - 7x + 2x$
6. $7p - 8 = -6p + 8$
7. **Rowboat Rentals:** \$5.00 per hour plus a \$100.00 deposit. *Deposit will be refunded if the boat is returned undamaged.*

   Which equation represents the total cost for renting and returning a row-boat undamaged? Let c be the total cost in dollars and t be the time in hours.

   a. $c = 5t + 100$
   b. $c = 500t$
   c. $c = 100t + 5$
   d. $c = 5t$

8. Ted wants to buy a \$400.00 bike. He has two options for payment.

   Option One: Ted can borrow the \$400.00 from his father and repay him \$40.00 a month for a year.

   Option Two: The bike shop will finance the bike for one year at a 15% annual interest rate. The formula for the total amount paid (a) is:

   $a = p + prt$, where p in the amount borrowed, r is the rate of interest, and t is the time in years.

   Which option would cost Ted the least amount of money?

   Explain how you determined your answer. Use words, symbols, or both in your explanation.

Inequalities

Hints/Guide:

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality as many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

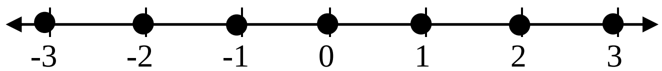
Inequality - a statement containing one of the following symbols:

$<$ is less than $\quad$ $>$ is greater than $\quad$ $\leq$ is less than or equal to

$\geq$ is greater than or equal to $\quad$ _ is not equal to

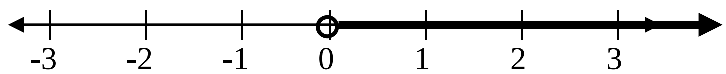
Examples:

1. Integers between -4 and 4.



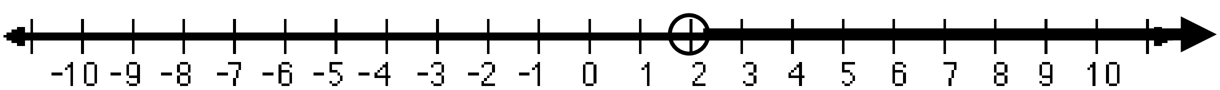
2. All numbers between -4 and 4.

3. The positive numbers.



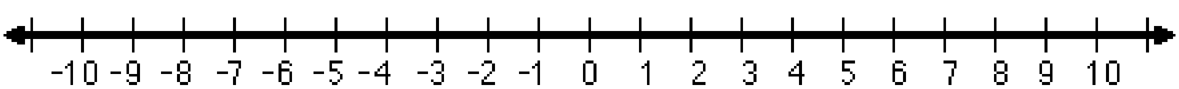
So, to solve the inequality -4x < -8 becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore x > 2 is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:

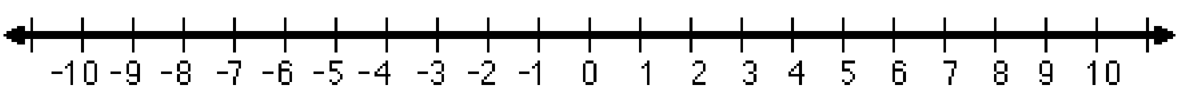


Exercises: Solve the following problems: No Calculators!

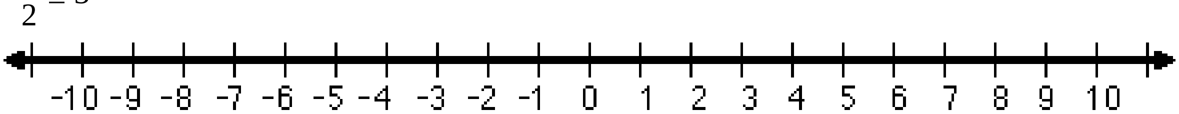
1. $4x > 9$



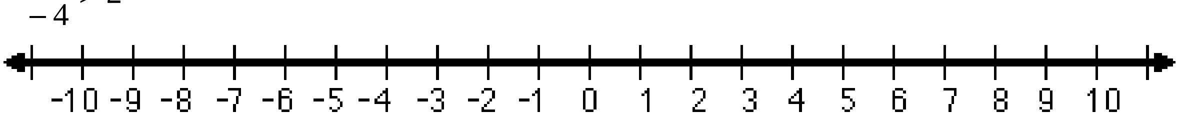
2. $-5t \geq -15$



3. $\frac{x}{2} \geq 3$



4. $\frac{x}{-4} > 2$



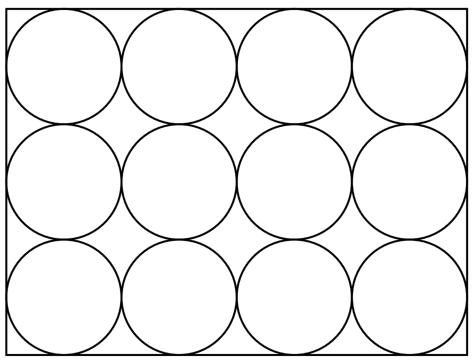
Irregular Area

Hints/Guide:

To solve problems involving irregular area, use either an additive or a subtractive method. In an additive area problem, break the object down into know shapes and then add the areas together. In a subtractive area problem, subtract the area of known shapes from a larger whole.

Exercises:

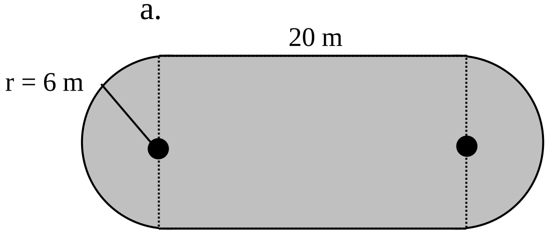
1. The baking sheet shown holds 12 cookies. Each cookie has a diameter of 3 inches.



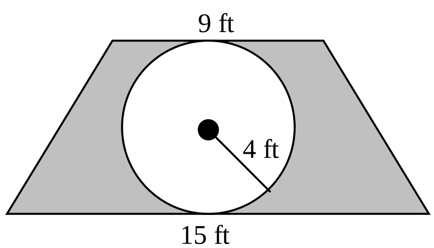
What is the area of the unused part of the baking sheet? Round your answer to the nearest square inch.

2. Find the area of the shaded regions.

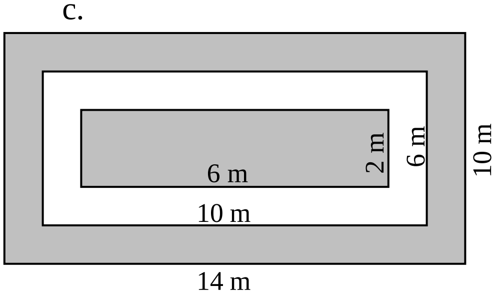
a.



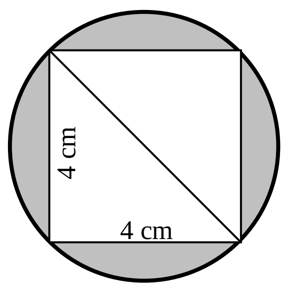
b.



c.



d.



Factoring Quadratic Equations

Hints/Guide:

Factoring a polynomial can make a problem easier to solve or allow one to easily find the roots of an equation. Factoring can be thought of as the opposite of distribution because terms are expanded, usually from a trinomial (three term) equation to an equation which is the product of two binomial (two) terms.

Examples: $x^2 + 5x + 6 = (x + 2)(x + 3)$
$2x^2 - 3x - 2 = (2x + 1)(x - 2)$

If these equations are set to zero, then we can solve for the roots of the equation by setting each binomial term to zero.

Example: $2x^2 - 3x - 2 = 0$ $\quad (2x + 1)(x - 2) = 0$
which means that $2x + 1 = 0$ or $x - 2 = 0$ because if the product is zero, then one of the factors must be zero.
therefore, $x = -0.5$ or $x = 2$.

Exercises: Find the roots of each equation.

1. $a^2 + a - 30 = 0$
2. $b^2 + 7b + 12 = 0$
3. $m^2 - 14m + 40 = 0$
4. $s^2 + 3s - 180 = 0$
5. $7a^2 + 22a + 3 = 0$
6. $2x^2 - 5x - 12 = 0$
7. $4n^2 - 4n - 35 = 0$
8. $72 - 26y + 2y^2 = 0$
9. $10 + 19m + 6m^2 = 0$
10. $x^2 - 2x = 15$
11. $2x^2 + x = 3$
12. $3x^2 - 4x = 4$

Solving Systems of Equations

Hints/Guide:

A system of equations occurs when a situation involves multiple components that can individually be described using equations. Where these equations intersect, their x and y values are the same. Graphically, this appears as an intersection of lines. Algebraically, the x and y values that solve simultaneous equations are the same. The three primary methods of solving systems of equations are graphically, by substitution, and by linear combination.

Exercises: Solve each system of equations using any method.

1. $3x - 4y = 3$
   $6x + 8y = 54$

2. $9 = 5x + 2y$
   $-31 = 3x - 4y$

3. $2x - 7y = 19$
   $-6x - 21y = -15$

4. $4x - 11y = -9$
   $-6x + 22y = 8$

5. Hanz and Mario went to a sale at a music store where all CDs were one price and all cassettes were another price. Hanz bought 2 CDs and 2 cassettes for $40.00, while Mario bought 1 CD and 4 cassettes for $44.00.

   The equations below represent these purchases, where x is the cost of a CD and y is the cost of a cassette.

   Hanz $2x + 2y = 40$ Mario $x + 4y = 44$

   What are the costs of a single CD and a single cassette? Solve the system of equations by either constructing a graph on a sheet of graph paper or by using an algebraic process. Explain how you determined the costs. Use words, symbols, or both in your explanation.

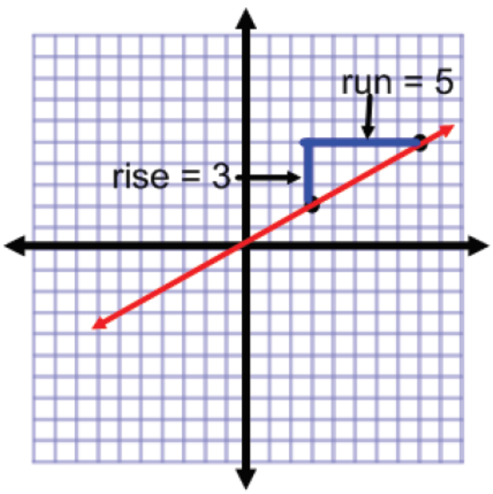
6. An exam will have 20 questions and be worth a total of 100 points. There will be a true/false section where the questions are worth 3 points each and a short essay section where the questions will be worth 11 points each. How many essay questions will there be on the test?

Graphing Linear Equations Reference Sheet

Hints/Guide:

Slope= $\frac{\text{rise}}{\text{run}}$

- Calculate the slope by choosing two points on the line.
- Count the rise (how far up or down to get to the next point?) This is the numerator.
- Count the run (how far left or right to get to the next point?) This is the denominator.
- Write the slope as a fraction.



Slope = 3/5

** Read the graph from left to right. If the line is **falling**, then the slope is **negative**.
If the line is **rising**, the slope is **positive**.

When counting the rise and run, if you count **down or **left**, then the number is **negative**. If you count **up** or **right**, the number is **positive**.

Slope Intercept Form

$y = mx + b$

m: Slope b: Y-intercept

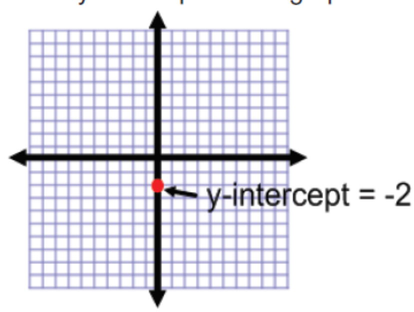
Graphing Using Slope Intercept Form

1. Identify the slope and y-intercept in the equation.

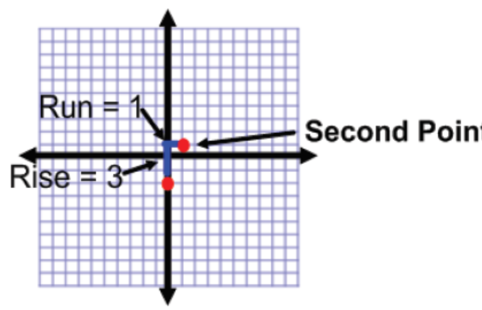
   $y = 3x - 2$

   3: Slope   -2: Y-intercept

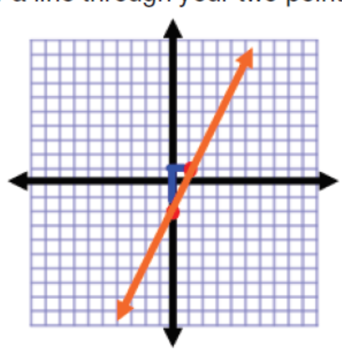
2. Plot the y-intercept on the graph.



3. From the y-intercept, count the rise and run for the slope. Plot the second point.



4. Draw a line through your two points.



Graphing Linear Equations Practice

Identify the slope and y-intercept of the following linear equations and then graph them on the coordinate plane.

1. $y = 2x + 1$

Slope:

y-int:

2. $y = 3x - 4$

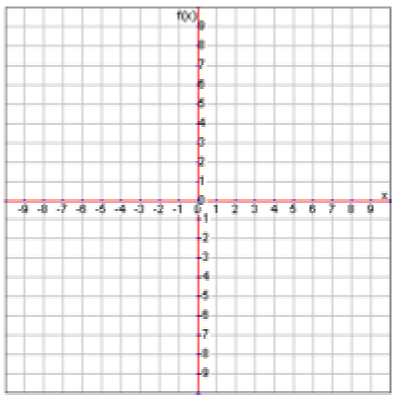
Slope:

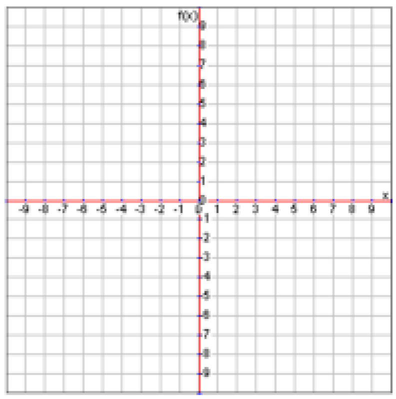
y-int:

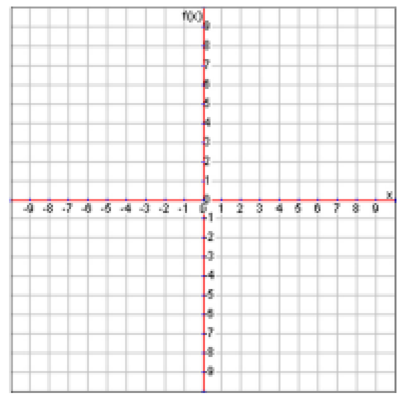
3. $y = -\frac{1}{3}x + 5$

Slope:

y-int:





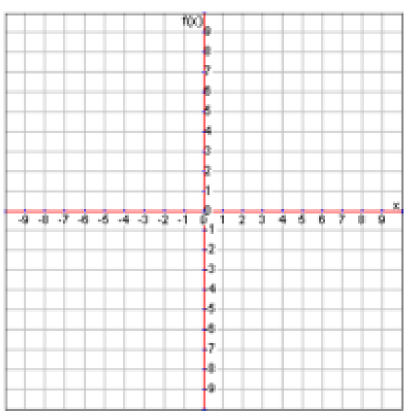


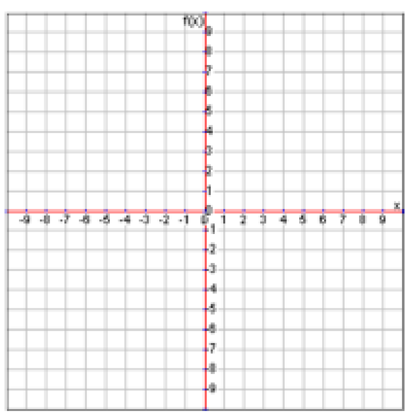
Put the following equations into slope-intercept form (solve for y) and then graph them on the coordinate plane.

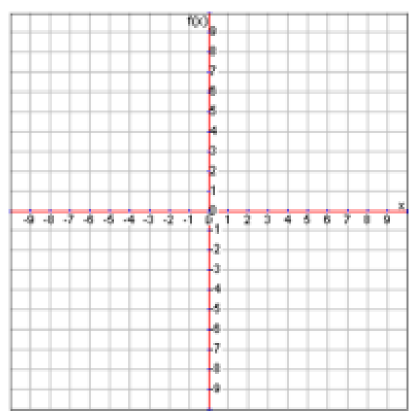
4. $2x + y = 2$

5. $-3x + y = 4$

6. $4x + y = -5$







Simplifying Radicals PREVIEW
(**This information will be explicitly taught in Honors Geometry)

Hints/Guide:
To simplify radicals, first factor the radicand as much as possible, then "pull out" square terms using the following rules:

$\sqrt{a^2} = a$ $\qquad$ $\sqrt{ab} = \sqrt{a}\sqrt{b}$ $\qquad$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ as long as $b \neq 0$

A radical is in simplest form when:

- there is no integer under the radical sign with a perfect square factor,
- there are no fractions under the radical sign, and
- there are no radicals in the denominator.

Exercises: Simplify each expression.

1. $\sqrt{\frac{15}{81}} =$
2. $\sqrt{24} + 5\sqrt{6} =$
3. $\sqrt{75} + \sqrt{243} =$
4. $4 + 2\sqrt{10} =$
5. $\sqrt{28} + \sqrt{7} =$
6. $\sqrt{\frac{27}{49}} =$
7. $5\sqrt{3} - \sqrt{75} =$
8. $4\sqrt{3} \bullet \sqrt{18} =$
9. $\sqrt{128} - \sqrt{8} =$
10. $(5\sqrt{3})^2 =$
11. $\sqrt{128} + \sqrt{50} =$
12. $\sqrt{75} \bullet \sqrt{27} =$
13. Nina says that $16 + 4\sqrt{2}$ cannot be simplified. George says that is can be simplified to $20\sqrt{2}$. Who is correct? Explain how you know.