

# Sail into Summer with Math!



## For Students Entering Investigations into Mathematics

This summer math booklet was developed to provide students in kindergarten through the eighth grade an opportunity to review grade level math objectives and to improve math performance.

Summer 2005

### **Student Responsibilities**

Students will be able to improve their own math performance by:

- Completing the summer math booklet
- Reviewing math skills throughout the summer.

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Student Signature

\_\_\_\_\_

Grade

\_\_\_\_\_

Date

### **Parent Responsibilities**

Parents will be able to promote student success in math by:

- Supporting the math goal of the cluster of schools,
- Monitoring student completion of the summer math booklet,
- Encouraging student use of math concepts in summer activities.

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Parent Signature

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Date

# IM Summer Mathematics Packet

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**Rename Fractions, Percents, and Decimals**

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as  $\frac{3}{5}$ , and divide the numerator (the top number of a fraction) by the denominator (the bottom number of a fraction). So:

$$\begin{array}{r} \phantom{5} \overline{) 3.0} \\ \underline{- 30} \\ 0 \end{array} \quad \text{and the fraction } \frac{3}{5} \text{ is equivalent to the decimal } 0.6$$

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a decimal by 100, so  $0.6 \times 100 = 60$  and

$$\frac{3}{5} = 0.6 = 60\%$$

To convert a percent to a decimal, we divide the percent by 100,  $60\% \div 100 = 0.6$  so  $60\% = 0.6$

To convert a fraction into a percent, we can use a proportion to solve,

$$\frac{3}{5} = \frac{x}{100}, \text{ so } 5x = 300 \text{ which means that } x = 60 = 60\%$$

Exercises:

No Calculators!

Rename each fraction as a decimal:

1.  $\frac{1}{5} =$

2.  $\frac{3}{4} =$

3.  $\frac{1}{2} =$

4.  $\frac{1}{3} =$

5.  $\frac{8}{10} =$

6.  $\frac{2}{3} =$

Rename each fraction as a percent:

7.  $\frac{1}{5} =$

8.  $\frac{3}{4} =$

9.  $\frac{1}{2} =$

10.  $\frac{1}{3} =$

11.  $\frac{8}{10} =$

12.  $\frac{2}{3} =$

Rename each percent as a decimal:

13.  $8\% =$

14.  $60\% =$

15.  $11\% =$

16.  $12\% =$

17.  $40\% =$

18.  $95\% =$

**Fraction Operations**

Hints/Guide:

When adding and subtracting fractions, we need to be sure that each fraction has the same denominator, then add or subtract the numerators together. For example:

$$\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{6}{8} = \frac{1+6}{8} = \frac{7}{8}$$

That was easy because it was easy to see what the new denominator should be, but what about if

it is not so apparent? For example:  $\frac{7}{12} + \frac{8}{15}$

For this example we must find the Lowest Common Denominator (LCM) for the two denominators. 12 and 15

$$12 = 12, 24, 36, 48, 60, 72, 84, \dots$$

$$15 = 15, 30, 45, 60, 75, 90, 105, \dots$$

$$\text{LCM}(12, 15) = 60$$

So,  $\frac{7}{12} + \frac{8}{15} = \frac{35}{60} + \frac{32}{60} = \frac{35+32}{60} = \frac{67}{60} = 1\frac{7}{60}$       Note: Be sure answers are in lowest terms

To multiply fractions, we multiply the numerators together and the denominators together, and then simplify the product. To divide fractions, we find the reciprocal of the second fraction (flip the numerator and the denominator) and then multiply the two together. For example:

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6} \quad \text{and} \quad \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$$

Exercises: Perform the indicated operation:

No calculators!

**SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.

1.  $\frac{1}{4} + \frac{3}{5} =$

2.  $\frac{6}{7} + \frac{2}{3} =$

3.  $\frac{2}{5} + \frac{8}{9} =$

4.  $\frac{3}{4} - \frac{2}{3} =$

5.  $\frac{2}{5} - \frac{2}{9} =$

6.  $\frac{9}{11} - \frac{2}{5} =$

7.  $\frac{1}{3} \cdot \frac{2}{3} =$

8.  $\frac{3}{4} \cdot \frac{3}{5} =$

9.  $\frac{7}{8} \cdot \frac{2}{5} =$

10.  $\frac{3}{8} \div \frac{3}{4} =$

11.  $\frac{1}{4} \div \frac{1}{4} =$

12.  $\frac{7}{11} \div \frac{3}{5} =$

### Multiply Fractions and Solve Proportions

Hints/Guide:

To solve problems involving multiplying fractions and whole numbers, we must first place a one under the whole number, then multiply the numerators together and the denominators together. Then we simplify the answer:

$$\frac{6}{7} \bullet 4 = \frac{6}{7} \bullet \frac{4}{1} = \frac{24}{7} = 3\frac{3}{7}$$

To solve proportions, one method is to determine the multiplying factor of the two equal ratios. For example:

$$\frac{4}{9} = \frac{24}{x} \text{ since 4 is multiplied by 6 to get 24, we multiply 9 by 6, so } \frac{4}{9} = \frac{24}{54}.$$

Since the numerator of the fraction on the right must be multiplied by 6 to get the numerator on the left, then we must multiply the denominator of 9 by 6 to get the missing denominator, which must be 54.

Exercises: Solve (For problems 8 - 15, solve for N):

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1.  $4 \bullet \frac{3}{4} =$

2.  $\frac{1}{5} \bullet 7 =$

3.  $8 \bullet \frac{1}{5} =$

4.  $6 \bullet \frac{3}{7} =$

5.  $\frac{4}{5} \bullet 4 =$

6.  $\frac{2}{3} \bullet 6 =$

7.  $7 \bullet \frac{1}{4} =$

8.  $\frac{1}{5} = \frac{n}{20}$

9.  $\frac{3}{n} = \frac{12}{28}$

10.  $\frac{1}{n} = \frac{5}{25}$

11.  $\frac{n}{4} = \frac{3}{12}$

12.  $\frac{3}{7} = \frac{12}{n}$

13.  $\frac{n}{9} = \frac{12}{27}$

14.  $\frac{2}{3} = \frac{18}{n}$

15.  $\frac{2}{7} = \frac{n}{21}$

**Add Mixed Numbers**

Hints/Guide:

When adding mixed numbers, we add the whole numbers and the fractions separately, then simplify the answer. For example:

$$\begin{array}{r} 4\frac{1}{3} = 4\frac{8}{24} \\ + 2\frac{6}{8} = 2\frac{18}{24} \\ \hline 6\frac{26}{24} = 6 + 1\frac{2}{24} = 7\frac{2}{24} = 7\frac{1}{12} \end{array}$$

First, we convert the fractions to have the same denominator, then add the fractions and add the whole numbers. If needed, we then simplify the answer.

Exercises: Solve in lowest terms:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

$$1. \begin{array}{r} 2\frac{1}{4} \\ + 8\frac{1}{2} \\ \hline \end{array}$$

$$2. \begin{array}{r} 3\frac{8}{15} \\ + 7\frac{1}{3} \\ \hline \end{array}$$

$$3. \begin{array}{r} 3\frac{3}{5} \\ + 5\frac{1}{2} \\ \hline \end{array}$$

$$4. \begin{array}{r} 5\frac{3}{8} \\ + 4\frac{1}{4} \\ \hline \end{array}$$

$$5. \begin{array}{r} 7\frac{3}{7} \\ + 6\frac{1}{2} \\ \hline \end{array}$$

$$6. \begin{array}{r} 5\frac{5}{9} \\ + 1\frac{1}{3} \\ \hline \end{array}$$

$$7. \begin{array}{r} 4\frac{1}{3} \\ + 6\frac{1}{4} \\ \hline \end{array}$$

$$8. \begin{array}{r} 1\frac{2}{3} \\ + 6\frac{1}{4} \\ \hline \end{array}$$

$$9. \begin{array}{r} 1\frac{2}{9} \\ + 5\frac{2}{3} \\ \hline \end{array}$$

**Subtract Mixed Numbers**

Hints/Guide:

When subtracting mixed numbers, we subtract the whole numbers and the fractions separately, then simplify the answer. For example:

$$\begin{array}{r} 7\frac{3}{4} = 7\frac{18}{24} \\ -2\frac{15}{24} = 2\frac{15}{24} \\ \hline 5\frac{3}{24} = 5\frac{1}{8} \end{array}$$

First, we convert the fractions to have the same denominator, then subtract the fractions and subtract the whole numbers. If needed, we then simplify the answer.

Exercises: Solve in lowest terms:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

$$1. \begin{array}{r} 4\frac{1}{3} \\ -2\frac{1}{4} \\ \hline \end{array}$$

$$2. \begin{array}{r} 6\frac{3}{4} \\ -\frac{2}{3} \\ \hline \end{array}$$

$$3. \begin{array}{r} 9\frac{2}{3} \\ -6\frac{1}{4} \\ \hline \end{array}$$

$$4. \begin{array}{r} 6\frac{3}{4} \\ -5\frac{1}{5} \\ \hline \end{array}$$

$$5. \begin{array}{r} 7\frac{1}{2} \\ -3\frac{1}{4} \\ \hline \end{array}$$

$$6. \begin{array}{r} 3\frac{1}{2} \\ -2\frac{3}{10} \\ \hline \end{array}$$

$$7. \begin{array}{r} 8\frac{1}{2} \\ -4\frac{7}{10} \\ \hline \end{array}$$

$$8. \begin{array}{r} 8\frac{1}{3} \\ -5\frac{5}{6} \\ \hline \end{array}$$

$$9. \begin{array}{r} 8\frac{5}{8} \\ -6\frac{3}{4} \\ \hline \end{array}$$

**Multiply Mixed Numbers**

Hints/Guide:

To multiply mixed numbers, we first convert the mixed numbers into improper fractions. This is done by multiplying the denominator by the whole number part of the mixed number and then adding the numerator to this product, and this is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the mixed number. For example:

$$3\frac{2}{5} \text{ leads to } 3 \cdot 5 + 2 = 17 \text{ so } 3\frac{2}{5} = \frac{17}{5}$$

Once the mixed numbers are converted into improper fractions, we multiply and simplify just as with regular fractions. For example:

$$5\frac{1}{5} \cdot 3\frac{1}{2} = \frac{26}{5} \cdot \frac{7}{2} = \frac{182}{10} = 18\frac{2}{10} = 18\frac{1}{5}$$

Exercises: Solve and place your answer in lowest terms:

No Calculators!

**SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.

1.  $3\frac{1}{3} \cdot 4\frac{1}{2} =$

2.  $2\frac{2}{3} \cdot 1\frac{1}{4} =$

3.  $1\frac{1}{9} \cdot 4\frac{3}{5} =$

4.  $4\frac{3}{4} \cdot 1\frac{1}{5} =$

5.  $3\frac{1}{3} \cdot 6\frac{4}{5} =$

6.  $6\frac{2}{3} \cdot 7\frac{3}{7} =$

7.  $1\frac{4}{5} \cdot 1\frac{2}{3} =$

8.  $2\frac{2}{5} \cdot 4\frac{2}{7} =$

9.  $4\frac{1}{3} \cdot 1\frac{1}{8} =$

**Divide Mixed Numbers**

Hints/Guide:

To divide mixed numbers, we must first convert to improper fractions using the technique shown in multiplying mixed numbers. Once we have converted to improper fractions, the process is the same as dividing regular fractions. For example:

$$2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \cdot \frac{3}{10} = \frac{15}{20} = \frac{3}{4} \qquad 3\frac{1}{2} \div 8\frac{2}{3} = \frac{7}{2} \div \frac{26}{3} = \frac{7}{2} \cdot \frac{3}{26} = \frac{21}{52}$$

Exercises: Solve and place your answer in lowest terms: No Calculators!  
**SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.

1.  $1\frac{1}{5} \div 4\frac{2}{5} =$

2.  $6\frac{1}{2} \div 4\frac{2}{3} =$

3.  $5\frac{1}{2} \div 6\frac{2}{3} =$

4.  $\frac{8}{9} \div 2\frac{3}{5} =$

5.  $3\frac{2}{3} \div 4\frac{3}{7} =$

6.  $4\frac{4}{7} \div \frac{4}{9} =$

7.  $6\frac{1}{5} \div 8\frac{2}{5} =$

8.  $4\frac{1}{4} \div \frac{5}{7} =$

9.  $6\frac{4}{7} \div 3\frac{3}{5} =$



**Summer Mathematics Packet**

**Find Percent of a Number**

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut as moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number. For example:

$$45\% \text{ of } 240 = 45\% \times 240 = 0.45 \times 240 = 108$$

Exercises: Solve for n:

No Calculators!

**SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.

1.  $30\% \text{ of } 450 = n$

2.  $7\% \text{ of } 42 = n$

3.  $10\% \text{ of } 321 = n$

4.  $15\% \text{ of } 54 = n$

5.  $65\% \text{ of } 320 = n$

6.  $80\% \text{ of } 64 = n$

7.  $9\% \text{ of } 568 = n$

8.  $15\% \text{ of } 38 = n$

9.  $25\% \text{ of } 348 = n$

10.  $85\% \text{ of } 488 = n$

11.  $90\% \text{ of } 750 = n$

12.  $6\% \text{ of } 42 = n$

13.  $60\% \text{ of } 78 = n$

14.  $4\% \text{ of } 480 = n$

15.  $10\% \text{ of } 435 = n$

16.  $24\% \text{ of } 54 = n$

**Solve Problems using Percent**

Hints/Guide:

When solving percent problems, we apply the rules for finding percent of a number in realistic situations. For example, to find the amount of sales tax on a \$450.00 item if the tax rate is 5%, we find 5% of 450 ( $.05 \times 450 = 22.5$ ), and then label our answer in dollars, getting \$22.50.

Exercises:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. Susie has just bought a pair of jeans for \$45.00, a sweater for \$24.00, and a jacket for \$85.00. The sales tax is 5%. What is her total bill?
2. Jack bought a set of golf clubs for \$250.00 and received a rebate of 20%. How much was the rebate?
3. A construction manager calculates it will cost \$2,890 for materials for her next project. She must add in 10% for scrap and extras. What will be the total cost?
4. The regular price for a video game system is \$164.50 but is on sale for 30% off. What is the amount of the discount?

What is the sale price?

5. Cindy earns a 15% commission on all sales. On Saturday, she sold \$980 worth of merchandise. What was the amount of commission she earned on Saturday?
6. The band had a fundraiser and sold \$25,000 worth of candy. They received 40% of this amount for themselves. How much did they receive?

## Summer Mathematics Packet

### Mean, Median, and Mode

Hints/Guide:

We need to define some terms to solve problems involving mean, median, and mode. Mean is the sum of the numbers being considered divided by the total number of numbers being considered (also called "average"). Median is the number in the middle of the data set after the numbers have been placed in order from least to greatest. If there is an even number of elements, the median is the mean of the two numbers in the middle of the data set. The mode is the number or numbers that occur most frequently in a data set. For example, with the data set of 56, 62, 67, 45, 81, 76:

Mean is  $56 + 62 + 67 + 45 + 81 + 76 = 387$  and  $387 \div 6 = 64.5$ , so the mean is 64.5

Median is (in order the data is 45, 56, 62, 67, 76, 81) the mean of 62 and 67, which is  $(62 + 67 = 129$  and  $129 \div 2 = 64.5)$  also 64.5.

There is no mode, because no number occurs more than once.

Exercises:

**SHOW ALL WORK.** Use a separate sheet of paper (if necessary) and staple to this page.

Find the mean, median, and mode of each of the following data sets:

1. 54, 65, 74, 35, 87

2. 54.6, 45.98, 67.4, 55.6, 45.7, 58.9

3. 122, 145, 156, 176, 198, 202

4. 11, 14, 16, 15, 32, 23, 27, 27, 23, 43

5. 6, 7, 8, 4, 6, 5, 8, 3, 6, 8, 5, 4

6. -4, 7, -3, 4, 8, 12, -5, -3, 8, 16, 9

7. 43, 56, 98, 67, 87

8. 12, 15, 14, 18, 33, 32, 24, 26, 27

9. 13.2, 17.6, 18.34, 12.54, 17.4, 15.8, 13.7, 17.6

10. Write a data set that has 5 numbers with a mean of 84 and a median of 86.

**Integers I**

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add its additive inverse.

For example  $6 - 11 = a$  becomes  $6 + -11 = a$  and solves as  $-5 = a$ .

Exercises: Solve the following problems:

No Calculators!

1.  $6 + (-7) =$

2.  $(-4) + (-5) =$

3.  $6 + (-9) =$

4.  $(-6) - 7 =$

5.  $6 - (-6) =$

6.  $7 - (-9) =$

7.  $5 + (-8) =$

8.  $-15 + 8 =$

9.  $14 + (-4) =$

10.  $-9 - (-2) =$

11.  $-7 - 6 =$

12.  $-8 - (-19) =$

13.  $29 - 16 + (-5) =$

14.  $-15 + 8 - (-19) =$

15.  $45 - (-13) + (-14) =$

16.  $-15 - 6 - 9 =$

17.  $-7 + (-6) - 7 =$

18.  $29 - 56 - 78 =$

19.  $17 + (-7) - (-5) =$

20.  $45 - (-9) + 5 =$

**Integers II**

Hints/Guide:

The rules for multiplying integers are:

Positive x Positive = Positive

Negative x Negative = Positive

Positive x Negative = Negative

Negative x Positive = Negative

The rules for dividing integers are the same as multiplying integers.

Exercises: Solve the following problems:

No Calculators!

1.  $4 \cdot (-3) =$

2.  $(-12) \cdot (-4) =$

3.  $(-8)(-3) =$

4.  $\frac{-14}{2} =$

5.  $\frac{28}{-4} =$

6.  $\frac{-36}{-6} =$

7.  $6(-5) =$

8.  $8(-4 - 6) =$

9.  $-6(9 - 11) =$

10.  $\frac{(-5)(-6)}{-2} =$

11.  $\frac{6(-4)}{8} =$

12.  $\frac{-56}{2^3} =$

13.  $\frac{-6 - (-8)}{-2} =$

14.  $-7 + \frac{4 + (-6)}{-2} =$

15.  $45 - 4(5 - (-3)) =$

16.  $(-4 + 7)(-5 + 3) =$

17.  $16 - (-3)(-7 + 5) =$

18.  $\frac{4 + (-6) - 5 - 3}{-6 + 4} =$

19.  $(-2)^3(-5 - (-6)) =$

20.  $13(-9 + 7) + 4 =$

**Solving Equations I**

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In one-step equations, we merely undo the operation - addition is the opposite of subtraction and multiplication is the opposite of division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side.

Examples:

1.  $x + 5 = 6$

$$\begin{array}{r} -5 \quad -5 \\ \hline x = 1 \end{array}$$

Check:  $1 + 5 = 6$

$6 = 6$

2.  $t - 6 = 7$

$$\begin{array}{r} +6 \quad +6 \\ \hline t = 13 \end{array}$$

Check:  $13 - 6 = 7$

$7 = 7$

3.  $\frac{4x}{4} = \frac{16}{4}$

$$\frac{4x}{4} = \frac{16}{4}$$

$x = 4$

Check:  $4(4) = 16$

$16 = 16$

4.  $6 \cdot \frac{r}{6} = 12 \cdot 6$

$r = 72$

Check:  $72 \div 6 = 12$

$12 = 12$

Exercises: Solve the following problems:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1.  $x + 8 = 13$

2.  $t - 9 = 4$

3.  $4t = -12$

4.  $\frac{r}{4} = 24$

5.  $y - 4 = 3$

6.  $h + 8 = 5$

7.  $\frac{p}{8} = -16$

8.  $-5k = 20$

9.  $9 - p = 17$

Solving Equations II

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In two-step equations, we must undo addition and subtraction first, then multiplication and division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side. Examples:

$$1. 4x - 6 = -14$$

$$\begin{array}{r} +6 \quad +6 \\ \hline 4x \quad = -8 \end{array}$$

$$\begin{array}{r} 4 \quad 4 \end{array}$$

$$x = -2$$

$$\text{Solve: } 4(-2) - 6 = -14$$

$$-8 - 6 = -14$$

$$-14 = -14$$

$$2. \frac{x}{-6} - 4 = -8$$

$$\begin{array}{r} +4 \quad +4 \end{array}$$

$$-6 \cdot \frac{x}{-6} = -4 \cdot -6$$

$$x = 24$$

$$\text{Solve: } (24/-6) - 4 = -8$$

$$-4 - 4 = -8$$

$$-8 = -8$$

Exercises: Solve the following problems:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

$$1. 4t - 6 = 22$$

$$2. \frac{m}{-5} + 6 = -4$$

$$3. -4r + 5 = 25$$

$$4. \frac{x}{3} - 7 = 6$$

$$5. 5g + 3 = -12$$

$$6. \frac{y}{-2} + (-4) = 8$$