

**Number Theory and Cryptography**

1. Compare the following Ciphers:

- a) Standard Alphabetic (Caesar)
- b) Decimation
- c) Multi-Alphabetic
- d) Matrix

Use the following table for questions 2-5

0	1	2	3	4	5	6	7	8	9	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2
										0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	,	.	!

2. Code or Decode using the Caesar Cipher:

- A. GLQQHUDWVLA
- B. SEND TROOPS AT DAWN
- C. LCZLOOCSDVVB
- D. STUDY WELL

3. Encode the following message using the matrix  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ . MATH IS FUN.

4. Decode D, B W using the modulo 30 inverse matrix  $\begin{bmatrix} 93 & -62 \\ -217 & 155 \end{bmatrix}$ .

5. Decode B E J H A T , I Y using the appropriate shift.

6. Determine if the following are true or false:

- A.  $-8 \equiv 4(\text{mod } 3)$
- B.  $0 \equiv 22(\text{mod } 11)$
- C.  $21 \equiv 3(\text{mod } 2)$
- D.  $-13 \equiv -6(\text{mod } 6)$
- E.  $-5 \equiv 12(\text{mod } 7)$
- F.  $-63 \equiv 21(\text{mod } 12)$
- G.  $-52 \equiv 26(\text{mod } 13)$
- H.  $72 \equiv -18(\text{mod } 20)$

7. Express in terms of the smallest positive values for each given modulus:

- A.  $13(\text{mod } 12)$
- B.  $-16(\text{mod } 29)$
- C.  $610(\text{mod } 8)$
- D.  $-47(\text{mod } 4)$

8. Solve the following:

A.  $16(\text{mod}5) + 23(\text{mod}5) =$

B.  $53(\text{mod}9) - 27(\text{mod}9) =$

C.  $112(\text{mod}5) - 99(\text{mod}5) =$

D.  $1079(\text{mod}8) - 810(\text{mod}8) =$

E.  $(77 \times 6)(\text{mod}18) =$

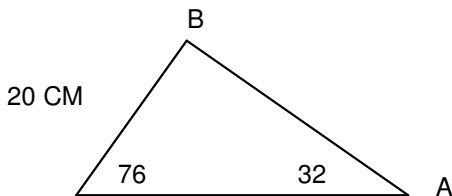
F.  $15(\text{mod}5) + 8(\text{mod}5) - 2(\text{mod}5) =$

G.  $16(\text{mod}12) \times 15(\text{mod}12) + 24(\text{mod}12) =$

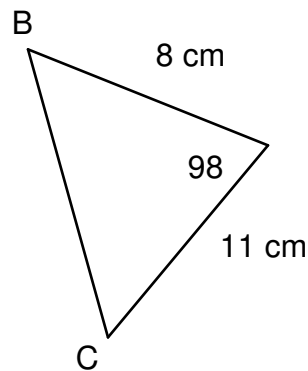
### Trigonometry

FOR THE FOLLOWING PROBLEMS (7 – 12) BE SURE YOUR CALCULATOR IS IN DEGREE MODE.  
TRIANGLES ARE NOT DRAWN TO SCALE.

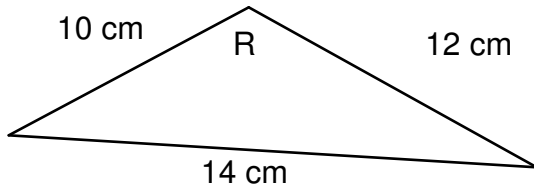
9. Find the distance from point B to point A.



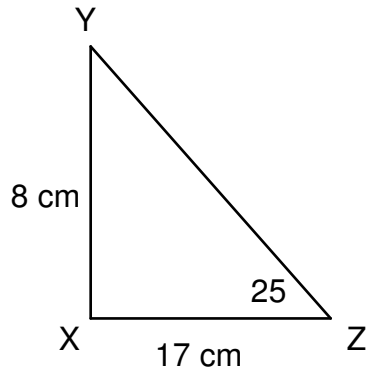
10. Find the length of  $\overline{BC}$ .



11. Find the Measure of  $\angle R$



12. In Right Triangle XYZ, which of the following equations are true?



A.  $x = \frac{17}{\sin 25^\circ}$

B.  $\tan 25^\circ = \frac{8}{17}$

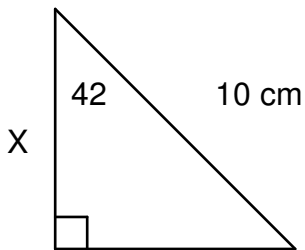
C.  $x = \frac{8}{\sin 25^\circ}$

D.  $x = \frac{17}{\cos 25^\circ}$

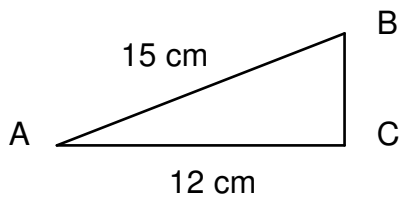
E.  $x = \frac{8}{\cos 25^\circ}$

F.  $\tan 25^\circ = \frac{17}{8}$

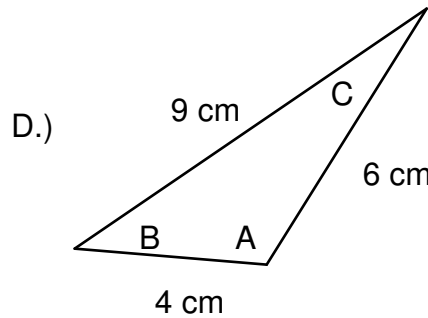
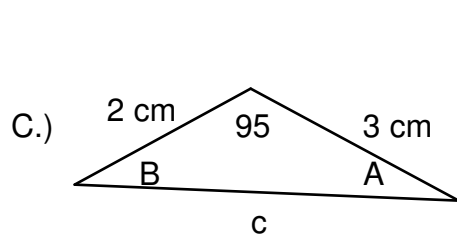
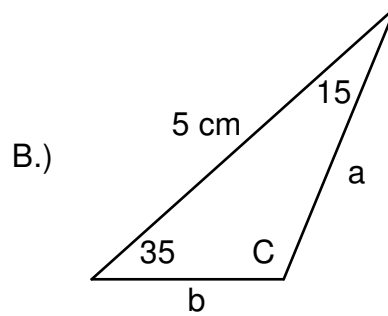
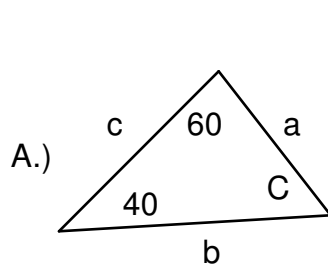
13. Find x



14. Find side BC in Right Triangle ABC.



15. Solve each triangle:



16. Given the equation  $y = 4 \cos(x - \pi) + 5$ , which of the following is/are true?

- A. Amplitude is 5.
- B. Period is  $2\pi$ .
- C. Vertical shift is  $2\pi$  (up).
- D. Vertical shift is 5 (up).
- E. Phase shift is  $\pi$  units (right).
- F. Period is 1.
- G. Phase shift is  $\pi$  units (left).

17. Joe is flying a kite at a  $40^\circ$  angle of elevation over his house, while standing 30 feet from his house. Write the trig function that determines the length of the kite string.

X = \_\_\_\_\_

18. Sally is flying her kite using 100 feet of string. If the angle of elevation from the ground is 65 degrees, how high is Sally's kite?

19. Sam is sitting in a tree 25 feet off the ground when he spots his friend Mary. If the angle of depression is 40 degrees, how far is Mary from the base of the tree?

20. Find the amplitude, period, and phase shift of each of the following:

$$y = -2 \sin 3 \left( x - \frac{\pi}{2} \right)$$

$$y = 3 \cos(x + \pi) + 2$$

### Game Theory

CONSIDER THE FOLLOWING MATRIX:

$$\begin{bmatrix} 4 & 1 & -4 \\ 2 & -5 & 6 \\ -3 & 2 & -2 \end{bmatrix}$$

21. What is the payoff if both players use their 3<sup>rd</sup> strategy? \_\_\_\_\_
22. What is the most aggressive strategy for player 1? \_\_\_\_\_
23. What is the most aggressive strategy for player 2? \_\_\_\_\_
24. What will be the payoff if both players use their most aggressive strategy? \_\_\_\_\_
25. What is the most conservative strategy for player 1? \_\_\_\_\_
26. What is the most conservative strategy for player 2? \_\_\_\_\_
27. What will be the payoff if both players use their most conservative strategy? \_\_\_\_\_
28. Write a game matrix for the situation below:

*Sally and Max each have a \$1.00, a \$5.00, and a \$10.00 bill. Simultaneously, they place a bill on the table. If the sum is \$2.00, \$6.00, or \$20.00, THEN Sally keeps both bills. If the sum is \$10.00, \$11.00, or \$15.00, then Max keeps both bills. Let Sally be player 1.*

29. Find a dominant strategy for the specified player:

A.  $\begin{bmatrix} -1 & 2 & 2 \\ -3 & 0 & 3 \\ -4 & 2 & -2 \end{bmatrix}$  for player 2

B.  $\begin{bmatrix} -3 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$  for player 1

30. Use the matrix  $\begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix}$  to calculate the average payoff if Player 1 uses his first strategy half the time and Player 2 uses his first strategy  $\frac{1}{4}$  of the time.

31. Find the optimal mixed strategy for player one.

$$\begin{bmatrix} 2 & -3 \\ -4 & 3 \end{bmatrix}$$

32. Reduce the matrix, then find the optimal mixed strategy for player two.

$$\begin{bmatrix} -3 & 3 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$

33. The Roman Numeral MCMXCI can be changed to what Hindu-Arabic Numeral?

34. Find the greatest common factor (GCF) of (16, 48, 144).

35. Solve  $(16 + 4) + 5$  in 12-clock.

36. Which number is composite?

- A. 67
- B. 88
- C. 73
- D. 3