Show all work on a separate sheet

1. Let R be the region in the first quadrant enclosed by the graphs of $y = x^2 - 2x + 2$ and $y = 1 + 2\sin x$.

   (a) Write an expression involving one or more integrals to find the length of the boundary of R.
   (b) Write an expression involving one or more integrals to find the volume of the solid generated when R is revolved about the x-axis.
   (c) The base of a solid is the region R. Each cross-section of the solid perpendicular to the x-axis is a semi-circle. Write an expression involving one or more integrals that gives the volume of the solid.

2. The sales of a small company are expected to grow at a rate given by $\frac{ds}{dt} = 100\left(3^{0.2t}\right)$, where t is measured in days and s in hundreds of dollars.

   (a) Determine the average rate of growth over the first ten days.
   (b) Show the geometric meaning of the average rate of growth on a graph of the rate function and explain their significance.
   (c) Find the accumulated sales of the company over the first 20 days.

3. Evaluate:

   (a) $\int x\cos x\,dx$ (b) $\int e^x \sin x\,dx$
   (c) $\int \frac{dx}{x^2 + x}$ (d) $\int \frac{2dx}{x^2 - 3x - 4}$

4. Use $u = 5x - 2$ to write an integral equivalent to $\int_0^3 x^2\sqrt{5x-2}\,dx$

5. Determine whether each integral converges or diverges.

   (a) $\int_1^\infty \frac{dx}{x^2}$ (b) $\int_{-\infty}^0 xe^x\,dx$ (c) $\int_0^2 \frac{dx}{(x-1)^2}$

6. Evaluate $\lim_{x\to 0^+} x^{\sin x}$

7. Write the first four non-zero terms of the power series representation for the functions:

   (a) $f(x) = \frac{1}{1+5x}$ (b) $g(x) = \frac{x}{1-2x}$

8. Write the first four non-zero terms of the MacLaurin series for $\sin\left(x^2\right)$.

9. Let $P(x) = 5 - 2(x-1) + 4(x-1)^2 - 3(x-1)^3 + (x-1)^4$ be the fourth degree Taylor polynomial for a function $f(x)$ about x = 1. Assume that f has derivatives for all orders for all real numbers.

(a) Find $f(1)$ *and* $f'''(1)$

(b) Write the second degree Taylor polynomial for g if $g(x) = f'(x)$ and use it to approximate $g(1.2)$.

(c) Write the third degree Taylor polynomial for h, if $h(x) = \int_1^x f(t)dt$ about x = 1.

10. What is the maximum possible error in approximating $\cos \pi$ using $\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

11. Determine the whteher each series is convergent or divergent. Justify your answer.

(a) $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$ (b) $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ (c) $\sum_{n=1}^{\infty} \left(\frac{\pi}{3}\right)^n$ (d) $\sum_{n=1}^{\infty} \frac{4^n}{n!}$

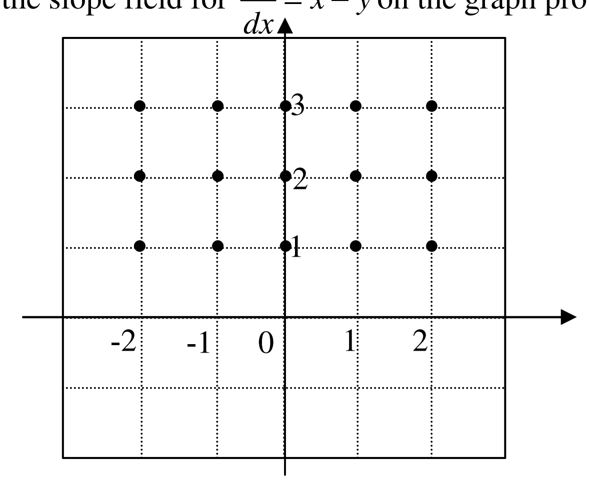
12. Determine the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$.

13. Solve the differential equations for y:

(a) $\frac{dy}{dx} = xy \quad y(4) = e$ (b) $\frac{dy}{dx} = \frac{x^2 - 1}{3y} \quad y(0) = 2$

14. If $\frac{dy}{dx} = \frac{3x}{y}$ *and* $f(1) = 2$, use Euler's method with $\Delta x = 1$ to approximate $f(3)$.

15. Construct the slope field for $\frac{dy}{dx} = x - y$ on the graph provided.



16. A population of grey wolves is modeled by the differential equation $\frac{dP}{dt} = \frac{P(500-P)}{640}$.

    (a) Solve the differential equation, if at $t = 0$ years, $P = 40$ wolves.
    (b) When will the population consist of 230 wolves?

17. The rate at which a student learns vocabulary words in given by the differential equation $\frac{dw}{dt} = (75-w)$.

    (a) Solve the differential equation, if at $t = 0$, $w = 10$ words are learned.
    (b) What is the maximum number of words learned?

18. Sales of widgets grow at a rate proportional to the amount of sales.

    (a) Set up a differential equation to model this problem.
    (b) Solve the differential equation, if at $t = 0$, $s = 20$ widgets have been sold, and at $t = 3$ days, $w = 500$ widgets have been sold.

19. Find the area bounded by one loop of the polar equation $r = 2\sin(3\theta)$.

20. A particle moves in the *xy*-plane so that its position at any time t, $0 \le t \le 2\pi$, is given by the equations $x(t) = t^2$, $y(t) = -2\cos(2t) + 3$.

    (a) Sketch the path of the particle in the xy-plane. Indicate the direction of the particle along the path.
    (b) Find the velocity and acceleration vectors of the particle.
    (c) Find the speed of the particle at t = π.