BC CALCULUS SEMESTER A REVIEW
Show all work on separate paper.

1. Evaluate each limit:

(a) $\lim_{x \to 3} \frac{x^2 - 9}{5x - x^2 - 6}$

(b) $\lim_{x \to 0} \frac{5x}{3 \sin x \cos x}$

(c) $\lim_{x \to 0^+} \frac{e^{1+x} - e}{x}$

(d) $\lim_{h \to 0} \frac{(3+h)^2 - (3-h)^2}{2h}$

(e) $\lim_{h \to 0} \frac{\sin(\frac{\pi}{6} + h) - \sin \frac{\pi}{6}}{h}$

(f) $\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{x}$

(g) $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$

(h) $\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$

2. Which of the following functions is/are continuous at x = 1?

$f(x) = \frac{e^{x-1}}{x-1}$ $\qquad g(x) = \ln(x^2 - 1)$ $\qquad h(x) = \frac{\sin x}{x}$

3. Find $\frac{dy}{dx}$ for each of the following functions:

(a) $y = 5(2x^3 + 1)^2 + 5\tan^{-1} 2x$

(b) $y = 4(3x + 5)\cos^2 x$

(c) $y = (2x + 1)^{\sin x}$

(d) $y = \frac{5}{\sqrt{x^3 + 5}}$

(e) $y = \int_{\pi}^{\cos x} \frac{dt}{\left(t^2 + 1\right)^{\frac{2}{3}}}$

(f) $x = \cos t$
$y = t^2 - 4t + 3$

4. Let $g(x) = f(x^2 + 1)$. Find each of the following:

(a) $g'(x)$

(b) $g'(2)$

(c) $g'(x^3)$

(d) $\frac{d}{dx}\left[g(x^3)\right]$

5. Given that $f$ is an even function and $\int_0^5 f(x)dx = 8$, g is an odd function and $\int_0^5 g(x)dx = 4$, Evaluate each of the following if possible:

(a) $\int_{-5}^{5} [f(x) + g(x)]dx$

(b) $\int_{-5}^{5} |g(x)|dx$

(c) $\int_{-5}^{5} \left[3f(x) + x^2\right]dx$

(d) $\int_0^5 \frac{f(x)}{g(x)}dx$

(e) $\int_{-5}^{5} [f(x) + 4]dx$

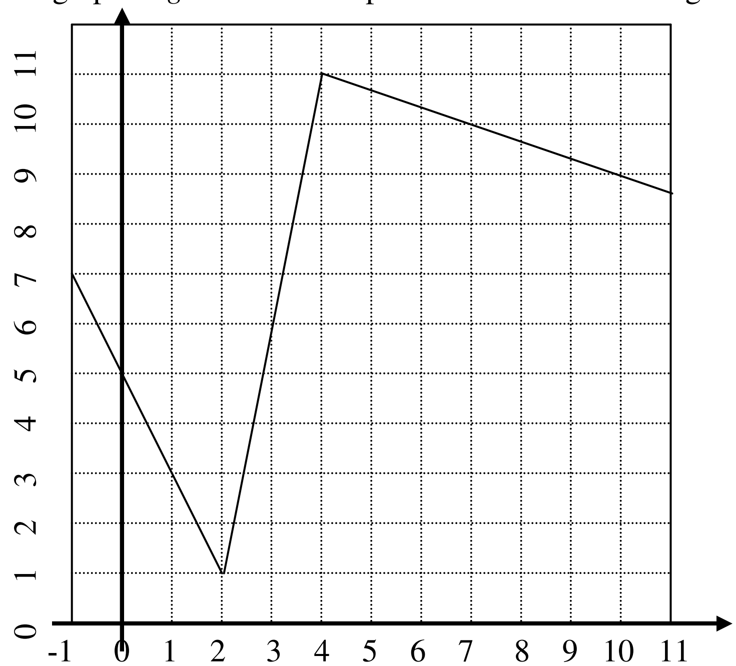
6. Let $f(x) = \int_1^{2x} \frac{dt}{\sqrt[3]{t^2+1}}$. Find each of the following:

(a) $f\left(\frac{1}{2}\right)$

(b) $f'(1)$

7. Suppose that $f$ is continuous on [-5, 5] and has the following properties: $f(0) = 2$, $f(3) = -2$, $f(5) = 1$, $f'' > 0$ on [-5, 0) and (1.5, 5], $f$ is decreasing when $x < 3$, and $f$ is increasing when $x > 3$. Sketch a possible graph of $f$.

8. Use the graph of *g* below to compute each of the following:




(a) $g'(1)$ (b) $f'(\sqrt{3}), if\ f(x) = g(x^2)$

(c) $f'(3), if\ f(x) = g(g(x))$

9. Selected values of a function *h* are given in the table below.

| *x* | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| *h(x)* | .8 | 1 | 1.4 | 2 | 2.8 |

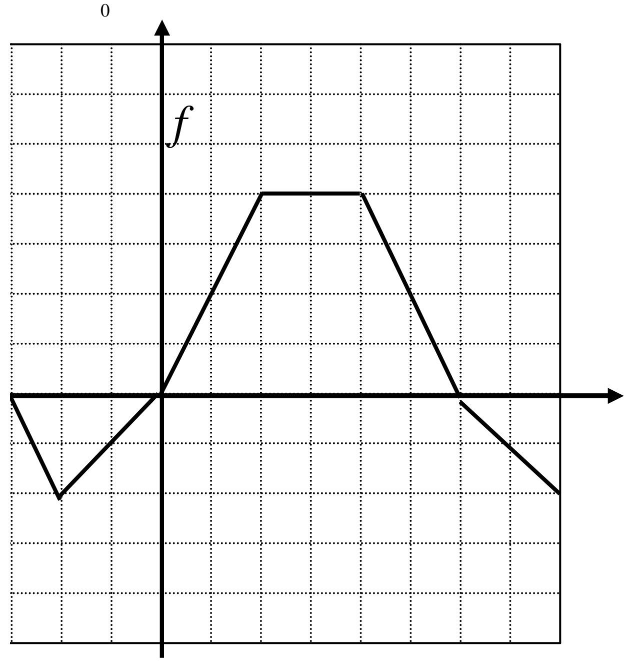
(a) Estimate the value of $h'(2)$ (b) Estimate the value of $h''(2)$.

10. Given $f(x) = \ln\left(3x^2 + 1\right)$, use the linearization of *f* at x = 1 to estimate *f(1.2)*

11. (a) Let $x^2 + 4xy + y^2 + 3 = 0$, find an expression for $\frac{dy}{dx}$.

(b) Are there any points on the curve where the tangent is horizontal or vertical? Justify your answer.

12. The graph of a function *f* is given below. One square represents one unit. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$



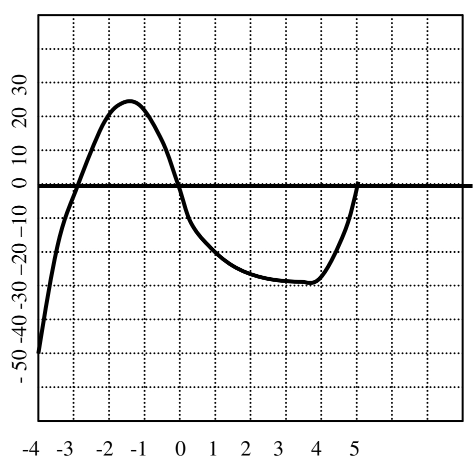

(a) Find each of the following: $g(0), g(3), g(-2)$

(b) Find all values of *x* for which g has a relative maximum on the open interval (-3,8). Justify your answer.

(c) Write the equation of the tangent line to the graph of g at $x = 3$.

(d) Find the *x*-coordinates of each point of inflection of the graph of *g* on the open interval (-3, 8). Justify your answer.

13. Given the graph of $f'$, ***the derivative of a function f*** on [-4,5]

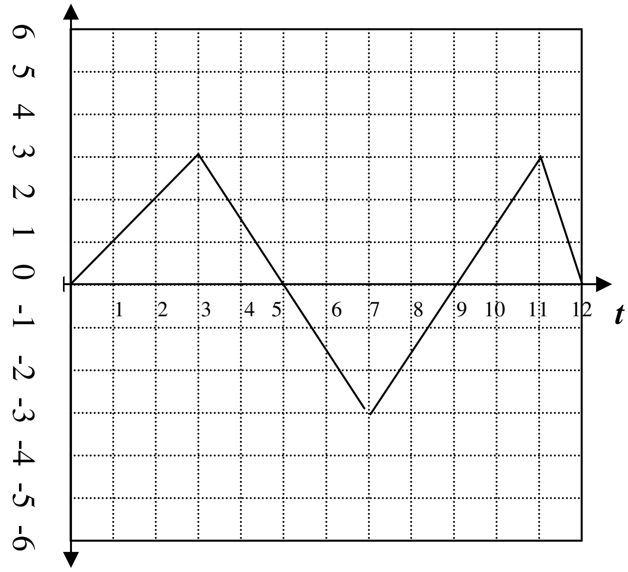



(a) On what interval(s) is *f* decreasing?

(b) Where does *f* have critical points?

(c) Where does *f* have its maximum?. Its minimum? Justify your answer.

14. The graph below shows the velocity of a particle moving along the x-axis. The particle is at $x = 0$ cm when $t = 0$ sec.




(a) Give the x-coordinate of the particle when $t = 12$.
(b) What is the total distance traveled by the particle on the interval [0,12] ?
(c) What is the average velocity of the particle on the interval [0,12]?

15. Chemicals from a storage tank are leaking into a pond. The rate of flow is measured at intervals and is recorded in the table below where $t$ is measured in hours and $R(t)$ in gallons per hour:

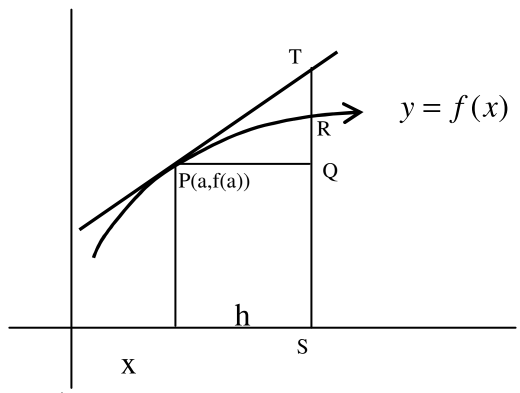
| *T* | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| *R(t)* | 40 | 38 | 36 | 30 | 26 | 18 | 8 | 6 | 3 |

(a) Make a sketch of the data to represent the rate of flow as a function of time.
(b) Write a definite integral that represents the total amount of chemical that entered the pond during the 16-hour period.
(c) Estimate the total amount of chemicals that entered the pond by the trapezoidal rule and by using Riemann sums. When using Riemann sums, find the area by using right endpoints, left endpoints, and midpoints. Use four equal subdivisions for each method and draw a diagram for each situation.

16. Let $f(x) = 3x^3 - 5x^2 + 2x + 2$
(a) Graph $f$ on [-1, 2].

(b) Use the graph of f to estimate the value(s) of "c" guaranteed by the Mean Value Theorem on the interval [-1,2].
(c) Find the value(s) of "c" analytically.
(d) Find the average value of f on [-1,2]

17. Refer to the diagram to answer the following questions:




(a) $f'(a) = ?$ (b) $f(a+h) = ?$

(c) A linear approximation of $f(a+h)$ is? (d) $\frac{RQ}{QP} = ?$

18. Let $f(x) = 3\sin\left(\frac{x}{3}\right) + 1$ and $g(x) = x^2 - 8x + 10$

(a) Sketch the graphs of f and g and find their points of intersection.
(b) Find the area enclosed by the graphs of f and g.

19. The acceleration of a particle is given by $a(t) = 2 + 5\sqrt{t}\, ft/s^2; t \geq 0$.
At t = 0, s(t)=0.v(t)=0

(a) What is the velocity of the particle at t = 9 seconds?
(b) What is the total distance covered by the object during the first 9 seconds?

20. At a given moment, two cars approach an intersection. Car A travels south at 30 mi/hr. Car B travels East at 40 mi/hr.
(a) How fast is the distance between the cars decreasing at the instant when A is 15 miles from the intersection and B is 10 miles from the intersection?
(b) How fast is the area of the triangle formed by the intersecting roads and the segment joining the cars changing at this instant?

21. Set up, but do not evaluate, one or more integral expressions that could be used to find the area between the curve $y = 3x^3 + 5x^2 - 2x$ and the x-axis.

22. Evaluate each integral:

(a) $\int \left(xe^{x^2-1} + \cos 3x\right)dx$  (b) $\int \sin^3 4x \cos 4x dx$

(c) $\int x^2 \sec^2\left(x^3\right)dx$  (d) $\int 9x\cos\left(3x^2\right)dx$

(e) $\int 5\sin(3x)e^{\cos(3x)}dx$  (f) $\int_0^1 \frac{5x^2}{8+3x^3}dx$

23. $f$ is continuous and differentiable on [-5,5] such that $f(-5) = -10$ and $f(5) = 10$. Decide which of the following statements are true and which are false. Justify your answer in each case.

(a) $f$ is an even function.
(b) $f(0) = 0$
(c) $f'(c) = 0$ for some c between –5 and 5.
(d) $f'(c) > 0$ for all x between –5 and 5.
(e) $-10 \le f(c) \le 10$ for all values of x between –5 and 5.
(f) $f(c) = 0$ for at least one c between –5 and 5.
(g) $f'(c) = 2$ for at least one c between –5 and 5.
(h) $f(c) = 9$ for at least one c between –5 and 5.

24. Given $y = 4x - 3$
(a) Find the minimum value of xy.
(b) Find the rate of change of xy with respect to x.
(c) Find the rate of change of xy with respect to y.

25. Given that $f$ is an even function, and $\int_2^5 f(x)dx = 12$, decide which statements are true and which are false. Justify your answer.

(a) $\int_{-5}^{-2} f(x)dx = -12$

(b) $\int_5^2 f(x)dx = 12$

(c) $\int_0^5 f(x)dx - \int_0^2 f(x)dx = 12$

(d) $\int_2^5 \left[f(x) - 2\right]dx = 6$