AP Calculus
June 1, 2008

Summer Study Guide

In order to be successful in AP Calculus, you must have certain prerequisite skills mastered. **You will be quizzed on the topics listed below during the first week of school. There will be no reassessment for this quiz. You will not be permitted to use a calculator during the quiz.** Study these topics over the summer, and do the examples. Answers to numbered examples are on the last page. If you have any questions on the course, please contact Mr. Kraft or Mr. Cangelosi at the addresses below.
(Please note that we may not access our school e-mail accounts frequently during the summer).

David_M_Kraft@mcpsmd.org Michael_S_Cangelosi@mcpsmd.org

1. Solve Equations

Example: Solve each equation for *x*.

1. $\sqrt{4+x} + 0.5x(4+x)^{-\frac{1}{2}} = 0$

2. $x^2e^{-x} - 2xe^{-x} = 0$

3. $2x^3 - 4x^2 + 5x + 3 = x^3 + x^2 + 19x + 3$

4. $3e^{2x} = 50$

5. $-\frac{1}{x+2} = \frac{1}{2} + \frac{1}{3}$

6. $\frac{x^2 - 5}{2x+1} = 0$

2. Simplify Algebraic Expressions

Example: Simplify each expression.

7. $\dfrac{\frac{1}{x}-\frac{1}{2}}{x}$ **8.** $\dfrac{x^2-4}{x+2}$ **9.** $\dfrac{\sqrt{2x+4}}{\sqrt{x+2}}$ **10.** $\dfrac{\frac{2x}{6+2x}}{\frac{6x^2}{3+x}}$

3. Trigonometry

- Know the unit circle. You must know the values of the six trigonometric functions (sine, cosine, tangent, cotangent, secant, and cosecant) for angles whose measures (in radians) are multiples of π/6 and π/4. In calculus we will always use radians rather than degrees.

Example: Give the exact value of each, without using your calculator.

sin(7π/6) = __________ **tan(3π/4) = _____________** **sec(π/3) = ______________**

cot(π/6) = ___________ **csc(3π/2) = _____________** **cos(2π/3) = _____________**

Hint: *Memorize trig facts for sine and cosine. Then use sine and cosine to find the trig. facts for tangent, cotangent, secant, and cosecant. For example, I know that cos(π/3) = ½ . By definition, secx = 1/cosx. Therefore, sec(π/3)=2. You must be able to do this quickly!*

- Know inverse trigonometric values.

Example: Give the exact value of each, in radians, without using your calculator.

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ __________________ $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$ __________________

$\cos^{-1} 0 =$ __________________

Note: *The little negative one is not an exponent. That is,* $\sin^{-1}(x)$ *is not the same as* $\dfrac{1}{\sin(x)}$ *. To find the inverse sine of* $\frac{\sqrt{2}}{2}$*, ask, "The sine of what angle is* $\frac{\sqrt{2}}{2}$*?" To find* $\tan^{-1} 1$*, ask, "The tangent of what angle is 1?"*

- Know how to simplify trigonometric expressions.

Example: Choose the equivalent expression.

11. $\dfrac{\sin^2 x}{\csc^2 x}$ $\quad a)\ 1 \quad b)\ \tan^2 x \quad c)\ \sin^2 x\cos^2 x \quad d)\ \sin^4 x \quad e)\ \csc^4 x$

12. $\tan x \csc x$ $\quad a)\ \sin x \quad b)\ \sin^2 x \quad c)\ \cos x \quad d)\ \cot x \quad e)\ \sec x$

4. Composition of Functions

Examples: Given that $f(x) = x^2 + 3x$ and $g(x) = \sin x$, find the following.

13. $f(g(x))$

14. $f(x+h)$

15. $f(f(x))$

16*. $h(1)$, where $h(x) = f^2(x+1)$

**Note:* $f^2(x) = (f(x))^2$

17. $j(\pi)$, where $j(x) = \sqrt{f(g(x)+2)}$

5. Properties of Exponents

1. $a^m \cdot a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $\left(a^m\right)^n = a^{mn}$
4. $a^{\frac{1}{n}} = \sqrt[n]{a}$
5. $a^{-n} = \dfrac{1}{a^n}$
6. $a^0 = 1$
7. $(ab)^n = a^n b^n$

Example: Rewrite each expression using properties of exponents.

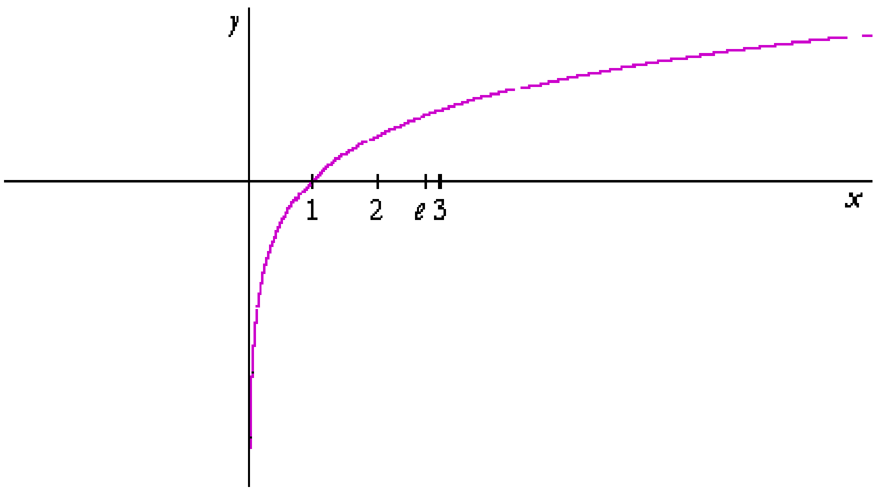
18. $x^n x^2$ $\qquad$ **19.** $\dfrac{x^3 x^{-5}}{x^{-4}}$ $\qquad$ **20.** $\dfrac{x\left(\sqrt{x}\right)}{\sqrt[4]{x}}$ $\qquad$ **21.** $(3x^4 y^3)^3$

6. Graphs of Exponential and Logarithmic Functions

You must be able to sketch and recognize the graphs of the functions listed below

- $y = e^x$, $y = e^{-x}$, $y = -e^x$
- $y = \ln x$, $y = -\ln x$

Below is the graph of $y = \ln x$. Note that $x = 0$ is a vertical asymptote of the graph. It is difficult to see this in the standard viewing window of your graphing calculator.



7. Definition and Properties of Logarithms

- **What is a logarithm?**

The logarithm is an exponent.

The equation: $Y = \log_b X$

is another way of saying: $b^Y = X$

Ex: $\mathbf{2^3 = 8}$ *means the same thing as* $\mathbf{3 = \log_2 8}$ (or $\mathbf{\log_2 8 = 3}$ of course)

Ex: $\mathbf{\log_4 16 = 2}$ *means the same thing as* $\mathbf{4^2 = 16}$

Example:

22. Find $\log_5 125$.

$\log_5 125 = x$ *means the same thing as* $5^x=125$.

Think, "Five to the what equals 125?"

23. Find $\log_2 16$. (Think, "Two to the what equals 16?")

24. Find $\log_2 2$.

25. Find $\log_2(1/2)$. Hint: Think negative exponents.

- **What is a *natural* logarithm?**

The expression lnx (the natural logarithm of x) means $\log_e x$.

Example: Without your calculator, find:

26. lne **27.** ln1 **28.** ln(1/e) **29.** $\ln(e^2)$

Try entering ln(-1) into your calculator. It will say "error." Why doesn't ln(-1) exist?
Hint: e is a positive number. Think, "Does **e** to the anything equal -1?"

Try entering ln(0) into your calculator. It will say "error." Why doesn't ln(0) exist?
Hint: Think, "Does **e** to the anything equal 0?"

- **Properties of Logarithms**

lna + lnb = ln(ab)

lna – lnb = ln(a/b)

$\ln(a^b)$ = blna

$$\log_b a = \frac{\ln a}{\ln b} \quad (\textit{change of base formula})$$

Example:

30. Simplify: ln6 – ln2

31. Simpify: ln5 + ln3

32. Choice: $\ln(3^4) =$

a) $(\ln 3)^4$ b) 4ln3

33. Choice: -ln2 =

a) ln(1/2) b) ln(-2)

34. Choice: $2\ln 5 =$

a) $\ln 25$ b) $\ln 10$

35. Rewrite the expression using the change of base formula.

$\log_7 x =$

- **Finally . . .**

$$\mathbf{e^{\ln a} = a} \qquad \mathbf{\ln e^{a} = a}$$

Example: Simplify each expression.

36. $e^{\ln(x+3)}$ **37.** $\ln e^{\sin x}$ **38*.** $e^{-2\ln x}$

* *Hint: Use a property of exponents.*

Answers:

1. Solve Equations 1. x = -8/3 2. x = 0 or 2 3. x = -2, 0, or 7 4. x = ½ln(50/3)

5. x = -16/5 6. x = $\pm\sqrt{5}$

2. Simplifying Algebraic Expressions

7. $\frac{\frac{1}{x}-\frac{1}{2}}{x}\cdot\frac{2x}{2x}=\frac{\frac{2x}{x}-\frac{2x}{2}}{2x^2}=\frac{2-x}{2x^2}$ 8. $\frac{(x-2)(x+2)}{x+2}=x-2$

9. $\sqrt{\frac{2x-4}{x-2}}=\sqrt{2}$, where $x\neq 2$ 10. $\frac{2x}{6+2x}\cdot\frac{3+x}{6x^2}=\frac{1}{3x}\cdot\frac{1}{2}=\frac{1}{6x}$

3. Trigonometry 11. d 12. e

4. Composition of Functions 13. $f(g(x))=\sin^2 x+3\sin x$ *Note:* $\sin^2 x=(\sin x)^2$

14. $f(x+h)=(x+h)^2+3(x+h)$ 15. $f(f(x))=(x^2+3x)^2+3(x^2+3x)$

16. $h(x)=f^2(x+1)=\left[f(x+1)\right]^2=\left[(x+1)^2+3(x+1)\right]^2$, so $h(1)=\left[(1+1)^2+3(1+1)\right]^2=100$

17. $j(x)=\sqrt{f(g(x)+2)}=\sqrt{(2+\sin x)^2+3(2+\sin x)}$, so $j(\pi)=\sqrt{(2+\sin\pi)^2+3(2+\sin\pi)}=\sqrt{10}$

5. Properties of Exponents 18. x^{n+2} 19. x^2 20. $x^{\frac{5}{4}}$ 21. $27x^{12}y^9$

7. Logarithms 22. 3 23. 4 24. 1 25. -1

26. Think, "**e** to the <u>what</u> equals **e**?" Answer: 1

27. Think, "**e** to the <u>what</u> equals **1**?" Answer: 0

28. Think, "**e** to the <u>what</u> equals **1/e** ?" Answer: -1

29. Think, "**e** to the <u>what</u> equals $\mathbf{e^2}$?" Answer: 2

30. ln3 31. ln15

32. b 33. a 34. a

35. $\frac{\ln x}{\ln 7}$

36. x+3

37. sin*x*

38. Since $\left(a^m\right)^n=a^{mn}$, we know that $e^{-2\ln x}=(e^{\ln x})^{-2}=x^{-2}=\frac{1}{x^2}$

Here's another way to think about it:

Since $\mathbf{\ln(a^b)=b\ln a}$, we know that $e^{-2\ln x}=e^{\ln x^{-2}}=x^{-2}=\frac{1}{x^2}$