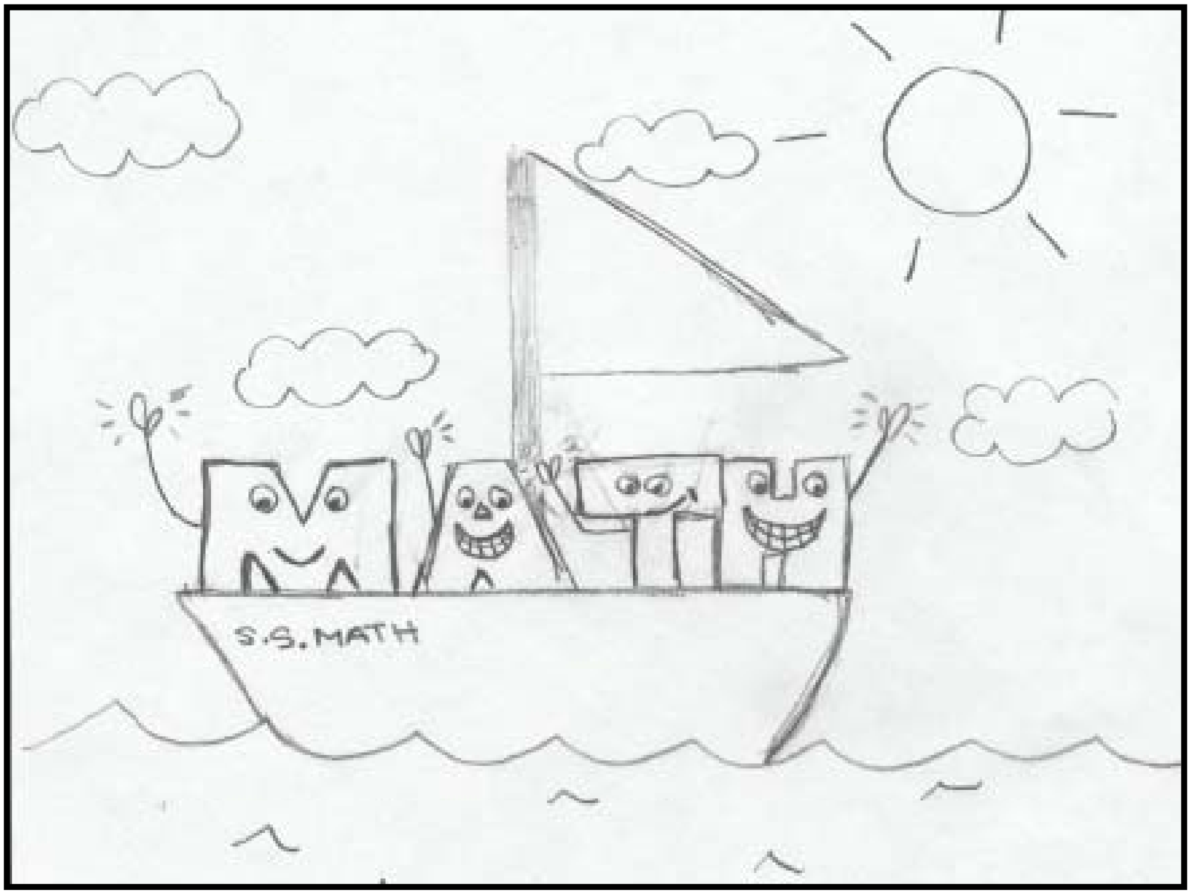
Sail into Summer with Math!



For Students Entering Math C

This summer math booklet was developed to provide students in kindergarten through the eighth grade an opportunity to review grade level math objectives and to improve math performance.

Summer

Student Responsibilities

Students will be able to improve their own math performance by:

- Completing the summer math booklet
- Reviewing math skills throughout the summer.

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Student Signature	Grade	Date

Parent Responsibilities

Parents will be able to promote student success in math by:

- Supporting the math goal of the cluster of schools,
- Monitoring student completion of the summer math booklet,
- Encouraging student use of math concepts in summer activities.

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Parent Signature	Date

Math C Summer Mathematics Packet

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Fraction Operations

Hints/Guide:

When adding and subtracting fractions, we need to be sure that each fraction has the same denominator, then add or subtract the numerators together. For example:

$$\frac{1}{8}+\frac{3}{4}=\frac{1}{8}+\frac{6}{8}=\frac{1+6}{8}=\frac{7}{8}$$

That was easy because it was easy to see what the new denominator should be, but what about if it is not so apparent? For example: $\frac{7}{12}+\frac{8}{15}$

For this example we must find the Lowest Common Denominator (LCM) for the two denominators. 12 and 15

12 = 12, 24, 36, 48, 60, 72, 84, ...

15 = 15, 30, 45, 60, 75, 90, 105, ...

LCM (12, 15) = 60

So, $\frac{7}{12}+\frac{8}{15}=\frac{35}{60}+\frac{32}{60}=\frac{35+32}{60}=\frac{67}{60}=1\frac{7}{60}$ Note: Be sure answers are in lowest terms

To multiply fractions, we multiply the numerators together and the denominators together, and then simplify the product. To divide fractions, we find the reciprocal of the second fraction (flip the numerator and the denominator) and then multiply the two together. For example:

$$\frac{2}{3}\bullet\frac{1}{4}=\frac{2}{12}=\frac{1}{6} \quad \text{and} \quad \frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\bullet\frac{4}{3}=\frac{8}{9}$$

Exercises: Perform the indicated operation: No calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $\frac{6}{7}+\frac{2}{3}=$

2. $\frac{8}{9}+\frac{3}{4}=$

3. $\frac{9}{11}-\frac{2}{5}=$

4. $\frac{5}{7}-\frac{5}{9}=$

5. $\frac{6}{11}\bullet\frac{2}{3}=$

6. $\frac{7}{9}\bullet\frac{3}{5}=$

7. $\frac{6}{7}\div\frac{1}{5}=$

8. $\frac{7}{11}\div\frac{3}{5}=$

9. $\left[\frac{2}{3}-\frac{5}{9}\right]\div\left[\frac{4}{7}+\frac{1}{6}\right]=$

10. $\frac{3}{4}+\frac{4}{5}\left(\frac{5}{9}+\frac{9}{11}\right)=$

11. $\left[\frac{3}{4}+\frac{4}{5}\right]\left[\frac{5}{9}+\frac{9}{11}\right]=$

Decimal Operations

Hints/Guide:

When adding and subtracting decimals, the key is to line up the decimals above each other, add zeros so all of the numbers have the same place value length, then use the same rules as adding and subtracting whole numbers, with the answer having a decimal point in line with the problem. For example:

```
                      34.5      34.500
34.5 + 6.72 + 9.045 =  6.72   =  6.720        AND        5 - 3.25 =  5.00
                     + 9.045   + 9.045                              - 3.25
                                50.265                                1.75
```

To multiply decimals, the rules are the same as with multiplying whole numbers, until the product is determined and the decimal point must be located. The decimal point is placed the same number of digits in from the right of the product as the number of decimal place values in the numbers being multiplied. For example:

8.54 x 17.2, since 854 x 172 = 146888, then we count the number of decimal places in the numbers being multiplied, which is three, so the final product is 146.888 (the decimal point comes three places in from the right).

To divide decimals by a whole number, the process of division is the same, but the decimal point is brought straight up from the dividend into the quotient. For example:

$3\overline{)51.06}$ = 17.02   The decimal point moves straight up from the dividend to the quotient.

Exercises: Solve: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. 15.709 + 2.34 + 105.06 =
2. 64.038 + 164.18 + 1005.7 =
3. 87.4 - 56.09 =
4. 500.908 - 4.72 =
5. 6108.09 - 2004.704 =
6. 9055.3 - 242.007 =
7. 63 x .04
8. .87 x .23
9. 8.904 x 2.1
10. 4.2 x .602
11. $35\overline{)70.35}$
12. $14\overline{)50.512}$
13. $23\overline{)74.888}$

Rename Fractions, Percents, and Decimals

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as $\frac{3}{5}$, and divide the numerator (the top number of a fraction) by the denominator (the bottom number of a fraction). So:

$$\begin{array}{r|l} & \ \ \ 6 \\ 5 & 3.0 \\ & -\ 30 \\ & \ \ \ 0 \end{array}$$ and the fraction $\frac{3}{5}$ is equivalent to the decimal 0.6

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a decimal by 100, so 0.6 x 100 = 60 and $\frac{3}{5} = 0.6 = 60\%$

To convert a percent to a decimal, we divide the percent by 100, 60% ÷ 100 = 0.6 so 60% = 0.6

To convert a fraction into a percent, we can use a proportion to solve,

$\frac{3}{5} = \frac{x}{100}$, so 5x = 300 which means that x = 60 = 60%

Exercises: Complete the chart:

	Fraction	Decimal	Percent
1		0.04	
2			125%
3	$\frac{2}{3}$		
4		1.7	
5			0.6%
6	$3\frac{1}{2}$		
7		0.9	
8			70%
9	$\frac{17}{25}$		
10		0.007	

Add and Subtract Mixed Numbers

Hints/Guide:

When adding and subtracting mixed numbers, we add the whole numbers and the fractions separately, then simplify the answer. For example:

$$\begin{array}{r} 4\frac{1}{3} = 4\frac{8}{24} \\ +\,2\frac{6}{8} = 2\frac{18}{24} \\ \hline 6\frac{26}{24} = 6 + 1\frac{2}{24} = 7\frac{2}{24} = 7\frac{1}{12} \end{array}$$

$$\begin{array}{r} 7\frac{3}{4} = 7\frac{18}{24} \\ -\,2\frac{15}{24} = 2\frac{15}{24} \\ \hline 5\frac{3}{24} = 5\frac{1}{8} \end{array}$$

First, we convert the fractions to have the same denominator, then add the fractions and add the whole numbers. If needed, we then simplify the answer.

Exercises: Solve in lowest terms: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $3\frac{1}{2} + 5\frac{3}{5}$

2. $6\frac{17}{25} + 8\frac{4}{7}$

3. $6\frac{2}{3} + 9\frac{7}{9}$

4. $3\frac{4}{5} + \frac{3}{11}$

5. $4\frac{3}{7} + 2\frac{1}{2}$

6. $3\frac{6}{7} + 3\frac{11}{15}$

7. $4\frac{1}{2} - 3\frac{5}{6}$

8. $5\frac{5}{6} - \frac{3}{8}$

9. $8\frac{7}{9} - 4\frac{8}{11}$

10. $8\frac{3}{10} - 6\frac{7}{9}$

11. $9\frac{7}{15} - 2\frac{7}{12}$

12. $12\frac{8}{9} - 7\frac{3}{4}$

Multiply and Divide Mixed Numbers

Hints/Guide:

To multiply mixed numbers, we first convert the mixed numbers into improper fractions. This is done by multiplying the denominator by the whole number part of the mixed number and then adding the numerator to this product, and this is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the mixed number. For example:

$$3\frac{2}{5} \text{ leads to } 3 \bullet 5 + 2 = 17 \text{ so } 3\frac{2}{5} = \frac{17}{5}$$

Once the mixed numbers are converted into improper fractions, we multiply and simplify just as with regular fractions. For example:

$$5\frac{1}{5} \bullet 3\frac{1}{2} = \frac{26}{5} \bullet \frac{7}{2} = \frac{182}{10} = 18\frac{2}{10} = 18\frac{1}{5}$$

Exercises: Solve and place your answer in lowest terms: No Calculators!
SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $6\frac{2}{3} \bullet 7\frac{3}{7} =$
2. $3\frac{1}{3} \bullet 6\frac{4}{5} =$
3. $7\frac{1}{8} \bullet 6 =$
4. $4\frac{3}{4} \bullet 1\frac{1}{5} =$
5. $7 \bullet 4\frac{2}{3} =$
6. $4\frac{1}{3} \bullet \frac{8}{9} =$

Hints/Guide:

To divide mixed numbers, we must first convert to improper fractions using the technique shown in multiplying mixed numbers. Once we have converted to improper fractions, the process is the same as dividing regular fractions. For example:

$$2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \bullet \frac{3}{10} = \frac{15}{20} = \frac{3}{4} \qquad 3\frac{1}{2} \div 8\frac{2}{3} = \frac{7}{2} \div \frac{26}{3} = \frac{7}{2} \bullet \frac{3}{26} = \frac{21}{52}$$

Exercises: Solve and place your answer in lowest terms: No Calculators!
SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $1\frac{1}{5} \div 4\frac{2}{5} =$
2. $4\frac{4}{7} \div \frac{4}{9} =$
3. $\frac{8}{9} \div 2\frac{3}{5} =$
4. $4\frac{1}{4} \div \frac{5}{7} =$
5. $3\frac{2}{3} \div 4\frac{3}{7} =$
6. $\frac{3}{4} \div 2\frac{3}{11} =$

Integers I

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign as the addends. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add the opposite of the second addend. For example,

$6 - 11 = 6 + -11 = -5$

Exercises: Solve the following problems:

1. (-4) + (-5) =
2. -9 - (-2) =
3. 6 + (-9) =
4. (-6) - 7 =
5. 7 - (-9) =
6. 15 - 24 =
7. (-5) + (-8) =
8. -15 + 8 - 8 =
9. 14 + (-4) - 8 =
10. 14.5 - 29 =
11. -7 - 6.85 =
12. -8.4 - (-19.5) =
13. 29 - 16 + (-5) =
14. -15 + 8 - (-19.7) =
15. 45.6 - (-13.5) + (-14) =
16. -15.98 - 6.08 - 9 =
17. -7.24 + (-6.8) - 7.3 =
18. 29.45 - 56.009 - 78.2 =
19. 17.002 + (-7) - (-5.23) =
20. 45.9 - (-9.2) + 5 =

Integers II

Hints/Guide:

The rules for multiplying integers are:

Positive x Positive = Positive | Negative x Negative = Positive
Positive x Negative = Negative | Negative x Positive = Negative

The rules for dividing integers are the same as multiplying integers.

Exercises: Solve the following problems:

1. 4 • (-3) • 6 =
2. 5(-12) • (-4) =
3. (4)(-2)(-3) =
4. $\frac{(-5)(-6)}{-2} =$
5. $\frac{6(-4)}{8} =$
6. $\frac{-56}{2^3} =$
7. 6 (-5 - (-6)) =
8. 8 (-4 - 6) =
9. -6 (9 - 11) =
10. $\frac{-14}{2} + 7 =$
11. $8 - \frac{-15}{-3} =$
12. $-3 + \frac{-12 \bullet -5}{4} =$
13. $\frac{-6-(-8)}{-2} =$
14. $-7 + \frac{4+(-6)}{-2} =$
15. 45 - 14 (5 - (-3)) =
16. (-4 + 7) (-16 + 3) =
17. 16 - (-13) (-7 + 5) =
18. $\frac{4+(-6)-5-3}{-6+4} =$
19. $(-2)^3(-5-(-6)) =$
20. 13 (-9 + 17) + 24 =

Solving Equations I

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In one-step equations, we merely undo the operation - addition is the opposite of subtraction and multiplication is the opposite of division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side.

Examples:

1. $x + 5 = 6$
   $\underline{-5 \quad -5}$
   $x = 1$
   Check: $1 + 5 = 6$
   $6 = 6$

2. $t - 6 = 7$
   $\underline{+6 \quad +6}$
   $t = 13$
   Check: $13 - 6 = 7$
   $7 = 7$

3. $\frac{4x}{4} = \frac{16}{4}$
   $x = 4$
   Check: $4(4) = 16$
   $16 = 16$

4. $6 \bullet \frac{r}{6} = 12 \bullet 6$
   $r = 72$
   Check: $72 \div 6 = 12$
   $12 = 12$

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $x + 8 = -13$
2. $t - (-9) = 4$
3. $-4t = -12$
4. $\frac{r}{4} = 24$
5. $y - 4 = -3$
6. $h + 8 = -5$
7. $\frac{p}{8} = -16$
8. $-5k = 20$
9. $-9 - p = 17$

Solving Equations II

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In two-step equations, we must undo addition and subtraction first, then multiplication and division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side. Examples:

1. 4x - 6 = -14

   + 6 + 6

   $\frac{4x}{4} = \frac{-8}{4}$

   x = -2

   Solve: 4 (-2) - 6 = -14

   -8 - 6 = -14

   -14 = -14

2. $\frac{x}{-6} - 4 = -8$

   + 4 + 4

   -6 • $\frac{x}{-6} = -4$ • -6

   x = 24

   Solve: (24/-6) - 4 = -8

   -4 - 4 = -8

   -8 = -8

Exercises: Solve the following problems: No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. -4t - 6 = 22
2. $\frac{m}{-5} + 6 = -4$
3. -4r + 5 = -25
4. $\frac{x}{-3} + (-7) = 6$
5. 5g + (-3) = -12
6. $\frac{y}{-2} + (-4) = 8$

Inequalities

Hints/Guide:

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality as many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

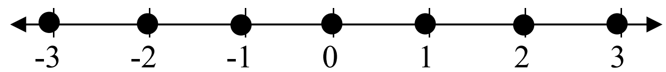
Inequality - a statement containing one of the following symbols:

$<$ is less than $>$ is greater than $\leq$ is less than or equal to

$\geq$ is greater than or equal to _ is not equal to

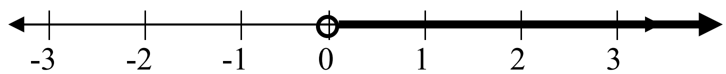
Examples:

1. Integers between -4 and 4.



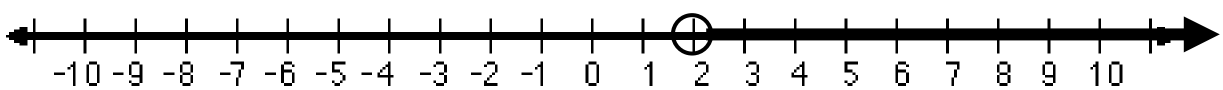
2. All numbers between -4 and 4.

3. The positive numbers.



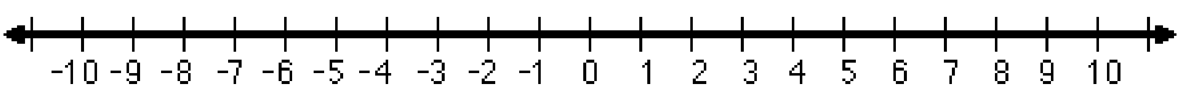
So, to solve the inequality -4x < -8 becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

and therefore x > 2 is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:

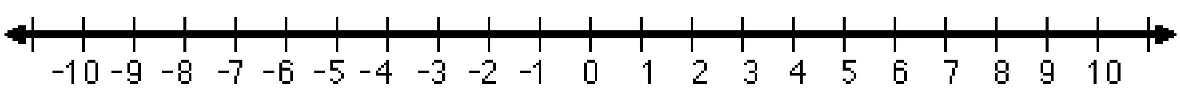


Exercises: Solve the following problems: No Calculators!

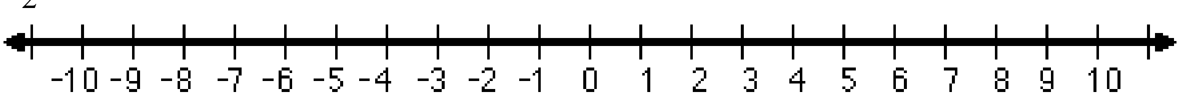
1. 4x > 9



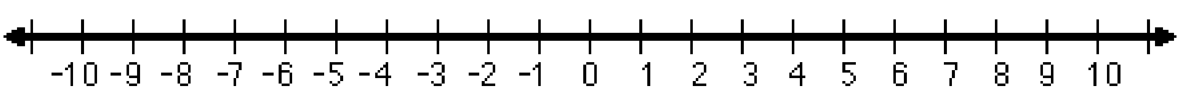
2. -5t $\geq$ -15



3. $\frac{x}{2} \geq 3$



4. $\frac{x}{-4} > 2$



Volume

Hints/Guide:

To find the volume of prisms (a solid figure whose ends are parallel and the same size and shape and whose sides are parallelograms) and cylinders, we multiply the area of the base times the height of the figure. The formulas we need to know are:

The area of a circle is $A = \pi r^2$ | The area of a rectangle is $A = bh$

The area of a triangle is $A = \frac{1}{2} b h$ | The volume of a prism is

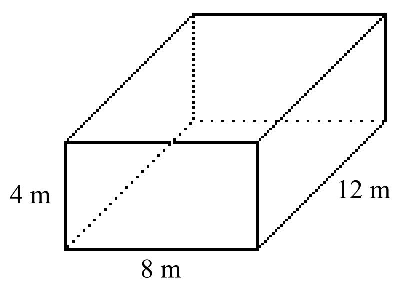
$V = (\text{Area of Base}) \cdot (\text{Height})$

So, the volume of a rectangular prism can be determined if we can find the area of the base and the perpendicular height of the figure.

Exercises: Find the volume of the following figures: Note: Use $\pi = 3.14$

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1.



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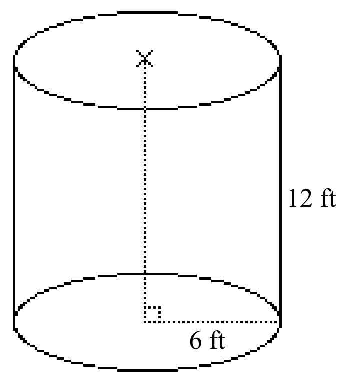

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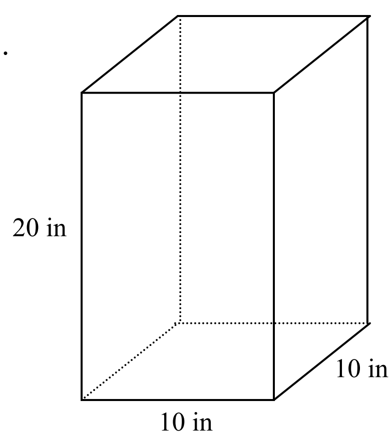

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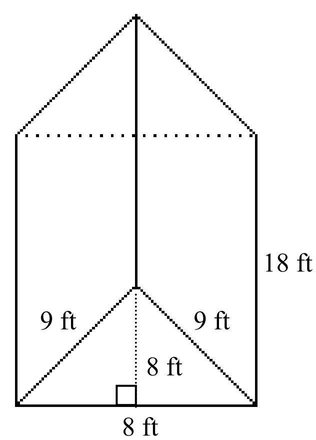
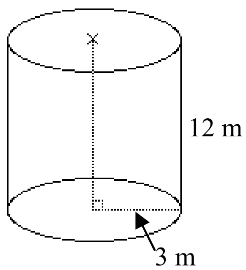


Surface Area

Hints/Guide:

To determine the surface area of an object, we must find the areas of each surface and add them together. For a rectangular prism, we find the area of each rectangle and then add them together. For a cylinder, we find the area of each base and then add the area of the rectangle (the circumference of the circular base times the height) which wraps around to create the sides of the cylinder. For example:



The area of each base is $A = \pi r^2 = 3.14 \cdot 3 \cdot 3 = 28.26\ m^2$

and the area of the cylinder "wrap" is

$A = 2\pi rh$ (which is the circumference of the circle times the height of the cylinder)

$= 2 \cdot 3.14 \cdot 3 \cdot 12$

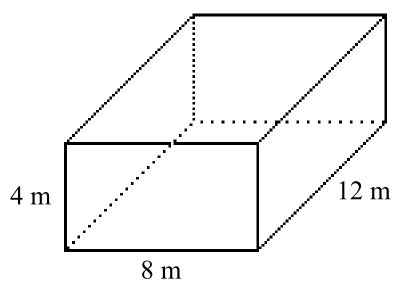
$= 226.08$

So the surface area is $28.26 + 28.26 + 226.08 = 282.6\ m^2$

Exercises: Determine the surface area of the following figures:

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1.



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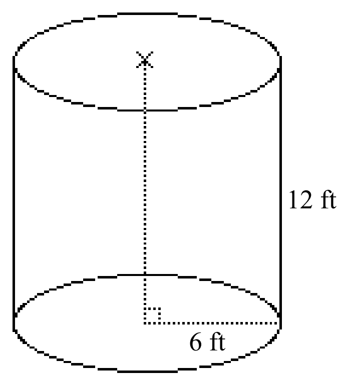

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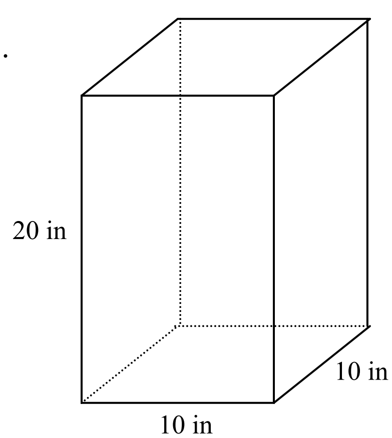

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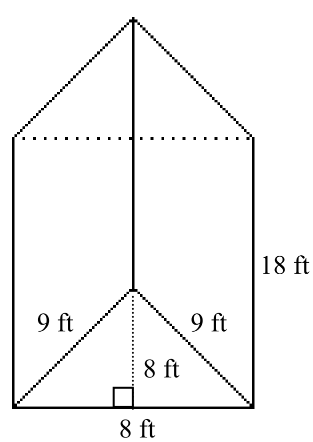


Geometry I

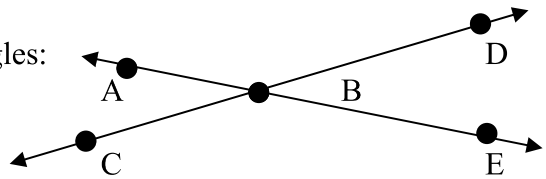
Hints/Guide:

In order to learn geometry, we first must understand so geometric terms:

Right Angle - an angle that measures 90 degrees.
Acute Angle - an angle that measures less than 90 degrees.
Obtuse Angle - an angle that measures more than 90 degrees, but less than 180 degrees.
Complementary - two angles that add together to equal 90 degrees.
Supplementary - two angles that add together to equal 180 degrees.
Vertical - Angles which are opposite from each other.
Adjacent - angles that are next to each other.

When two lines intersect, they form four angles:

$\angle$ABC $\angle$ABD
$\angle$DBE $\angle$EBC



Vertical angles, such as $\angle$ABC and $\angle$DBE, are equal in measure and adjacent angles, such as $\angle$ABD and $\angle$DBE, are supplementary.

Exercises:

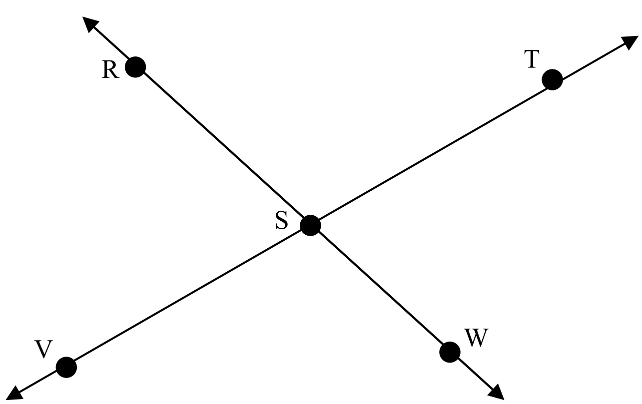
1. In the above example, list two acute angles and two obtuse angles

   Acute ______________, ____________ Obtuse _____________, ____________

2. If you have a 43º angle, what is the measure of the angle which is complementary to it?

3. If you have a 43º angle, what is the measure of the angle which is supplementary to it?

4. Using the figure, list two pairs of vertical angles and two pairs of adjacent angles.



Geometry II

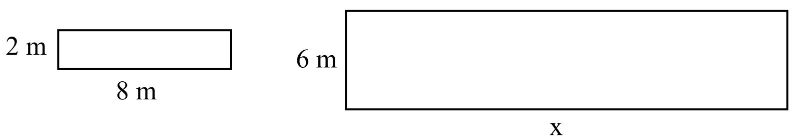
Hints/Guide:

In order to add to our knowledge of geometry, here are some additional terms:

Congruent - two figures which are the same shape and the same size.

Similar - two figures which are the same shape but different size.

In similar triangles, congruent angles in the same location in the figure are called corresponding angles. The sides opposite corresponding angles are called corresponding sides. The measures of corresponding angle or of corresponding sides of similar triangles are proportional. For example:

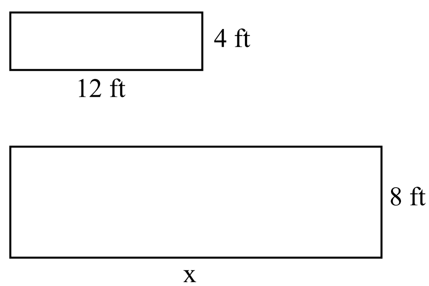


$\frac{2}{6} = \frac{8}{x}$ so since 2x = 48

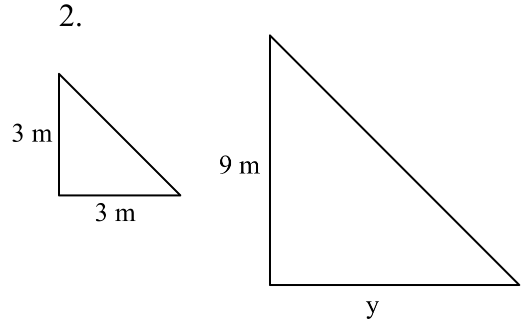
x = 24 m

Exercises: Solve for the indicated variables (All figures are similar):

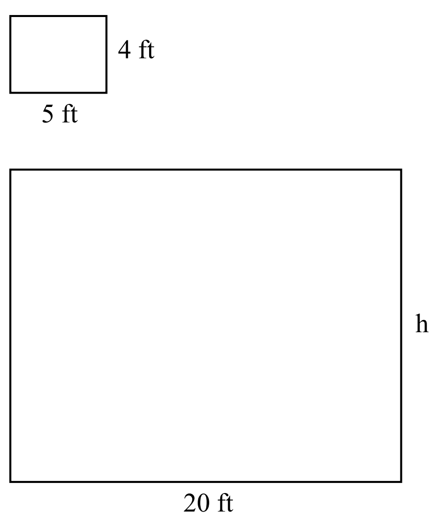
1.



2.



3.



4.