PRECALCULUS/HONORS PRECALCULUS SUMMER PACKET 2011

Name: ___________________________

This packet contains REVIEW material. These are expected skills for PreCalculus and will NOT be the focus of class. Refer to your Geometry and Algebra 2 notebooks for additional support.

Algebra

1. Simplify: $[2^3 + 4(7 - 3)] \div 8$

2. Evaluate: $3y^3+2y^2-y+7$ if $y = -4$

Solve each equation.

3. $5 - (4 - 2x) = 2x + 8$

4. $3(1 + x) = 2[3(x + 2) - (x + 1)]$

5. $\frac{3}{4}(8x - 12) - \frac{2}{3}(15x + 4) = -6x + 14$

6. $5|2x - 5| + 4 = 29$

7. $x^2 = 16$

8. $25x^2 = 225$

9. Solve for h: $T = 2\pi rh + 2\pi r^2$

Multiply: 10. $(2r - 3s)(5r + 7s)$

11. $(h^2 + 6h - 7)(2h^2 + 7h + 10)$

Systems of Equations:

Substitution:

Solve 1 equation for 1 variable.
Rearrange.
Plug into 2nd equation.
Solve for the other variable.

Elimination:

Find opposite coefficients for 1 variable.
Multiply equation(s) by constant(s).
Add equations together (lose 1 variable).
Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

Ex: $3x + y = 6$
$2x + 2y = 4$

$3x + y = 6$	solve 1st equation for y
$2x - 2(6 - 3x) = 4$	plus into 2nd equation
$2x - 12 + 6x = 4$	distribute
$8x = 16$	simplify
$x = 2$	

Ex: $3x + y = 6$
$2x + 2y = 4$

$6x = 2y = 12$	multiply 1st equation by 2
$2x - 2y = 4$	coefficients of y are opposite
$8x = 16$	add
$x = 2$	simplify

Plug $x = 2$ back into original $3(2) + y = 6$
$6 + y = 6$
$y = 0$

Solution: (2, 0)

Solve each system of equations. Use any method.

1. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

2. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

3. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Functions - To evaluate a function for a given value, substitute the value into the function for x and simplify.

Recall: $(f \circ g)(x) = f\big(g(x)\big)$ read "f of g of x" Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case $f(x)$).

Ex: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $(f \circ g)(x)$.

$$(f \circ g)(x) = f\big(g(x)\big) = f(x - 4)$$

$$2(x - 4)^2 + 1$$

$$2(x^2 - 8x + 16) + 1$$

$$2(x^2 - 8x + 16) + 1$$

$$2x^2 - 16x + 32 + 1$$

$$(f \circ g)(x) = f\big(g(x)\big) = 2x^2 - 16x + 33$$

Given $f(x) = 7 - x^2$ and $(x) = x - 4$, find each value.

1. $f(-4)$

2. $g(-4)$

3. $f(g(-2))$

4. $f(4a)$

5. $f(t+1)$

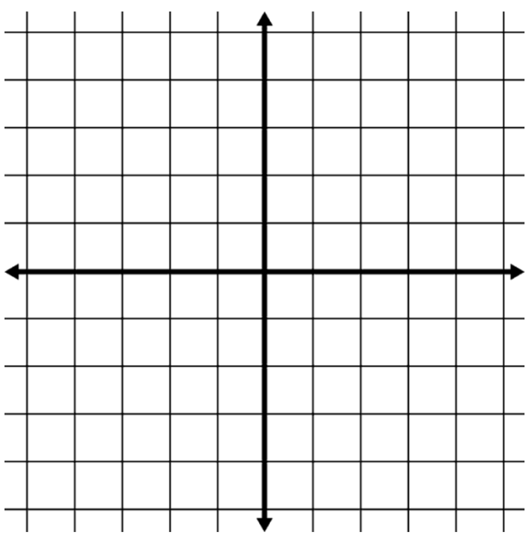
6. $(f \circ g)(x)$

7. $(g \circ f)(x)$

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________

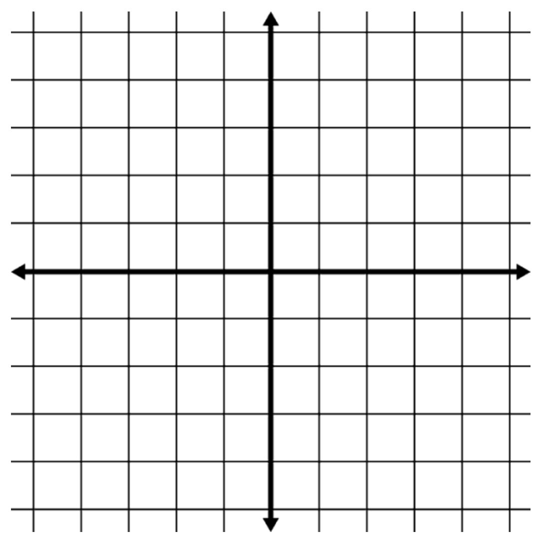
Graph each function and describe the transformations.

$\boldsymbol{g(x) = (x+2)^2}$



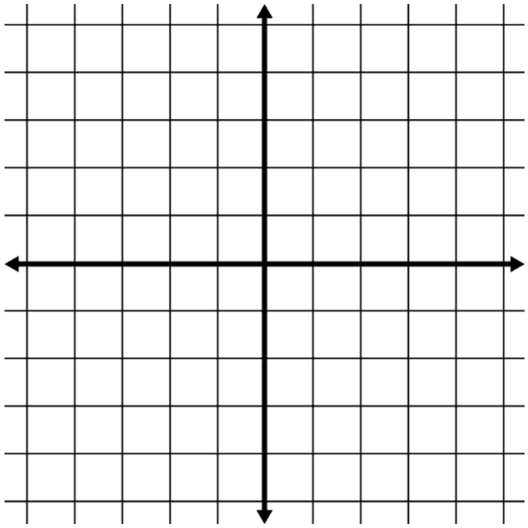
Describe the transformation of $g(x)$ from the parent function $f(x) = x^2$

$\boldsymbol{g(x) = |x| - 3}$



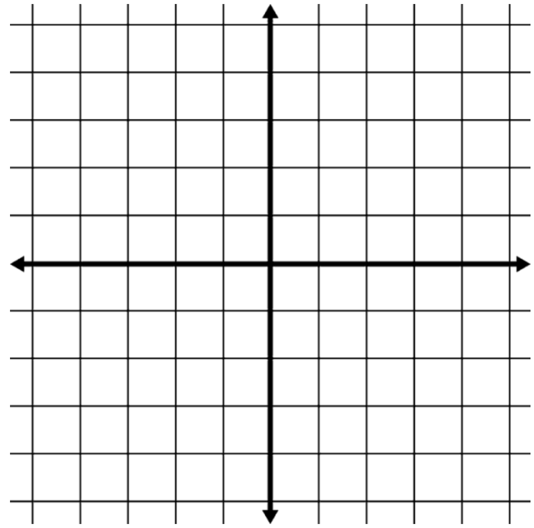
Describe the transformation of $g(x)$ from the parent function $f(x) = |x|$

$g(x) = \sqrt{x+3} - 2$



Describe the transformation of $g(x)$ from the parent function $f(x) = \sqrt{x}$

$h(x) = 2|x+1| + 1$



Describe the transformation of $h(x)$ from the parent function $f(x) = |x|$

Radicals:

To simplify means that 1) no radical has a perfect square factor <u>and</u>
2) there is no radical in the denominator (rationalized).

Recall- the Product Property $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the Quotient Property $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24x^4} = \sqrt{4} \cdot \sqrt{6} \cdot \sqrt{x^4}$ find a perfect square factor
$= 2x^2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$ multiply straight across

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60} \cdot \sqrt{105}$

7. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

Rationalize.

8. $\frac{1}{\sqrt{2}}$

9. $\frac{2}{\sqrt{3}}$

10. $\frac{3}{2-\sqrt{5}}$

Exponents:

Rules of 1: $a^1 = a$ and $1^a = 1$

Zero Rule $a^0 = 1$

Product Rule $a^m \cdot a^n = a^{m+n}$

Quotient Rule $\frac{a^m}{a^n} = a^{m-n}$

Power Rule $(a^m)^n = a^{m \cdot n}$

Negative Exponents $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Root Rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Simplify. All answers must have <u>positive</u> exponents.

1. $x^5 \cdot x^7$

2. $(x^7)^5$

3. $\frac{x^7}{x^5}$

4. $\frac{x^5}{x^7}$

5. $\frac{(3x^2)^3}{(9xy)^5}$

6. $2(3x^3)(5x^2)^2$

7. $\left(\frac{3}{4}\right)^{-2}$

8. $(-3x^2y^3)^4$

9. $(5x^4y^3)^2$

10. $(-4xy^5) \cdot (2x^3y^4)$

11. $\frac{36x^6y^7}{4x^3y^9}$

12. $\frac{15x^3y^{-2}}{3x^2y^3}$

Evaluate.

13. $9^{\frac{1}{2}} \cdot 81^{\frac{3}{4}}$

14. $27^{\frac{-2}{3}}$

15. $\left(\frac{3^4}{7^4}\right)^{\frac{-1}{4}}$

16. $\left(2^{\frac{1}{5}} \cdot 2^{\frac{1}{2}}\right)^{10}$

Factoring - follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a) if you have one (other than 1) factor it out front. Ex: $3x^2 - 6 = 3(x^2 - 2)$
b) if you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

<u>2 Terms</u>

a) Is it a difference of two squares? $a^2 - b^2 = (a+b)(a-b)$ Ex: $x^2 - 25 = (x+5)(x-5)$
b) Is it a sum or difference of two cubes? $a^3 - b^3 = (a+b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

<u>3 Terms</u>: $ax^2 + bx + c$

a) $a = 1$ --- Find 2 numbers that multiply to c and add to b Ex: $x^2 + 7x + 12 = (x+3)(x+4)$

b) $a \neq 1$ --- Follow these Steps: Ex: $8x^2 + 14x + 3$

1) Multiply $a \cdot c$ — $8 \cdot 3 = 24$
2) Find 2 numbers that multiply to this number and add to b — $12 \cdot 2 = 24$; $12 + 2 = 14$
3) Divide each of these numbers by a and simplify — $\frac{12}{8} = \frac{3}{2}$; $\frac{2}{8} = \frac{1}{4}$
4) Write factors from the bottom up — $(2x+3)(4x+1)$

<u>4 Terms</u>: $ax^3 + bx^2 + cx + d$ Ex: $x^3 + 3x^2 + 9x + 27$

a) Pair up the first two terms and last two terms — $(x^3 + 3x^2) + (9x + 27)$
b) Factor out front of the parentheses the terms have in common — $\boldsymbol{x^2}(x+3) + \mathbf{9}(x+3)$
c) Put the leftover terms in the parentheses — $(x+3)(\boldsymbol{x^2} + \mathbf{9})$

Factor completely. You can check your answer by multiplying.

1. $3x^3 - 15x$ ____________________

2. $x^2 + 5x - 14$ ____________________

3. $x^2 - 16$ ____________________

4. $25x^2 - 9y^2$ ____________________

5. $3x^2 + 9x - 30$ ____________________

6. $4x^2 - 21x - 18$ ____________________

7. $x^2 + 16x + 64$ ____________________

8. $x^3 - y^3$ ____________________

9. $6x^3 - 30x^2$ ____________________

10. $5x^3y^2 + 15x^2y^3$ ____________________

11. $x^2 - 17x + 30$ ____________________

12. $2x^2 + 9x - 5$ ____________________

13. $x^2 - 81$ ____________________

14. $25x^2 - 64$ ____________________

15. $a^2 - 14a + 49$ ____________________

16. $x^2 + 20x + 100$ ____________________

17. $25x^2 - 10x + 1$ ____________________

18. $2x^2 - 11x + 5$ ____________________

19. $5x^2 - 42x - 27$ ____________________

20. $25x^2 + 60x + 36$ ____________________

21. $7x^2 - 11x + 4$ ____________________

22. $12x^2 + 24x + 9$ ____________________

Solving Quadratics - To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the *quadratic formula*.

Ex:

$x^2 - 4x = 20$	set equal to zero FIRST
$x^2 - 4x - 20 = 0$	factor (if possible)
$(x+3)(x-7) = 0$	set each factor equal to zero
$x + 3 = 0 \quad x - 7 = 0$	solve each for x.
$x = -3 \quad x = 7$	

Quadratic Formula- allows you to solve any quadratic for all its real and imaginary roots. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex: Solve $x^2 + 2x + 3 = 0$

$x^2 + 2x + 3 = 0$	cannot be factored
a=1 b=2 c=3	determine the values for a, b, and c.
$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}$	substitute a, b, and c into the quadratic formula

$$x = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = -1 \pm i\sqrt{2}$$

$x = -1 + i\sqrt{2}$ and $x = -1 - i\sqrt{2}$

Solve.

1. $x^2 + 10 = 7x$

2. $x^2 - 10 = -3x$

3. $3x^2 - 7x + 8 = 0$

4. $4x^2 - 6x + 1 = 0$

5. $2x^2 - 9x = 5$

6. $9x^2 + 12x + 68 = 0$

Geometry

Coordinate Geometry Formulas
Let (x_1, y_1) and (x_2, y_2) be two points in the plane.
slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$ midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Determine the distance between the following pairs of points.

1. (2, 6) and (5, 9)
2. (-3, 6) and (2, 1)
3. (-1, -1) and (-2, -5)

Long Division – can be used when dividing any polynomials.
Synthetic Division – can ONLY be used when dividing a polynomial by a linear (degree one) polynomial.

Ex: $\frac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$\frac{2x^3+3x^2-6x+10}{x+3}$

$$
\begin{array}{r}
2x^2 - 3x + 3 + \frac{1}{x+3} \\
x + 3 \,\big)\overline{\;2x^3 + 3x^2 - 6x + 10} \\
\underline{(-)\;(2x^3 + 6x^2)} \\
-3x^2 - 6x + 10 \\
\underline{(-)\;(-3x^2 - 9x)} \\
3x + 10 \\
\underline{(-)\;(3x + 9)} \\
1
\end{array}
$$

Synthetic Division

$\frac{2x^3+3x^2-6x+10}{x+3}$

$-3]$	2	3	-6	10
	↓	-6	9	-9
	2	-3	3	1

$= 2x^2 - 3x + 3 + \frac{1}{x+3}$

Divide each polynomial using long division OR synthetic division.

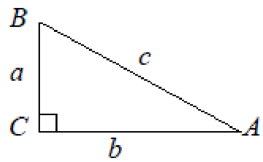
1. $\frac{c^3-3c^2+18c-16}{c^2+3c-2}$

2. $\frac{x^4-2x^2-x+2}{x+2}$

3. $(x^3 - 7x^2 + 14x - 8) \div (x - 4)$

Special Triangles

Formulas for Right Triangles



Pythagorean Theorem:

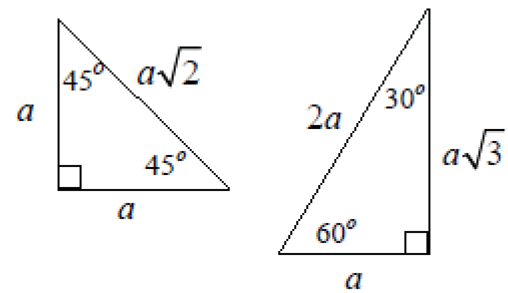
$a^2 + b^2 = c^2$

$\sin A = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos A = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$

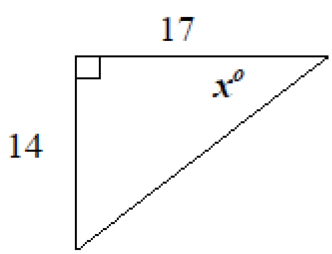
$\tan A = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$

Special Triangles

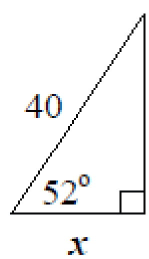


Determine the value of x and/or y in each figure below.

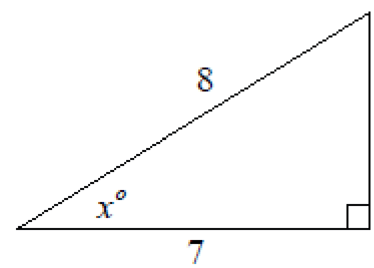
1.



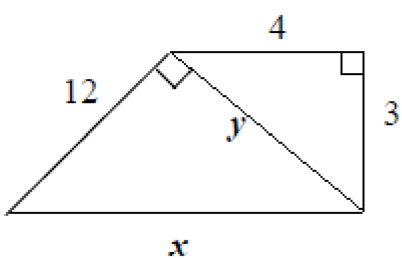
2.



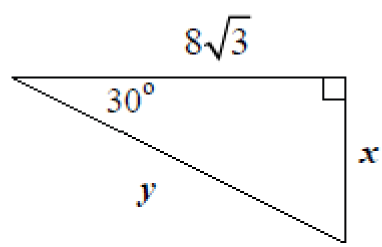
3.



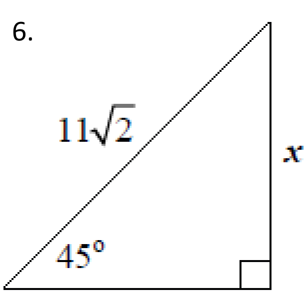
4.



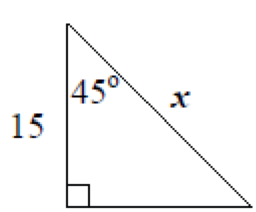
5.



6.



7.



8.