AP Calculus AB
Summer Review Packet

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1. This packet is to be handed in to your Calculus teacher on the first day of the school year.
2. All work must be shown on separate paper attached to the packet.
3. There will be a test on all of the material in this packet during the first week of school.

I. Evaluating Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$
\begin{aligned}
f(g(x)) &= f(x-4) \\
&= 2(x-4)^2 + 1 \\
&= 2(x^2 - 8x + 16) + 1 \\
&= 2x^2 - 16x + 32 + 1 \\
f(g(x)) &= 2x^2 - 16x + 33
\end{aligned}
$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$*, and let h represent some constant. Evaluate each function for the specified value.*

1. $g(-3)$
2. $f(t+1)$
3. $f[g(-2)]$
4. $g[f(m+2)]$

Let $f(x) = x^2$, g(x) = 2x + 5, and $h(x) = x^2 - 1$. *Evaluate each function for the specified value.*

5. $f[g(x-1)]$
6. $g[h(x^3)]$

Find $\dfrac{f(x+h) - f(x)}{h}$ *for each given function f(x). Let h represent some constant.*

7. $f(x) = 9x + 3$
8. $f(x) = 5 - 2x$

II. Intercepts

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$x - \text{int}.\ (Let\ y = 0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x - \text{intercepts}\ (-1,0)\ and\ (3,0)$

$y - \text{int}.\ (Let\ x = 0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept}\ (0,-3)$

Find the x and y intercepts for each equation.

9. $y = 2x - 5$

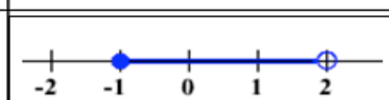
10. $y = x^2 + x - 2$

11. $y = x\sqrt{16 - x^2}$

12. $y^2 = x^3 - 4x$

III. Interval Notation

13. Complete the table using the appropriate notation or graphical representation.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each equation, stating your answer in BOTH interval notation and a graphical representation (number line).

14. $-4 \leq 2x - 3 < 4$

15. $\dfrac{x}{2} - \dfrac{x}{3} > 5$

IV. Domain and Range

Remember, the domain is the set of all possible values for x (everything you can input), and the range is the set of all possible values for y (every possible output).

Find the domain and range of each function. Write your answer in interval notation.

16. $f(x) = x^2 - 5$

17. $f(x) = -\sqrt{x+3}$

18. $f(x) = 3\sin x$

19. $f(x) = \dfrac{2}{x-1}$

V. Equation of a Line

Slope-Intercept form: $y = mx + b$ (where m is the slope and b is the y-intercept)

Point-Slope form: $y - y_1 = m(x - x_1)$ (where m is the slope and (x_1, y_1) is a point on the line)

Vertical Line: $x = c$ (slope is undefined, c is x-intercept)

Horizontal Line: $y = c$ (slope is zero, c is y-intercept)

20. Write the equation of a line with a slope of 3 and a y-intercept of 5 in slope-intercept form.

21. Write the equation of a line passing through the point (0, 5) with a slope of 2/3 in point-slope form.

22. Write the equation of a line passing through the points (-3, 6) and (1, 2).

23. Write the equation of a line passing through the point (-4, 2) with a slope of zero.

24. Write the equation of a line passing through the point (-4, 2) with an undefined slope.

25. Write the equation of a line passing through the point (2, 8) and parallel to the line $y = \dfrac{5}{6}x - 1$

VI. Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.	Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

26. Convert to degrees:

a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63

27. Convert to radians:

a. 45° b. -17° c. 237°

VII. Reference Triangles

28. Sketch the angle in standard position. Draw the reference triangle and label both sides, if possible.

a. $\frac{2}{3}\pi$ b. 225°

c. $-\frac{\pi}{4}$ d. 30°

VIII. Unit Circle

29. $\sin 180^\circ$

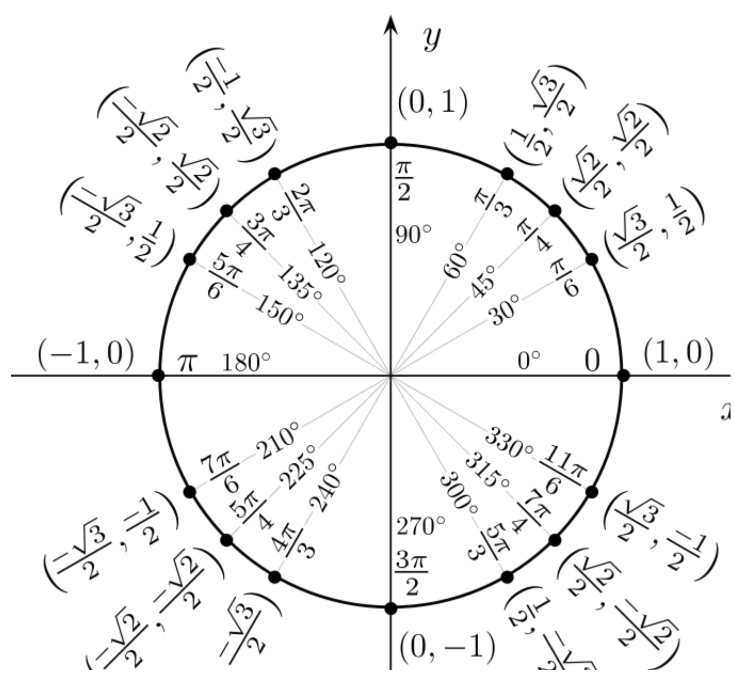
30. $\cos 60^\circ$

31. $\tan 120^\circ$

32. $\tan 45^\circ$

33. $\sin(-135^\circ)$

34. $\cos 270^\circ$



IX. Trigonometric Equations

Solve each of the equations for $0 \le x < 2\pi$ *. Isolate the variable, sketch a reference triangle, and find all of the solutions within the given domain. (EXACT SOLUTIONS – NO DECIMALS!) Remember to double the domain when solving for a double angle!*

35. $\sin x = -\frac{1}{2}$

36. $2\cos x = \sqrt{3}$

37. $\sin 2x = -\frac{\sqrt{3}}{2}$

38. $\sin^2 x = \frac{1}{2}$

39. $4\cos^2 x - 3 = 0$

40. $\cos 2x = \frac{1}{\sqrt{2}}$

X. <u>Inverse Trigonometric Functions</u>

Inverse Trig Functions can be written two different ways: $\arctan x$ or $\tan^{-1} x$

Remember that some inverse trig functions have restricted domains!!!

The domain of $\arcsin x$ is $-1 \le x \le 1$

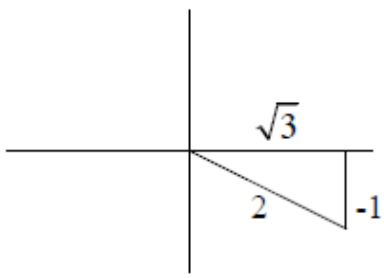
The domain of $\arccos x$ is $-1 \le x \le 1$

The domain of $\arctan x$ is $-\infty < x < \infty$

Example:

Express the value of "y" in radians.

$y = \arctan \frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, $y = -\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: $y = -\frac{\pi}{6}$

For each of the following, express the value of "y" in radians.

41. $y = \arcsin \frac{-\sqrt{3}}{2}$

42. $y = \arccos(-1)$

43. $y = \arctan(-1)$

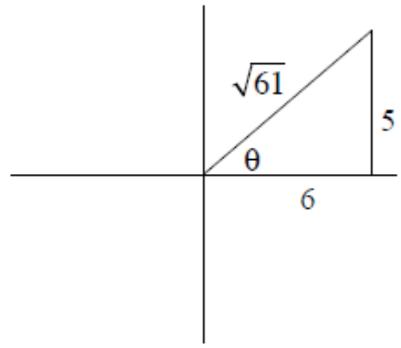
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

44. $\tan\left(\arccos\frac{2}{3}\right)$

45. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

46. $\sin\left(\arctan\frac{12}{5}\right)$

47. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

XI. Vertical Asymptotes

Determine the vertical asymptote(s) for the function. The vertical asymptote occurs where the function is undefined (where the denominator is equal to zero).

48. $f(x) = \frac{1}{x^2}$

49. $f(x) = \frac{x^2}{x^2 - 4}$

50. $f(x) = \frac{2 + x}{x^2(1 - x)}$

XII. Horizontal Asymptotes

Determine the horizontal asymptote using the three cases below.

Case 1: The degree of the numerator is less than the degree of the denominator. The function has a horizontal asymptote at $y = 0$.

Case 2: The degree of the numerator is equal to the degree of the denominator. The function has a horizontal asymptote equal to the ratio of the leading coefficients.

Case 3: The degree of the numerator is greater than the degree of the denominator. The function has no horizontal asymptote.

Determine the horizontal asymptote for the function.

51. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

52. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

53. $f(x) = \frac{4x^5}{x^2 - 7}$

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0