

Sail into Summer with Math!



For Students Entering Investigations into Mathematics

This summer math booklet was developed to provide students in kindergarten through the eighth grade an opportunity to review grade level math objectives and to improve math performance.

Summer 2008

Sail into Summer with Math!

One goal of Gaithersburg Middle School is to promote increased math performance at all grade levels. Completing the summer math booklet allows schools, students, and parents to work together to achieve this goal. Students who complete the summer math booklet will be able to:

- Increase retention of math concepts,
- Improve and raise the level of math MSA performance,
- Work toward closing the gap in student performance,
- Apply math concepts to performance tasks, and
- Successfully complete Algebra 1.

Student Responsibilities



Students will be able to improve their own math performance by:

- Completing the summer math booklet
- Reviewing math skills throughout the summer, and
- **Returning the math booklet to next year's math teacher.**

Student Signature

Grade

Date

Parent Responsibilities

Parents will be able to promote student success in math by:

- Supporting the math goal of the cluster of schools,
- Monitoring student completion of the summer math booklet,
- Encouraging student use of math concepts in summer activities, and
- **Insuring the return of the math booklet to school in the fall.**



Parent Signature

Date

The "Sail into Summer with Math!" booklets were developed by:
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The cover of the IM summer math booklet was created by
Marie Stump, a Seventh Grade student at
John Poole Middle School.

Rename Fractions, Percents, and Decimals

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as $\frac{3}{5}$, and divide the numerator (the top number of a fraction) by the denominator (the bottom number of a fraction). So:

$$\begin{array}{r} \overline{) 3.0} \\ \underline{- 30} \\ 0 \end{array} \quad \text{and the fraction } \frac{3}{5} \text{ is equivalent to the decimal } 0.6$$

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a decimal by 100, so $0.6 \times 100 = 60$ and $\frac{3}{5} = 0.6 = 60\%$)

To convert a percent to a decimal, we divide the percent by 100, $60\% \div 100 = 0.6$ so $60\% = 0.6$

To convert a fraction into a percent, we can use a proportion to solve,

$$\frac{3}{5} = \frac{x}{100}, \text{ so } 5x = 300 \text{ which means that } x = 60 = 60\%$$

Exercises:

No Calculators!

Rename each fraction as a decimal:

1. $\frac{1}{5} =$

2. $\frac{3}{4} =$

3. $\frac{1}{2} =$

4. $\frac{1}{3} =$

5. $\frac{8}{10} =$

6. $\frac{2}{3} =$

Rename each fraction as a percent:

7. $\frac{1}{5} =$

8. $\frac{3}{4} =$

9. $\frac{1}{2} =$

10. $\frac{1}{3} =$

11. $\frac{8}{10} =$

12. $\frac{2}{3} =$

Rename each percent as a decimal:

13. $8\% =$

14. $60\% =$

15. $11\% =$

16. $12\% =$

17. $40\% =$

18. $95\% =$

Fraction Operations

Hints/Guide:

When adding and subtracting fractions, we need to be sure that each fraction has the same denominator, then add or subtract the numerators together. For example:

$$\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{6}{8} = \frac{1+6}{8} = \frac{7}{8}$$

That was easy because it was easy to see what the new denominator should be, but what about if it is not so apparent? For example: $\frac{7}{12} + \frac{8}{15}$

For this example we must find the Lowest Common Denominator (LCM) for the two denominators. 12 and 15

$$12 = 12, 24, 36, 48, 60, 72, 84, \dots$$

$$15 = 15, 30, 45, 60, 75, 90, 105, \dots$$

$$\text{LCM}(12, 15) = 60$$

So, $\frac{7}{12} + \frac{8}{15} = \frac{35}{60} + \frac{32}{60} = \frac{35+32}{60} = \frac{67}{60} = 1\frac{7}{60}$ Note: Be sure answers are in lowest terms

To multiply fractions, we multiply the numerators together and the denominators together, and then simplify the product. To divide fractions, we find the reciprocal of the second fraction (flip the numerator and the denominator) and then multiply the two together. For example:

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6} \quad \text{and} \quad \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$$

Exercises: Perform the indicated operation:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $\frac{1}{4} + \frac{3}{5} =$

2. $\frac{6}{7} + \frac{2}{3} =$

3. $\frac{3}{4} - \frac{2}{3} =$

4. $\frac{2}{5} - \frac{2}{9} =$

5. $\frac{1}{3} \cdot \frac{2}{3} =$

6. $\frac{3}{4} \cdot \frac{3}{5} =$

7. $\frac{7}{8} \cdot \frac{2}{5} =$

8. $\frac{3}{8} \div \frac{3}{4} =$

Multiply Fractions and Solve Proportions

Hints/Guide:

To solve problems involving multiplying fractions and whole numbers, we must first place a one under the whole number, then multiply the numerators together and the denominators together. Then we simplify the answer:

$$\frac{6}{7} \bullet 4 = \frac{6}{7} \bullet \frac{4}{1} = \frac{24}{7} = 3\frac{3}{7}$$

To solve proportions, one method is to determine the multiplying factor of the two equal ratios. For example:

$$\frac{4}{9} = \frac{24}{x} \text{ since 4 is multiplied by 6 to get 24, we multiply 9 by 6, so } \frac{4}{9} = \frac{24}{54}.$$

Since the numerator of the fraction on the right must be multiplied by 6 to get the numerator on the left, then we must multiply the denominator of 9 by 6 to get the missing denominator, which must be 54.

Exercises: Solve (For problems 8 - 15, solve for N):

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4 \bullet \frac{3}{4} =$

2. $\frac{1}{5} \bullet 7 =$

3. $8 \bullet \frac{1}{5} =$

4. $6 \bullet \frac{3}{7} =$

5. $\frac{1}{5} = \frac{n}{20}$

6. $\frac{3}{n} = \frac{12}{28}$

7. $\frac{1}{n} = \frac{5}{25}$

8. $\frac{n}{4} = \frac{3}{12}$

Add Mixed Numbers

Hints/Guide:

When adding mixed numbers, we add the whole numbers and the fractions separately, then simplify the answer. For example:

$$\begin{array}{r} 4\frac{1}{3} = 4\frac{8}{24} \\ + 2\frac{6}{8} = 2\frac{18}{24} \\ \hline 6\frac{26}{24} = 6 + 1\frac{2}{24} = 7\frac{2}{24} = 7\frac{1}{12} \end{array}$$

First, we convert the fractions to have the same denominator, then add the fractions and add the whole numbers. If needed, we then simplify the answer.

Exercises: Solve in lowest terms:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

$$1. \begin{array}{r} 2\frac{1}{4} \\ + 8\frac{1}{2} \\ \hline \end{array}$$

$$2. \begin{array}{r} 3\frac{8}{15} \\ + 7\frac{1}{3} \\ \hline \end{array}$$

$$3. \begin{array}{r} 3\frac{3}{5} \\ + 5\frac{1}{2} \\ \hline \end{array}$$

$$4. \begin{array}{r} 5\frac{3}{8} \\ + 4\frac{1}{4} \\ \hline \end{array}$$

$$5. \begin{array}{r} 7\frac{3}{7} \\ + 6\frac{1}{2} \\ \hline \end{array}$$

$$6. \begin{array}{r} 5\frac{5}{9} \\ + 1\frac{1}{3} \\ \hline \end{array}$$

Subtract Mixed Numbers

Hints/Guide:

When subtracting mixed numbers, we subtract the whole numbers and the fractions separately, then simplify the answer. For example:

$$\begin{array}{r} 7\frac{3}{4} = 7\frac{18}{24} \\ -2\frac{15}{24} = 2\frac{15}{24} \\ \hline 5\frac{3}{24} = 5\frac{1}{8} \end{array}$$

First, we convert the fractions to have the same denominator, then subtract the fractions and subtract the whole numbers. If needed, we then simplify the answer.

Exercises: Solve in lowest terms:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

$$1. \begin{array}{r} 4\frac{1}{3} \\ -2\frac{1}{4} \\ \hline \end{array}$$

$$2. \begin{array}{r} 6\frac{3}{4} \\ -\frac{2}{3} \\ \hline \end{array}$$

$$3. \begin{array}{r} 9\frac{2}{3} \\ -6\frac{1}{4} \\ \hline \end{array}$$

$$4. \begin{array}{r} 6\frac{3}{4} \\ -5\frac{1}{5} \\ \hline \end{array}$$

$$5. \begin{array}{r} 7\frac{1}{2} \\ -3\frac{1}{4} \\ \hline \end{array}$$

$$6. \begin{array}{r} 3\frac{1}{2} \\ -2\frac{3}{10} \\ \hline \end{array}$$

Multiply Mixed Numbers

Hints/Guide:

To multiply mixed numbers, we first convert the mixed numbers into improper fractions. This is done by multiplying the denominator by the whole number part of the mixed number and then adding the numerator to this product, and this is the numerator of the improper fraction. The denominator of the improper fraction is the same as the denominator of the mixed number. For example:

$$3\frac{2}{5} \text{ leads to } 3 \cdot 5 + 2 = 17 \text{ so } 3\frac{2}{5} = \frac{17}{5}$$

Once the mixed numbers are converted into improper fractions, we multiply and simplify just as with regular fractions. For example:

$$5\frac{1}{5} \cdot 3\frac{1}{2} = \frac{26}{5} \cdot \frac{7}{2} = \frac{182}{10} = 18\frac{2}{10} = 18\frac{1}{5}$$

Exercises: Solve and place your answer in lowest terms:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $3\frac{1}{3} \cdot 4\frac{1}{2} =$

2. $2\frac{2}{3} \cdot 1\frac{1}{4} =$

3. $1\frac{1}{9} \cdot 4\frac{3}{5} =$

4. $4\frac{3}{4} \cdot 1\frac{1}{5} =$

5. $3\frac{1}{3} \cdot 6\frac{4}{5} =$

6. $6\frac{2}{3} \cdot 7\frac{3}{7} =$

Divide Mixed Numbers

Hints/Guide:

To divide mixed numbers, we must first convert to improper fractions using the technique shown in multiplying mixed numbers. Once we have converted to improper fractions, the process is the same as dividing regular fractions. For example:

$$2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \cdot \frac{3}{10} = \frac{15}{20} = \frac{3}{4} \qquad 3\frac{1}{2} \div 8\frac{2}{3} = \frac{7}{2} \div \frac{26}{3} = \frac{7}{2} \cdot \frac{3}{26} = \frac{21}{52}$$

Exercises: Solve and place your answer in lowest terms:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $1\frac{1}{5} \div 4\frac{2}{5} =$

2. $6\frac{1}{2} \div 4\frac{2}{3} =$

3. $5\frac{1}{2} \div 6\frac{2}{3} =$

4. $\frac{8}{9} \div 2\frac{3}{5} =$

5. $3\frac{2}{3} \div 4\frac{3}{7} =$

6. $4\frac{4}{7} \div \frac{4}{9} =$

Find Percent of a Number

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut as moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number. For example:

$$45\% \text{ of } 240 = 45\% \times 240 = 0.45 \times 240 = 108$$

Exercises: Solve for n:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $30\% \text{ of } 450 = n$

2. $7\% \text{ of } 42 = n$

3. $10\% \text{ of } 321 = n$

4. $15\% \text{ of } 54 = n$

5. $65\% \text{ of } 320 = n$

6. $80\% \text{ of } 64 = n$

Find Elapsed Time

Hints/Guide:

The key to understanding time problems is to think about time revolving around on a clock. If a problem starts in the morning (a.m.) and ends in the afternoon (p.m.), count the amount of time it takes to get to 12 noon, then count the amount of time it takes until the end. For example:

Joanne is cooking a large turkey and puts it in the oven at 10:15 in the morning. Dinner is planned for 4:30 in the evening and this is when Joanne will take the turkey out of the oven. How long will the turkey cook?

From 10:15 to 12:00 noon is 1 hour 45 minutes. From 12:00 noon to 4:30 p.m. is 4 hours 30 minutes. To add the times together:

$$\begin{array}{r} 1 \text{ h } 45 \text{ m} \\ + \quad 4 \text{ h } 30 \text{ m} \\ \hline 5 \text{ h } 75 \text{ m} = 5 \text{ h } + 1 \text{ h } 15 \text{ m} = 6 \text{ h } 15 \text{ m} \end{array}$$

The turkey will cook for 6 hours and 15 minutes.

Exercises:

1. The school day begins at 7:55 a.m. and ends at 2:40 p.m. How long are you in school?
2. If you go to sleep at 9:30 p.m. and wake up at 6:30 a.m. the next morning, how long did you sleep?
3. If you want to cook a chicken that takes 4 hours and 30 minutes to completely cook and you are planning dinner for 6:00 p.m., what time do you need to start cooking the chicken?
4. If you ride your bike for 2 hours and 45 minutes and you started riding at 11:30 a.m., at what time will you finish your riding?
5. If you go to a basketball game at the MCI Center to see the Washington Wizards, and the game begins at 7:05 p.m. and ends at 10:35 p.m., how long was the game?

Solve Money Problems

Hints/Guide:

Solving money problems is merely applying the rules of decimals in a real life setting. When reading the problems, we need to determine whether we add (such as depositing money or determining a total bill), subtract (checks, withdrawals, and the difference in pricing), multiply (purchasing multiple quantities of an item), or divide (distributing money evenly, loan payments). Once we have determined which operation to use, we apply the rules for decimal operations and solve the problem and label our answer appropriately.

Exercises:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. Frank works at Apartment Depot and earns \$8.50 per hour. Last week, he worked 36 hours. What was his total pay?
2. Harry went to Rent-a-Center and rented a pneumatic nailer for \$45.00, a power sander for \$39.95, and a radial arm saw for \$57.90. What was his total bill, excluding tax?
3. Joe is planning a trip to Houston and has calculated \$450.95 for lodging, \$98.00 for food, and \$114.50 for gasoline. How much will his trip cost?
4. Susan has \$350 in her checking account. She writes checks for \$45.70 for flowers, \$78.53 for books, and \$46.98 for CD's. How much money is left in her checking account?
5. In order to pay off the car she bought, Lauri had to make 34 more payments of \$145.98. How much does she still owe?
6. Jared earns \$455.00 per week as manager of the Save-Mart. What will be his income over 12 weeks?
7. The Jennings family paid \$371.40 for the year for their cable service. If their payments were the same each month, how much was their monthly bill?

Solve Problems using Percent

Hints/Guide:

When solving percent problems, we apply the rules for finding percent of a number in realistic situations. For example, to find the amount of sales tax on a \$450.00 item if the tax rate is 5%, we find 5% of 450 ($.05 \times 450 = 22.5$), and then label our answer in dollars, getting \$22.50.

Exercises:

No Calculators!

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. Susie has just bought a pair of jeans for \$45.00, a sweater for \$24.00, and a jacket for \$85.00. The sales tax is 5%. What is her total bill?
2. Jack bought a set of golf clubs for \$250.00 and received a rebate of 20%. How much was the rebate?
3. A construction manager calculates it will cost \$2,890 for materials for her next project. She must add in 10% for scrap and extras. What will be the total cost?
4. The regular price for a video game system is \$164.50 but is on sale for 30% off. What is the amount of the discount?

What is the sale price?

5. Cindy earns a 15% commission on all sales. On Saturday, she sold \$980 worth of merchandise. What was the amount of commission she earned on Saturday?
6. The band had a fundraiser and sold \$25,000 worth of candy. They received 40% of this amount for themselves. How much did they receive?

Mean, Median, and Mode

Hints/Guide:

We need to define some terms to solve problems involving mean, median, and mode. Mean is the sum of the numbers being considered divided by the total number of numbers being considered (also called "average"). Median is the number in the middle of the data set after the numbers have been placed in order from least to greatest. If there is an even number of elements, the median is the mean of the two numbers in the middle of the data set. The mode is the number or numbers that occur most frequently in a data set. For example, with the data set of 56, 62, 67, 45, 81, 76:

Mean is $56 + 62 + 67 + 45 + 81 + 76 = 387$ and $387 \div 6 = 64.5$, so the mean is 64.5

Median is (in order the data is 45, 56, 62, 67, 76, 81) the mean of 62 and 67, which is $(62 + 67 = 129$ and $129 \div 2 = 64.5)$ also 64.5.

There is no mode, because no number occurs more than once.

Exercises:

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

Find the mean, median, and mode of each of the following data sets:

You may use a calculator for this page.

1. 54, 65, 74, 35, 87

Mean:

Median:

Mode:

2. 54.6, 45.98, 67.4, 55.6, 45.7, 58.9

Mean:

Median:

Mode:

3. 122, 145, 156, 176, 198, 202

Mean:

Median:

Mode:

4. 11, 14, 16, 15, 32, 23, 27, 27, 23, 43

Mean:

Median:

Mode:

5. 6, 7, 8, 4, 6, 5, 8, 3, 6, 8, 5, 4

Mean:

Median:

Mode:

6. -4, 7, -3, 4, 8, 12, -5, -3, 8, 16, 9

Mean:

Median:

Mode:

Integers I

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add its additive inverse.

For example $6 - 11 = a$ becomes $6 + -11 = a$ and solves as $-5 = a$.

Exercises: Solve the following problems:

No Calculators!

1. $6 + (-7) =$

2. $(-4) + (-5) =$

3. $6 + (-9) =$

4. $(-6) - 7 =$

5. $6 - (-6) =$

6. $7 - (-9) =$

7. $5 + (-8) =$

8. $-15 + 8 =$

9. $14 + (-4) =$

10. $-9 - (-2) =$

11. $-7 - 6 =$

12. $-8 - (-19) =$

13. $29 - 16 + (-5) =$

14. $-15 + 8 - (-19) =$

15. $45 - (-13) + (-14) =$

16. $-15 - 6 - 9 =$

Integers II

Hints/Guide:

The rules for multiplying integers are:

Positive x Positive = Positive

Negative x Negative = Positive

Positive x Negative = Negative

Negative x Positive = Negative

The rules for dividing integers are the same as multiplying integers.

Remember order of operations (PEMDAS) when there is more than one operation.

Exercises: Solve the following problems:

No Calculators!

1. $4 \cdot (-3) =$

2. $(-12) \cdot (-4) =$

3. $(-8)(-3) =$

4. $\frac{-14}{2} =$

5. $\frac{28}{-4} =$

6. $\frac{-36}{-6} =$

7. $6(-5) =$

8. $8(-4 - 6) =$

9. $-6(9 - 11) =$

10. $\frac{(-5)(-6)}{-2} =$

11. $\frac{6(-4)}{8} =$

12. $\frac{-56}{2^3} =$

13. $\frac{-6 - (-8)}{-2} =$

14. $-7 + \frac{4 + (-6)}{-2} =$

15. $45 - 4(5 - (-3)) =$

16. $(-4 + 7)(-5 + 3) =$

Geometry I

Hints/Guide:

In order to learn geometry, we first must understand so geometric terms:

Right Angle - an angle that measures 90 degrees.

Acute Angle - an angle that measures less than 90 degrees.

Obtuse Angle - an angle that measures more than 90 degrees, but less than 180 degrees.

Complementary - two angles that add together to equal 90 degrees.

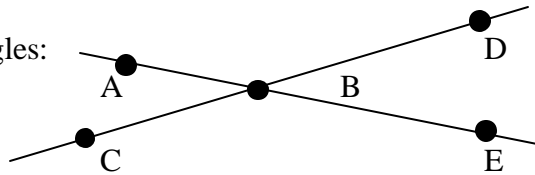
Supplementary - two angles that add together to equal 180 degrees.

Vertical - Angles which are opposite from each other.

Adjacent - angles that are next to each other.

When two lines intersect, they form four angles:

$\angle ABC$ $\angle ABD$
 $\angle DBE$ $\angle EBC$



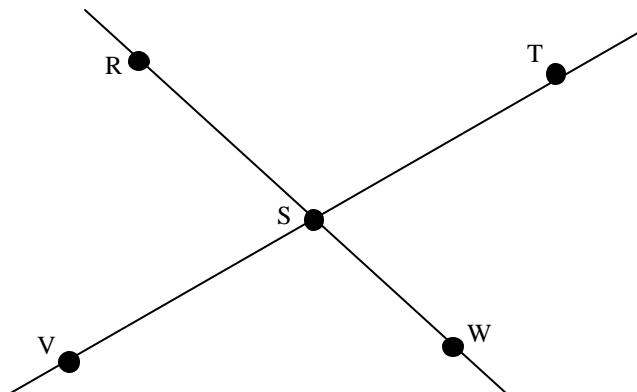
Vertical angles, such as $\angle ABC$ and $\angle DBE$, are equal in measure and adjacent angles, such as $\angle ABD$ and $\angle DBE$, are supplementary.

Exercises:

1. In the above example, list two acute angles and two obtuse angles

Acute _____, _____ Obtuse _____, _____

2. If you have a 43° angle, what is the measure of the angle which is complementary to it?
3. If you have a 43° angle, what is the measure of the angle which is supplementary to it?
4. Using the figure, list two pairs of vertical angles and two pairs of adjacent angles.



Geometry II

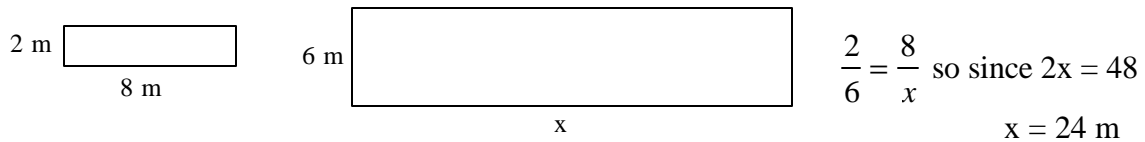
Hints/Guide:

In order to add to our knowledge of geometry, here are some additional terms:

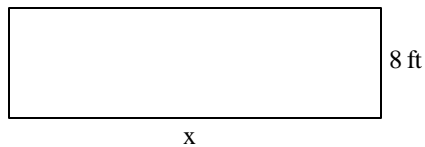
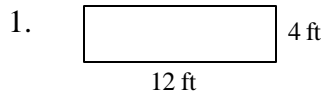
Congruent - two figures which are the same shape and the same size.

Similar - two figures which are the same shape but different size.

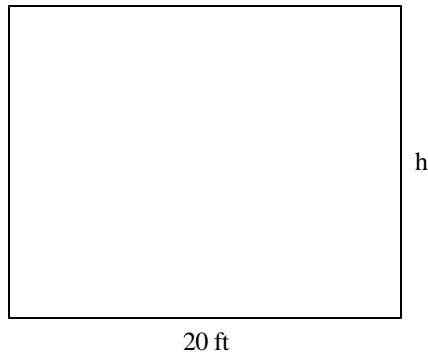
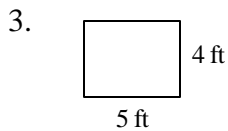
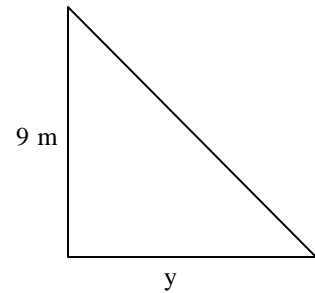
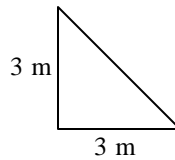
In similar triangles, congruent angles in the same location in the figure are called corresponding angles. The sides opposite corresponding angles are called corresponding sides. The measures of corresponding angle or of corresponding sides of similar triangles are proportional. For example:



Exercises: Solve for the indicated variables (All figures are similar):



2.



4.

