Formulas to Know

Exponent Formulas:

$$a^0 = 1 \qquad a^x \cdot a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y} \qquad (a^x)^y = a^{xy}$$

$$(ab)^x = a^x b^x \qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$a^{-x} = \frac{1}{a^x}$$

Factoring Formulas:

$$a^2 - b^2 = (a-b)(a+b)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

OR $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

OR $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

X_1 and x_2 are the roots of ax^2+bx+c

Radical Formulas:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (b \neq 0)$$

$$\left(\sqrt[n]{a}\right)^k = \sqrt[n]{a^k}$$

$$\sqrt[n]{\sqrt[k]{a}} = \sqrt[nk]{a}$$

$$\sqrt[n]{a} = \sqrt[nk]{a^k}$$

$$\left(\sqrt[n]{a}\right)^n = a \quad (a \geq 0)$$

$$\sqrt[n]{a} < \sqrt[n]{b}, \quad if \quad 0 \leq a < b$$

$$\sqrt{a^2} = |a| = \begin{cases} a & if \quad a \geq 0 \\ -a & if \quad a < 0 \end{cases}$$

$$\sqrt[2n]{a^{2n}} = |a|$$

$$\sqrt[2n+1]{-a} = -\sqrt[2n+1]{a} \quad (a \geq 0)$$

Absolute Value Formulas:

$$|a| = a, \text{ if } a \ge 0$$

$$|a| = -a, \text{ if } a < 0$$

Sample Problems - Simplifying Using Formulas

1. Simplify

$$\frac{\sqrt{a^2-4ab+4b^2}}{\sqrt{a^2+4ab+4b^2}} - \frac{8ab}{a^2-4b^2} + \frac{2b}{a-2b}$$

$$0 < a < 2b \qquad a^2-4b^2 = (a-2b)(a+2b)$$

$$\sqrt{a^2-4ab+4b^2} = \sqrt{(a-2b)^2} = |a-2b| = 2b-a$$

$$\sqrt{a^2+4ab+4b^2} = |a+2b| = a+2b$$

$$\frac{\sqrt{a^2-4ab+4b^2}}{\sqrt{a^2+4ab+4b^2}} = \frac{2b-a}{2b+a}$$

$$\frac{2b-a}{2b+a} - \frac{8ab}{(a-2b)(a+2b)} + \frac{2b}{a-2b} =$$

$$\frac{(2b-a)(a-2b)-8ab+2b(a+2b)}{(a-2b)(a+2b)} =$$

$$\frac{2ba-4b^2-a^2+2ab-8ab+2ab+4b^2}{(a-2b)(a+2b)} =$$

$$\frac{-a^2-2ab}{(a-2b)(a+2b)} =$$

$$\frac{-a(a+2b)}{(a-2b)(a+2b)} =$$

$$\frac{-a}{a-2b} = \frac{a}{2b-a}$$

2. Simplify $\dfrac{x^2+4x-5+(x-5)\sqrt{x^2-1}}{x^2-4x-5+(x+5)\sqrt{x^2-1}}; \quad x > 1$

$$\frac{(x+5)(x-1)+(x-5)\sqrt{x^2-1}}{(x-5)(x+1)+(x+5)\sqrt{x^2-1}}$$

Because $x > 1$; $\begin{cases} x-1 = \sqrt{(x-1)^2} \\ x+1 = \sqrt{(x+1)^2} \end{cases}$ AND $\sqrt{x^2-1} = \sqrt{(x-1)(x+1)} = \sqrt{x-1}\cdot\sqrt{x+1}$

$$\frac{\sqrt{x-1}\left((x+5)\sqrt{x-1}+(x-5)\sqrt{x+1}\right)}{\sqrt{x+1}\left((x-5)\sqrt{x+1}+(x+5)\sqrt{x-1}\right)} =$$

$$\frac{\sqrt{x-1}\left(x\sqrt{x-1}+5\sqrt{x-1}+x\sqrt{x+1}-5\sqrt{x+1}\right)}{\sqrt{x+1}\left(x\sqrt{x+1}-5\sqrt{x+1}+x\sqrt{x-1}+5\sqrt{x-1}\right)} =$$

$$\frac{\sqrt{x-1}}{\sqrt{x+1}} = \sqrt{\frac{x-1}{x+1}}$$

3. Prove that... $\frac{\sqrt{7+4\sqrt{3}}\cdot\sqrt{19-8\sqrt{3}}}{4-\sqrt{3}}-\sqrt{3}=2$

$$\frac{\sqrt{7+4\sqrt{3}}\cdot\sqrt{19-8\sqrt{3}}}{4-\sqrt{3}}=2+\sqrt{3}$$

Square both sides of the equation

$$\frac{\left(7+4\sqrt{3}\right)\left(19-8\sqrt{3}\right)}{\left(4-\sqrt{3}\right)^2}=\left(2+\sqrt{3}\right)^2$$

$$\frac{\left(7+4\sqrt{3}\right)\left(19-8\sqrt{3}\right)}{16-8\sqrt{3}+3}=4+4\sqrt{3}+3$$

$$\frac{\left(7+4\sqrt{3}\right)\left(19-8\sqrt{3}\right)}{\left(19-8\sqrt{3}\right)}=7+4\sqrt{3}$$

$$7+4\sqrt{3}=7+4\sqrt{3}$$

4. If $\sqrt{24-t^2}-\sqrt{8-t^2}=2$

Then $\sqrt{24-t^2}+\sqrt{8-t^2}=?$

$$a+b=\frac{a^2-b^2}{a-b}=\frac{(a-b)(a+b)}{(a-b)}$$

Let $\sqrt{24-t^2}$ be a and $\sqrt{8-t^2}$ be b

$$\sqrt{24-t^2}+\sqrt{8-t^2}=\frac{24-t^2-8+t^2}{2}=\frac{16}{2}=8$$

Practice Problems

1. Simplify

$$\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\left(1+\frac{b^2+c^2-a^2}{2bc}\right)\div\frac{a-b-c}{abc}$$

2. Simplify

$$\left(\frac{1}{t^2+3t+2}+\frac{2t}{t^2+4t+3}+\frac{1}{t^2+5t+6}\right)\cdot\frac{(t-3)^2+12t}{2}$$

3. Simplify

$$\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^3}+\frac{1}{b^3}}\div\frac{a^2b^2}{(a+b)^2-3ab}\left(\frac{ab}{a^2-b^2}\right)$$

4. Simplify

$$\frac{1}{b(abc+a+c)}-\frac{1}{a+\frac{1}{b+\frac{1}{c}}}\div\frac{1}{a+\frac{1}{b}}$$

5. Simplify

$$\left(2-x+4x^2+\frac{5x^2-6x+3}{x-1}\right)\div\left(2x+1+\frac{2x}{x-1}\right)$$

6. Simplify

$$\frac{\left(\frac{a}{b}+\frac{b}{a}+1\right)\left(\frac{1}{a}-\frac{1}{b}\right)^2}{\frac{a^2}{b^2}+\frac{b^2}{a^2}-\left(\frac{a}{b}+\frac{b}{a}\right)}$$

7. Simplify

$$\frac{\sqrt{x-a}}{\sqrt{x+a}+\sqrt{x-a}}+\frac{x-a}{\sqrt{x^2-a^2}-x+a}\div\sqrt{\frac{x^2}{a^2}-1};\quad x>a>0$$

8. Simplify

$$5\sqrt{48\sqrt[3]{\frac{2}{3}}}+\sqrt{32\sqrt[3]{\frac{9}{4}}}-11\sqrt[3]{12\sqrt{8}}$$

9.
Find a sum of cubes of two numbers (a and b), if a + b = 11 and ab = 21.

More Formulas to Know

$$ax^2+bx+c=0 \qquad a\neq 0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$D=b^2-4ac$

If $D>0$ *then the equation has* 2 *solutions.*

If $D=0$ *then the equation has* 1 *solutions.*

If $D<0$ *then the equation has* 0 *solutions or*

$x\in\varnothing$; *x belongs to the empty set.*

Sample Problems

1. $\frac{2}{3-x}+\frac{1}{2}=\frac{6}{x(3-x)}\rightarrow \qquad \frac{2}{3-x}+\frac{1}{2}-\frac{6}{x(3-x)}=0$

$$\frac{2\cdot 2x+x(3-x)-6\cdot 2}{2x(3-x)}=0 \quad\rightarrow\quad \frac{4x+3x-x^2-12}{2x(3-x)}=0$$

$$\frac{7x-x^2-12}{2x(3-x)}=0 \quad\rightarrow\quad 2x(3-x)\neq 0,\ x\neq 0,\ x\neq 3$$

$7x-x^2-12=0$ or $x^2-7x+12=0 \rightarrow x_1=3 \quad x_2=4$

If x = 3 denominator is equal to 0 → **X = 4**.

2. $\sqrt{x-2}=x-4$

$$\left(\sqrt{x-2}\right)^2=(x-4)^2$$

$$x-2=x^2-8x+16$$

$$x^2-9x+18=0 \rightarrow x_1=3 \quad x_2=6$$

Check both cases.

$$if \quad x=3 \rightarrow \sqrt{3-2}=3-4 \rightarrow \sqrt{1}=-1 \rightarrow 1\neq -1$$

$$if \quad x=6 \rightarrow \sqrt{6-2}=6-4 \rightarrow 2=2 \rightarrow \boxed{x=6}$$

3. $\sqrt{1-4x}+2=\sqrt{(2x+1)^2-8x}$

$$\begin{cases}1-4x\geq 0 \;\&\; x\leq \frac{1}{4}\end{cases}$$

$$\sqrt{1-4x}+2=\sqrt{4x^2+4x+1-8x}$$

$$\sqrt{1-4x}+2=\sqrt{4x^2-4x+1}$$

$$\sqrt{1-4x}+2=\sqrt{(2x-1)^2} \rightarrow \sqrt{1-4x}+2=|2x-1|$$

Because $x\leq \frac{1}{4}$, $\quad 2x-1<0$

If $2x-1<0$, then $|2x-1|=1-2x$

$$\sqrt{1-4x}+2=1-2x$$

$$\sqrt{1-4x}=-2x-1$$

$$1-4x=1+4x+4x^2$$

$$0=4x^2+8x$$

$$x_1=0 \quad x_2=-2$$

Check the answers...

$$x=0 \rightarrow \sqrt{1}+2=\sqrt{1} \rightarrow 2\neq 0$$

$$\boxed{x=-2} \rightarrow \sqrt{1+8}+2=\sqrt{(3)^2+16} \rightarrow \sqrt{9}+2=$$

$$\sqrt{9+16} \rightarrow 3+2=\sqrt{25} \rightarrow 3+2=5$$

4. $(x+1)(x^2+2)+(x+2)(x^2+1)=2$

$$x^3+2x+x^2+2+x^3+x+2x^2+2=2$$

$$2x^3+3x^2+3x+2=0$$

$$2(x^3+1)+3x(x+1)=0$$

$$2(x+1)(x^2-x+1)+3x(x+1)=0$$

$$(x+1)(2(x^2-x+1)+3x)=0$$

$$x+1=0 \qquad x=-1$$

$$2x^2-2x+2+3x=0$$

$$2x^2+x+2=0$$

Discriminant $\quad D=b^2-4ac$

$$D=1^2-4\cdot 2\cdot 2=1-16<0$$

$$x\in\varnothing$$

Final answer is $\boxed{x=-1}$

5. $$7\left(x+\frac{1}{x}\right)-2\left(x^2+\frac{1}{x^2}\right)=9$$

$$Let\ z=x+\frac{1}{x}\quad then$$

$$z^2=\left(x+\frac{1}{x}\right)^2 \qquad z^2=x^2+2+\frac{1}{x^2}$$

$$x^2+\frac{1}{x^2}=z^2-2 \qquad 7z-2(z^2-2)=9$$

$$7z-2z^2+4=9 \qquad 2z^2-7z+5=0$$

$$z_1=\frac{5}{2} \qquad z_2=1 \qquad x+\frac{1}{x}=\frac{5}{2}$$

$$2x^2-5x+2=0 \qquad x_1=2 \qquad x_2=\frac{1}{2}$$

$$x+\frac{1}{2}=1 \qquad x^2-x+1=0 \qquad D<0$$

$$x\in\varnothing$$

$$x_1=2 \qquad x_2=\frac{1}{2}$$

6. $$\sqrt{x+1}+\sqrt{4x+13}=\sqrt{3x+12} \qquad x\geq -1$$

$$x\geq -1 \qquad x\geq -\frac{13}{4} \qquad x\geq -4$$

Square both parts of equation

$$x+1+4x+13+2\sqrt{(x+1)(4x+13)}=3x+12$$

$$5x+14+2\sqrt{(x+1)(4x+13)}=3x+12$$

$$2\sqrt{(x+1)(4x+13)}=-2x-2$$

$$\sqrt{(x+1)(4x+13)}=-(x+1)$$

$$-(x+1)\geq 0$$

$$x+1\leq 0$$

$$x\leq -1$$

x should be greater or equal to -1 *and less or equal to* -1. *It is possible only if* $x=-1$.

$$\boxed{x=-1}$$

7. $2x^2+6-2\sqrt{2x^2-3x+2}=3x+3$

$$(2x^2-3x+2)-2\sqrt{2x^2-3x+2}+1=0$$

$$z=\sqrt{2x^2-3x+2} \qquad z\ge 0$$

$$z^2-2z+1=0$$

$$(z-1)^2=0$$

$$z=1$$

$$\sqrt{2x^2-3x+2}=1$$

$$2x^2-3x+2=0$$

$$x_1=\frac{1}{2} \qquad x_2=1$$

8. $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$

$$\sqrt{x-1}=z \qquad z\ge 0, \quad x\ge 1$$

$$x^2=x-1 \qquad x=z^2+1$$

$$\sqrt{x+3-4\sqrt{x-1}}=\sqrt{z^2+1+3-4z}$$

$$=\sqrt{z^2-4z+4} \quad =\sqrt{(z-2)^2} \quad =|z-2|$$

$$\sqrt{x+8-6\sqrt{x-1}}=\sqrt{z^2+1+8-6z}$$

$$=\sqrt{z^2-6z+9} \quad =\sqrt{(z-3)^2} \quad =|z-3|$$

$$|z-2|+|z-3|=1 \qquad \begin{matrix} z-2\ge 0 \\ z\ge 2 \\ z-3\ge 0 \\ z\ge 3 \end{matrix}$$

a. $z<2$

$2-z+3-z=1$

$z=2$ *but* $z<2$ *it means that* $z\in\varnothing$.

b. $2\le z<3$

$z-2+3-z=1$

$1=1$ z *can be any number.*

It means that in this case the answer is $2\le z<3$.

c. $z\ge 3$

$z-2+z-3=1 \qquad z=3$

Final answer for z *is* $2\le z\le 3$

$z=\sqrt{x-1} \qquad 2\le\sqrt{x-1}\le 3$

$$4\le x-1\le 9$$

$$5\le x\le 10$$

9. Solve the system

$$\begin{cases} 2x+y+z=7 \\ x+2y+z=8 \\ x+y+2z=9 \end{cases}$$

Add three equations and divide by 4

$$4x+4y+4z=24$$

$$x+y+z=6$$

$$x=1$$

Use the second and new equation

$$x+2y+z=8$$

$$x+y+z=6$$

$$y=2$$

Use the third and new equation

$$x+y+2z=9$$

$$x+y+z=6$$

$$z=3$$

$$x=1 \quad y=2 \quad z=3$$

10. $$x\sqrt{y}+y\sqrt{x}=6$$

$$x^2y+y^2x=20$$

$$x\sqrt{y}=z \quad y\sqrt{x}=t$$

$$x^2y=z^2 \quad y^2x=t^2$$

$$z+t=6 \quad t=6-z$$

$$z^2+t^2=20 \quad z^2+(6-z)^2=20$$

$$z^2+36-12z+z^2=20$$

$$2z^2-12z+16=0 \qquad a. \quad z_1=4 \quad t_1=2$$

$$z^2-6z+8=0 \qquad b. \quad z_2=2 \quad t_2=4$$

$$a. \quad x\sqrt{y}=4 \quad \sqrt{y}=\frac{4}{x}$$

$$y\sqrt{x}=2 \quad \frac{16}{x^2}\sqrt{x}=2$$

$$16\sqrt{x}=2x^2$$

$$8\sqrt{x}=x^2$$

$$64x=x^4$$

$$64=x^3 \quad x=4 \quad \sqrt{y}=1$$

$$y=1$$

$$\boxed{\begin{array}{l} x=4 \\ y=1 \end{array}}$$

b. $x\sqrt{y}=2$ $\quad \sqrt{y}=\frac{2}{x}$

$y\sqrt{x}=4$ $\quad \frac{4}{x^2}\sqrt{x}=4$

$$4\sqrt{x}=4x^2$$

$$\sqrt{x}=x^2$$

$$x=x^4$$

$1=x^3$ $\quad x-1$ $\quad \sqrt{y}=\frac{2}{1}$

$\sqrt{y}=1$ $\quad y=4$

Practice Problems

1. $\frac{x-3}{x-1}+\frac{x+3}{x+1}=\frac{x+6}{x+2}+\frac{x-6}{x-2}$

2. Solve for x $\quad \frac{ax^2}{x-1}=(a+1)^2$

3. Solve for x $\quad \frac{x+n}{m+n}-\frac{m-n}{x-n}=\frac{x+p}{m+p}-\frac{m-p}{x-p}$

4. Solve for x $\quad \frac{x+2}{x+1}+\frac{x+6}{x+3}+\frac{x+10}{x+5}=6$

5. Solve for x $\quad \left(\frac{x^2+6}{x^2-4}\right)^2=\left(\frac{5x}{4-x^2}\right)^2$

6. Solve for x $\quad (x-a)^3-(x-b)^3=b^3-a^3$

7. Solve for x $\quad 3\left(x+\frac{1}{x^2}\right)-7\left(1+\frac{1}{x}\right)=0$

8. Solve for x $\quad \frac{x^2}{a^3}+\frac{b^3}{x^2}=\frac{b}{a}+\frac{b^2}{a^2}$

9. Solve for x $\quad \frac{(x-a)^2+x(x-a)+x^2}{(x-a)^2-x(x-a)+x^2}=\frac{19}{7}$

10. Solve for x $\quad \sqrt{15-x}+\sqrt{3-x}=6$

11. Solve for x $$\sqrt{x^2+9}-\sqrt{x^2-7}=2$$

12. Solve for x $$\frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}-\sqrt[3]{x}}=3$$

13. Solve the system $$\begin{cases} x-y=1 \\ x^3-y^3=7 \end{cases}$$

14. Solve the system $$\begin{cases} \dfrac{x^2+y^2}{x+y}=\dfrac{10}{3} \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{3}{4} \end{cases}$$

15. Solve the system $$\begin{cases} x^4-y^4=15 \\ x^3y-xy^3=6 \end{cases}$$

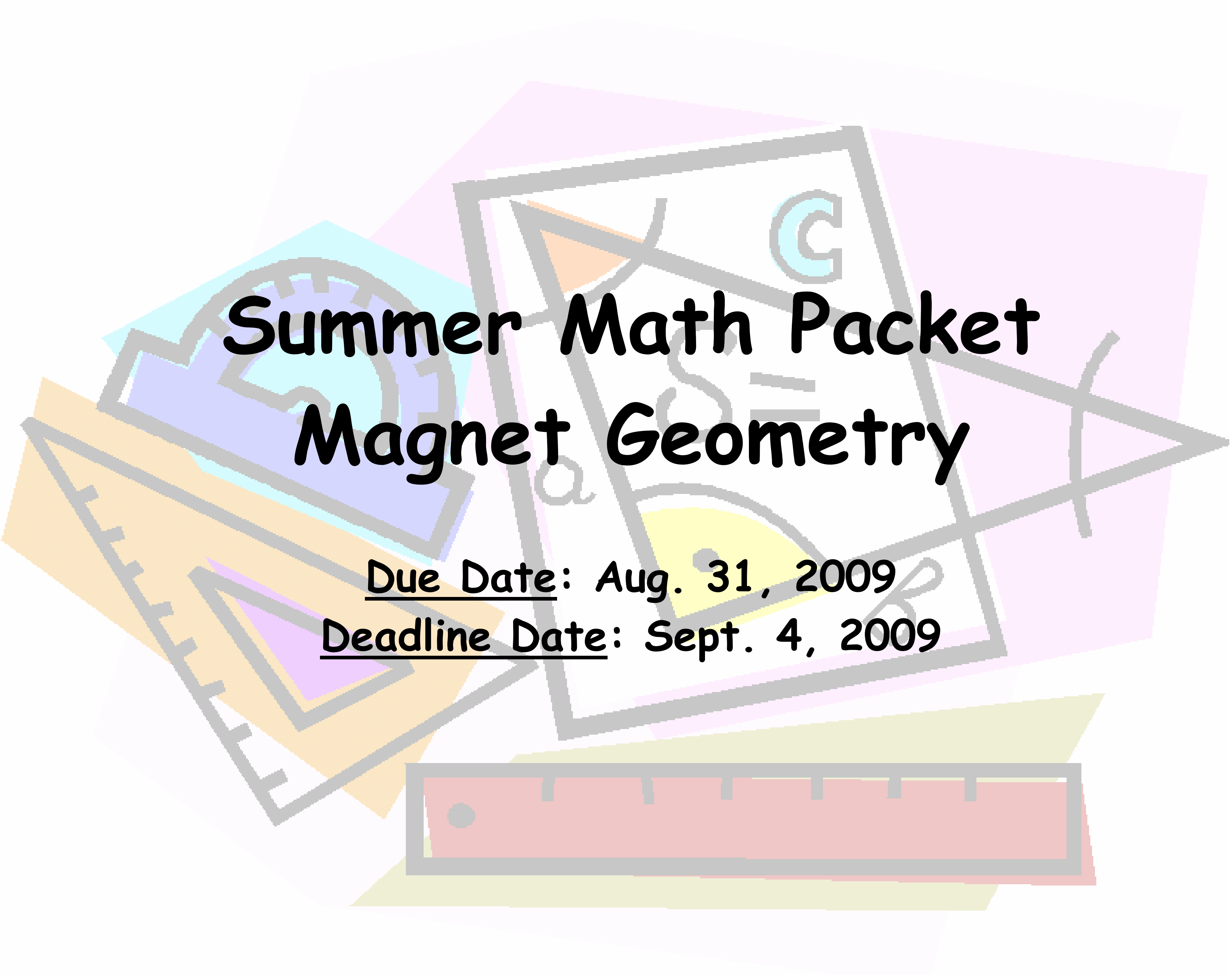
16. Solve for u and v $$\begin{cases} u^3+v^3+1=m \\ u^3v^3=-m \end{cases}$$

17. Solve the system $$\begin{cases} x+2y+3z=3 \\ 3x+y+2z=7 \\ 2x+3y+z=2 \end{cases}$$

18. Solve for x and y $$\begin{cases} \sqrt{\dfrac{x+a}{y}}+\sqrt{\dfrac{y}{x+a}}=2 \\ x+y=xy+a \end{cases}$$

19. Solve for x and y $$\begin{cases} \sqrt[3]{x}+\sqrt[3]{y}=3 \\ \sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}=3 \end{cases}$$

20. Solve for x and y $$\frac{(x-1)(x-2)(x-3)(x-4)}{(x+1)(x+2)(x+3)(x+4)}=1$$



Summer Math Packet
Magnet Geometry

<u>Due Date</u>: Aug. 31, 2009

<u>Deadline Date</u>: Sept. 4, 2009

Dear Students,

In order to help prepare for Magnet Geometry it is essential to keep up on Algebra laws and formulas learned in Magnet Algebra. In order to not take away time from the Magnet Geometry curriculum, these concepts should be practiced over the summer and will be review during the first two weeks of school.

This packet will be kept by the students and referred to throughout the school year.

Mr. Easley
Math Content Specialist
Roberto Clemente Middle School