

Discontinuities Worksheet

$$1. \quad f(x) = \begin{cases} 2x, & \text{if } x < 3 \\ 5x - 9, & \text{if } x > 3 \end{cases}$$

Complete the following:

- $\lim_{x \rightarrow 3^-} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow 3^+} f(x) = \underline{\hspace{2cm}}$
- Is there is discontinuity at $x = 3$? _____
- If there is a discontinuity at $x = 3$, what type is it? _____
- If the discontinuity at $x = 3$ is removable, give the x - and y -coordinates of the point that will remove the discontinuity. _____

$$2. \quad f(x) = \begin{cases} x^2, & \text{if } x < 1 \\ 4x - 3, & \text{if } x > 1 \end{cases}$$

Complete the following:

- $\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}}$
- Is there is discontinuity at $x = 1$? _____
- If there is a discontinuity at $x = 1$, what type is it? _____
- If the discontinuity at $x = 1$ is removable, give the x - and y -coordinates of the point that will remove the discontinuity. _____

$$3. \quad f(x) = \begin{cases} \frac{1}{x-2}, & \text{if } x < 4 \\ \frac{x}{8}, & \text{if } x > 4 \end{cases}$$

Complete the following:

- $\lim_{x \rightarrow 4^-} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow 4^+} f(x) = \underline{\hspace{2cm}}$
- Is there is discontinuity at $x = 4$? _____
- If there is a discontinuity at $x = 4$, what type is it? _____
- If the discontinuity at $x = 4$ is removable, give the x - and y -coordinates of the point that will remove the discontinuity. _____

Power Function Problems

1. The weight (in pounds) of a certain type of fish varies directly as the square root of the length of the fish cubed.
 - a. Express this variation as a power function. $W(L) = kL^{\frac{3}{2}}$
 - b. If the length of one of these fish is 25 inches, the fish weighs 12.5 pounds. Find the constant of variation. $k = 0.1$
 - c. How much will a fish that is 36 inches long weigh? **21.6 pounds**
 - d. A fish weighs 6.4 pounds. How long, in inches is the length of the fish?
16 inches

2. The rate (in gallons per minute) at which water flows from a hose varies directly as the square root of the water pressure (in pounds per square inch) at the faucet.
 - a. Express this variation as a power function. $R(w) = kw^{\frac{1}{2}}$
 - b. If water flows at the rate of 2 gallons per minute when the pressure at the faucet is 9 pounds per square inch, find the constant of variation. $k = \frac{2}{3}$
 - c. If the pressure at the faucet is 25 pounds per square inch, at what rate is water flowing from the hose? $\frac{10}{3} \text{ gal / min}$
 - d. A gardener needs a flow of 6 gallons per minute. What pressure is needed at the faucet? **81 lbs / in²**

3. A shoe manufacturer knows that the price to make a pair of shoes varies inversely as the square root of the number of pairs made.
 - a. Express this variation as a power function. $P(n) = kn^{-\frac{1}{2}}$
 - b. Suppose that when 100 pairs of shoes are produced, that each pair will cost \$60 to make. Find the constant of variation. $k = 600$
 - c. What will the price per pair be if 400 pairs are produced? **\$30 per pair**
 - d. In order for the price per pair is to be \$20, how many pairs of shoes should be produced? **900 pairs**

Graphing Piece-wise Functions

Sketch the graph of each piece-wise function. In each case, give any points of discontinuity.

$$1. \quad f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

$$2. \quad f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ e^x & \text{if } x > 0 \end{cases}$$

$$3. \quad f(x) = \begin{cases} x^3 & \text{if } x < 1 \\ \ln x & \text{if } x \geq 1 \end{cases}$$

$$4. \quad f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$$

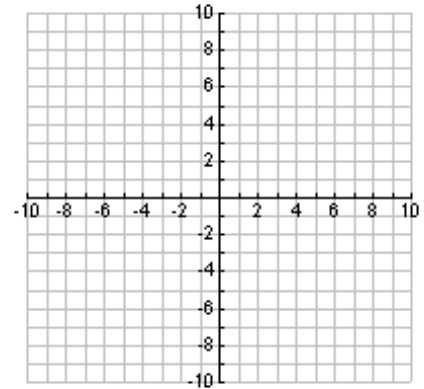
$$5. \quad f(x) = \begin{cases} x & \text{if } x < -2 \\ 3 & \text{if } -2 \leq x \leq 4 \\ \text{int}(x) & \text{if } x > 4 \end{cases}$$

Name _____

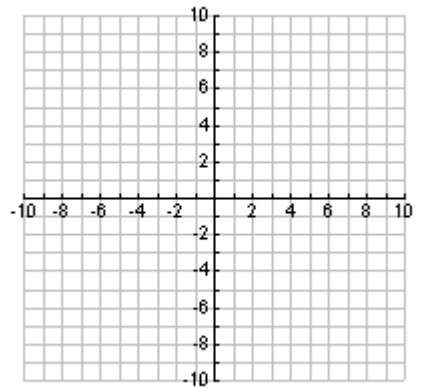
Power Functions

Complete the following for each power function below.

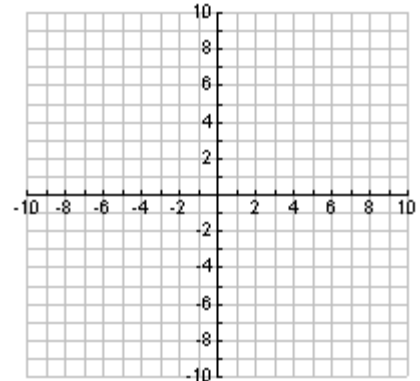
1. $f(x) = x^4$
 Domain: _____ Range: _____
 Continuous on what interval(s)? _____
 Increasing on what interval(s)? _____
 Decreasing on what interval(s)? _____
 Even, odd, or neither? _____
 Symmetries? _____
 Local extrema? _____
 End behavior? _____
 Asymptotes? _____
 Sketch the graph.



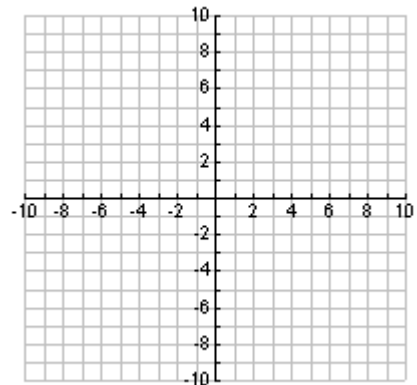
2. $f(x) = x^{-2}$
 Domain: _____ Range: _____
 Continuous on what interval(s)? _____
 Increasing on what interval(s)? _____
 Decreasing on what interval(s)? _____
 Even, odd, or neither? _____
 Symmetries? _____
 Local extrema? _____
 End behavior? _____
 Asymptotes? _____
 Sketch the graph.



3. $f(x) = 2\sqrt[3]{x}$
 Domain: _____ Range: _____
 Continuous on what interval(s)? _____
 Increasing on what interval(s)? _____
 Decreasing on what interval(s)? _____
 Even, odd, or neither? _____
 Symmetries? _____
 Local extrema? _____
 End behavior? _____
 Asymptotes? _____
 Sketch the graph.



4. $f(x) = \frac{1}{\sqrt{x}}$
 Domain: _____ Range: _____
 Continuous on what interval(s)? _____
 Increasing on what interval(s)? _____
 Decreasing on what interval(s)? _____
 Even, odd, or neither? _____
 Symmetries? _____
 Local extrema? _____
 End behavior? _____
 Asymptotes? _____
 Sketch the graph.



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