The semester B examinations for Precalculus/Honors Precalculus will consist of two parts. Part 1 will be selected response on which a calculator will not be allowed. Part 2 will be short answer on which a calculator will be allowed.

The following symbols apply to this review:

Indicates that a problem like this may be on the no-calculator, selected response section **or** the calculator section of the exam.

H Indicates an Honors Precalculus problem. These are found at the end of the review.

Compound Interest Formulas:

Continuous compounding: $A(t) = Pe^{rt}$ compounded k times per year: $A(t) = P\left(1+\frac{r}{k}\right)^{k\,t}$

Newton's Law of Cooling: $T(t) = T_m + (T_0 - T_m)e^{-kt}$ **Honors Only**

Parametric equations for projectile motion:
$$x = (v_0 \cos\theta)t$$
$$y = -16t^2 + (v_0 \sin\theta)t + h_0$$

Distance from the point (x_1, y_1) to the line $Ax + By + C = 0$: $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

If $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$, then $\vec{u} \bullet \vec{v} = u_1 v_1 + u_2 v_2$

If $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$, then $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ **Honors only**

If $\vec{u} = \langle u_1, u_2 \rangle$, then

$|\vec{u}| = \sqrt{u_1^{\,2} + u_2^{\,2}}$ and the direction of $\vec{u} = \tan^{-1}\left|\frac{u_2}{u_1}\right|$, placed in the appropriate quadrant.

The angle between $\vec{u}$ and $\vec{v} = Cos^{-1}\left(\frac{\vec{u}\bullet\vec{v}}{|\vec{u}||\vec{v}|}\right)$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Sum of an infinite geometric series:

$S = \frac{a_1}{1-r}$, if $|r| < 1$

Binomial Theorem:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

where $\binom{n}{r} = {}_nCr = \frac{n!}{r!(n-r)!}$

OR

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \ldots + y^n = \sum_{r=0}^{n}\frac{n!}{r!(n-r)!}x^{n-r}y^r$$

Decimal approximations should be given to 3 places after the decimal point.

For problems 1 through 3, sketch the graph of the function. Write the equations of any asymptotes, and the coordinates of any intercepts.

1. $f(x)=\dfrac{3}{x^2-3x-10}$

2. $f(x)=\dfrac{x-4}{x^2+2x-8}$

3. $f(x)=2+\dfrac{5}{x-3}$

4. Which of the following is the equation of the horizontal asymptote of the graph of the function $f(x)=\dfrac{4x^2-2}{x^2-5}$

A $x=\dfrac{2}{5}$ **B** $x=5$ **C** $y=\dfrac{1}{2}$ **D** $y=4$

In problems 5 through 7, solve the equations. Check for any extraneous roots.

5. $\dfrac{3x}{x-1}=\dfrac{12}{x^2-1}+2$

6. $1=\dfrac{16}{y}-\dfrac{39}{y^2}$

7. $\dfrac{x}{x-2}+\dfrac{3x}{x-4}=\dfrac{32-2x}{x^2-6x+8}$

8. Which of the following statements is false?

A $f(x) = \ln x$ has a domain of all positive real numbers

B $f(x) = \ln x$ increases without bound as x increases without bound.

C The graph of $f(x) = e^{-x}$ is the graph of $g(x) = e^{x}$ reflected about the y-axis.

D $f(x) = e^{-x}$ has a range of all real numbers.

9. Which of the following is equivalent to $\log_5\left(\frac{5}{x^3}\right)$?

A $5 - 3\log_5 x$

B $1 - 3\log_5 x$

C $-3\log_5 x$

D $2\log_5 x$

10. Which of the following is equivalent to $\log\left(\frac{y^2}{\sqrt{x}}\right)$?

A $2\log y - 2\log x$

B $2\log y - \frac{1}{2}\log x$

C $\log(2y) - \log\left(\frac{1}{2}x\right)$

D $\frac{\log\left(y^2\right)}{\log\left(\sqrt{x}\right)}$

11. Which of the following is equivalent to $3\ln x + \ln z - \ln y$?

A $3\ln\left(\frac{xz}{y}\right)$

B $\ln\left(\frac{xz}{y}\right)^3$

C $\ln\left(\frac{x^3 z}{y}\right)$

D $\ln\left(\frac{(xz)^3}{y}\right)$

12. Which of the following is equivalent to $\frac{\log 20}{\log 2}$?

A $\log 10$
B $\log 18$
C $\log_2 20$
D $\log_{20} 2$

13. Write an expression equivalent to the following.

a. $e^{\ln(5x)}$ b. $e^{\ln x + 2\ln y}$

For problems 14 and 15, evaluate. Write your answer correct to three decimal places.

14. $\log_7 6904$

15. $\log_4 0.0123$

16. The graph of $g(x)$ is the graph of $f(x) = 3^x$ reflected about the x-axis and translated 5 units to the left. What is the function rule for $g(x)$?

17. The graph of $g(x)$ is the graph of $f(x) = 9^x$ reflected about the y-axis and stretched horizontally by a factor of 7. What is the function rule for $g(x)$?

For problems 18 through 26, solve. If necessary, write your answer correct to three places after the decimal point.

18. $3e^{2x} - 7 = 50$

19. $5^{2x+1} = 800$

20. $2.8^x = 345$

21. $5 \cdot 2^{3x-1} + 20 = 520$

22. $\ln(x+4)=2$

23. $\log_7(3x-1)=2$

24. $\log_5 x+\log_5(x-4)=1$

25. $\log_2(x+5)-\log_2(x-1)=2$

26. $2-\log x=\log 20$

27. A colony of insects has an initial population of 600. The number of insects triples every 4 weeks.
    a. Write a function for the number of insects after *t* weeks.
    b. What will the number of insects be after 7 weeks?
    c. After how many weeks will the number of insects be 12000?

28. A population that grows continuously has an initial value of 400. After $t = 8$ years, the population is 1000. What is the continuous growth rate of this population?

29. Jack puts $5000 in the bank at 6% interest compounded monthly. How many years will it take the amount of money in the account to become $6000 ?

30. The population of a certain city t years after 2000 is given by the function $P(t)=1300e^{.04t}$.
    a. What was the initial population in 2000?
    b. What is the rate of growth of the population?
    c. What will the population be in 2010?
    d. At what time *t* will the population be 2500?

31. The percentage of adult height obtained by a boy *x* years old is modeled by the function $f(x)=29+49\ \log(x+1)$.

    a. Approximately what percentage of his adult height is a 9 year-old?

    b. At what age will a boy reach 90% of his adult height?

32. A house has an initial value of $100,000. Its value increases by 8% a year. Write a function that represents the value of the house after *t* years.

For problems 33 and 34, eliminate the parameter to find the relationship between x and y.

33. $x = 3t$
$y = 6t - 7$

34. $x = 2t + 4$
$y = 4t + 5$

35. A golfer hits the ball from the ground at a 20^o angle with an initial velocity of 132 feet per second.
   a. Give the motion of the ball as a set of parametric equations representing the horizontal movement of the ball after t seconds.

   b. Give the location of the ball after 2 seconds.

36. A punter kicks a football from at height of 2 feet with an initial velocity of 82 feet per second at an angle of 63 degrees with the horizontal.
   a. Write parametric equations for the horizontal and vertical distances that the ball is from the punter after t seconds.

   b. What is the position of the ball at $t = 1.5$ seconds.

   c. A player 6 feet tall is standing 50 yards (150 feet) from the punter directly in the path of the kick. Will the ball go over his head? Why or why not?

   d. Suppose that the kick returner lets the ball hit the ground. When will the ball hit the ground, and how far from the punter will the ball hit the ground?

37. Write the vector and parametric equations of the line that passes through the points $(1,4)$ and $(3,9)$.

38. Given the points $A(2,-1)$ and $B(0,4)$. What is the vector $\overrightarrow{AB}$ in component form?

   A $\langle 2,-5 \rangle$

   B $\langle -2,5 \rangle$

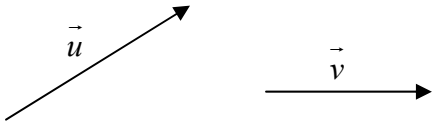
   C $\langle 2,3 \rangle$

   D $\langle -2,-3 \rangle$

39. What is the magnitude and direction of the vector $\langle -6,8 \rangle$?

40. What is the distance from the point $(2,-3)$ and the line $3x - 4y + 12 = 0$?

Look at the vectors $\vec{u}$ and $\vec{v}$ below



41. Represent the graphically the vector $\vec{u}+\vec{v}$ using both the head-to-tail method and the parallelogram method of addition.

42. Represent the vector $\vec{u}-\vec{v}$ graphically.

In problems 43 through 48, use the vectors $\vec{a}=2\vec{i}-3\vec{j}$, $\vec{b}=6\vec{i}+5\vec{j}$, and $\vec{c}=r\vec{i}-9\vec{j}$

43. $\vec{a}-\vec{b}=$

44. $3\vec{a}+5\vec{b}=$

45. $\vec{a}\bullet\vec{b}=$

46. The angle between $\vec{a}$ and $\vec{b}=$

47. Find the value of *r* such that the vectors $\vec{a}$ and $\vec{c}$ are parallel.

48. Find the value of *r* such that the vectors $\vec{b}$ and $\vec{c}$ are perpendicular.

49. A plane is headed due west at 400 mph. A wind is blowing towards the northwest at 60 mph.
    a. What is the ground speed of the plane?
    b. In what direction is the plane traveling?

In problems 50 and 51, represent the series using summation notation.

50. $8+16+32+64+...$

51. $11+15+19+23+27+31+35$

In problems 52 through 54, determine the sum, if any, of the series.

52. $9+3+1+\frac{1}{3}+...$

53. $\sum_{n=1}^{\infty} 3\cdot 2^n$

54. $\sum_{n=1}^{\infty} 8\left(\frac{3}{4}\right)^n$

In problems 55 and 56, evaluate.

55. $\sum_{n=1}^{4} (3n+1)$

56. $\sum_{n=5}^{8} 4n$

57. Expand. $(x-2)^4$

58. What is the coefficient of the x^3 term in the expansion of $(x+2)^5$?

H **These review problems are for <u>HONORS Precalculus only</u>.**

For problems 59 and 60, sketch the graph of the function. Write the equations of any asymptotes, the domain and range, and the x- and y-coordinates of any removable discontinuities, and the coordinates of any intercepts.

H  59. $f(x)=\dfrac{x^2-7x+6}{x^2-36}$

H  60. $f(x)=\dfrac{x^2-8x+7}{x-6}$

For problems 61 and 62, use partial fraction decomposition to write an expression equivalent to the given expression.

H 61. $\dfrac{13x-31}{x^2-5x+6}$

H 62. $\dfrac{-9}{x^2-x-2}$

H 63. $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2+8x-20}=$

For problems 64 and 65, solve the inequalities.

H  64. $\dfrac{x}{x-2}<0$

H  65. $\dfrac{x-3}{x^2+7x+10}\geq 0$

H  66. For the function $g(x)=-\ln(x+2)-5$,

a. What are the domain and range?
b. What are the equations of any asymptotes?
c. Describe the transformations to the graph of $f(x)=\ln x$ that results in the graph of $g(x)$.

H 67. The logistic function $f(t)=\dfrac{30000}{1+19e^{-1.5t}}$ describes the number of people $f(t)$ that have become ill with the flu t weeks after its initial outbreak.

a. How many people became ill when the outbreak first began?
b. After approximately how many weeks were there 10000 people ill?
c. What is the maximum number of people that can become ill?

H 68. A cup of hot chocolate with a temperature of 90° C is placed in a room that is 20°C. After 1 minute, the hot chocolate has cooled to a temperature of 82°C. How long will it take for the hot chocolate to cool to a temperature of 50°C?

H  69. Eliminate the parameter to find the relationship between x and y: $\begin{aligned} x &= 5\cos t \\ y &= 8\sin t \end{aligned}$

H  70. A vector in space has an initial point of $A(-2,-3,4)$ and $B(-1,4,-5)$.

a. Write $\overrightarrow{\boldsymbol{AB}}$ in component form.
b. What is the magnitude of $\overrightarrow{\boldsymbol{AB}}$?

H 71. Let vectors $\vec{\boldsymbol{u}}=\langle -4,1,5\rangle$ and $\vec{\boldsymbol{v}}=\langle -3,2,-1\rangle$

a. Determine the angle between the vectors.
b. Determine the cross product of the vectors.
c. Describe the geometric meaning of the cross product that you found in part (b) above.
d. Write the vector equation of the line that is parallel to $\vec{\boldsymbol{u}}$ and passes through the point $P(2,4,-3)$.

H  72. Sketch the graph of $x+2y+3z=6$ in a 3-dimensional plane .

In problems 73 and 74, write each rectangular equation in polar form.

H  73. $x+y=9$

H  74. $x=7$

In problems 75 and 76, write each polar equation in rectangular form.

H 

75. $r\sin\theta = 6$

H 

76. $r = \dfrac{3}{2\cos\theta + 5\sin\theta}$

H 

77. A plane has the equation $5x + 2y - 4z = 20$ .What are the *x*-, *y*-, and *z*-intercepts of the plane?

In problems 78 and 79, graph each equation.

H 78. $r = 1 - 2\cos\theta$

H 79. $r = 4\sin 2\theta$

In problems 80 and 81, determine the coordinates (r,θ) that are <u>solutions</u> for each system, where $0 \le \theta < 360^o$.

H 80.
$$r = 2\cos\theta$$
$$r = 1 - 3\cos^2\theta$$

H 81.
$$r = \sin\theta$$
$$r = \sin(2\theta)$$

In problems 82 and 83, write the equivalent rectangular coordinates for each point in polar form.

H 

82. $\left(8, \dfrac{\pi}{3}\right)$

H 

83. $\left(-4, -\dfrac{7\pi}{4}\right)$

In problems 84 and 85, write the equivalent polar form for each point in rectangular form.

H 

84. $\left(4, -4\sqrt{3}\right)$

H 

85. $(-1, -1)$

H 86. Which of the following represents the same point as $\left(4,-\frac{\pi}{3}\right)$?

A $\left(4,\frac{\pi}{3}\right)$

B $\left(-4,\frac{2\pi}{3}\right)$

C $\left(4,\frac{2\pi}{3}\right)$

D $\left(-4,\frac{5\pi}{3}\right)$

In problems 87 and 88, write the complex number in polar form.

H 87. $-6\sqrt{3}-6i$

H 88. $-5+4i$

In problems 89 and 90, write the complex number in rectangular form.

H 89. $6\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

H 90. $-2\left(\cos 7\pi+i\sin 7\pi\right)$

In problems 91 and 92, simplify. Write your answers in polar form.

H 91. $\dfrac{\left[3\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]}{6\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}$

H 92. $\left[3\left(\cos 55^{o}+i\sin 55^{o}\right)\right]^{4}$

H 93. Find the four fourth roots of $8+8i\sqrt{3}$. Give your answers in polar form.

H 94. What is the sum of the series $1+x+x^{2}+x^{3}+x^{4}+\ldots$?

H 

95. What is the coefficient of the $x^9 y$ term in the expansion of $\left(x^3 + 5y\right)^4$?

H 

96. Expand. $\sum_{n=0}^{5} (-1)^n \frac{n!}{3^{n+2}}$