

The semester A examination for Algebra 2 will consist of two parts. Part 1 will be selected response. Part 2 will be short answer.

- Students may use a calculator.
- If a calculator is used to find points on a graph, the appropriate calculator function (i.e. zero, intersect, minimum or maximum) should be used. The trace function should not be used.
- Decimal approximations must be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of any function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

The formulas below will be provided in the examination booklet.

Quadratic Formula: If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Factoring:  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$        $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

For questions 1 through 8, use the following functions:

$$f(x) = x - 3$$

$$g(x) = 2x - 8$$

$$h(x) = x^2 - 2$$

Evaluate.

1.  $f(g(3))$

2.  $h(f(-7))$

Perform the following operations.

3.  $f(x) + g(x)$

4.  $f(x) - g(x)$

5.  $f(x) \cdot g(x)$

6.  $\frac{f(x)}{g(x)}$

7.  $g(h(x))$

8.  $h(f(x))$

For questions 9 through 12, graph each function in a standard window. State whether the function is one to one.

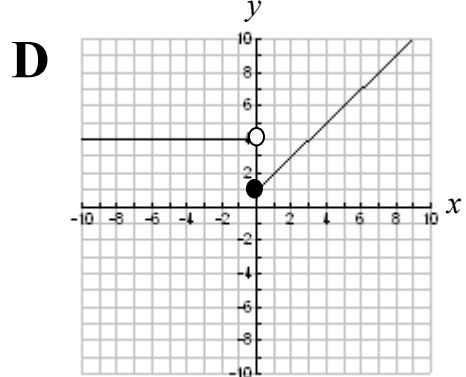
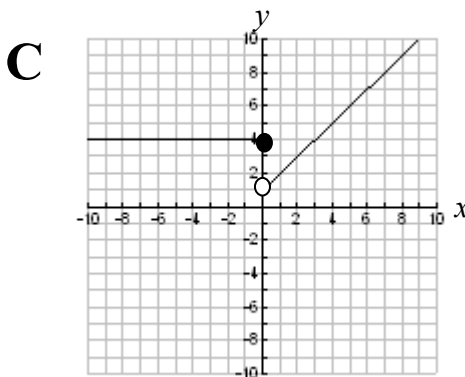
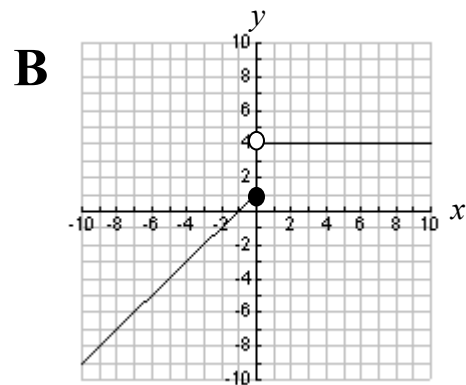
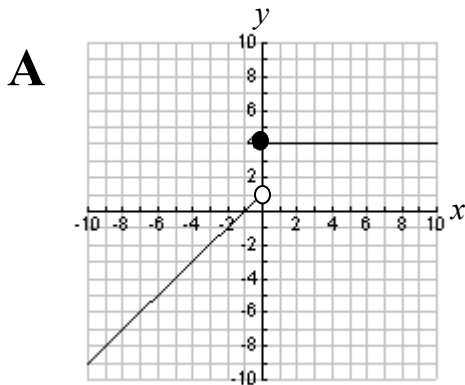
9.  $f(x) = 2x - 5$

10.  $f(x) = -x^2$

11.  $f(x) = |x|$

12.  $f(x) = x^3 - 9x$

13. Which of the following is the graph of  $f(x) = \begin{cases} x+1, & \text{if } x < 0 \\ 4, & \text{if } x \geq 0 \end{cases}$  ?

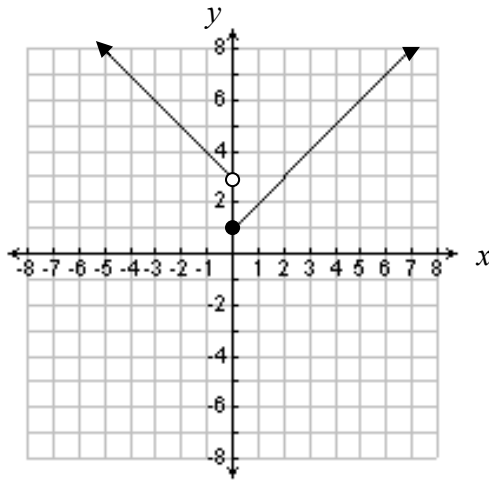


14. Let  $g(x) = 3|x+2| - 4$
- Sketch the graph of  $g(x)$
  - What is the domain of  $g(x)$ ?
  - What is the range of  $g(x)$ ?
  - What is the vertex of the graph of  $g(x)$ ?
  - What is the axis of symmetry of the graph of  $g(x)$ ?
  - What is the minimum value of  $g(x)$ ?
  - If  $f(x) = |x|$ , describe the transformations of  $f(x)$  that were used to create  $g(x)$ .
15. When ordering items from a catalog, the buyer has the option of having items gift-wrapped. The shipping charge for every order is \$12. If less than 50 items are wrapped, the charge is \$5.00 per item wrapped. If 50 or more items are wrapped, the charge is \$3.00 per item wrapped.
- Write a piece-wise function for the total charge of gift-wrapping  $x$  items, including the shipping charge.

$$f(x) = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

- If 35 items are gift wrapped, what is the total charge including shipping?

16. Write the function represented by the graph below.



$$f(x) = \left\{ \right.$$

17. Let  $f(x) = x^2$ . Describe the transformations of  $f(x)$  that will produce the graphs of the following functions.
- $g(x) = 4x^2 + 1$
  - $h(x) = \frac{1}{2}(x+5)^2 + 9$
18. Verify algebraically that  $f(x) = 7x - 6$  and  $g(x) = \frac{x+6}{7}$  are inverse functions.
19. If  $f(x) = \frac{1}{3}x + 5$ , Which of the following represents the inverse function,  $f^{-1}(x)$ ?

**A**  $f^{-1}(x) = -\frac{1}{3}x - 5$

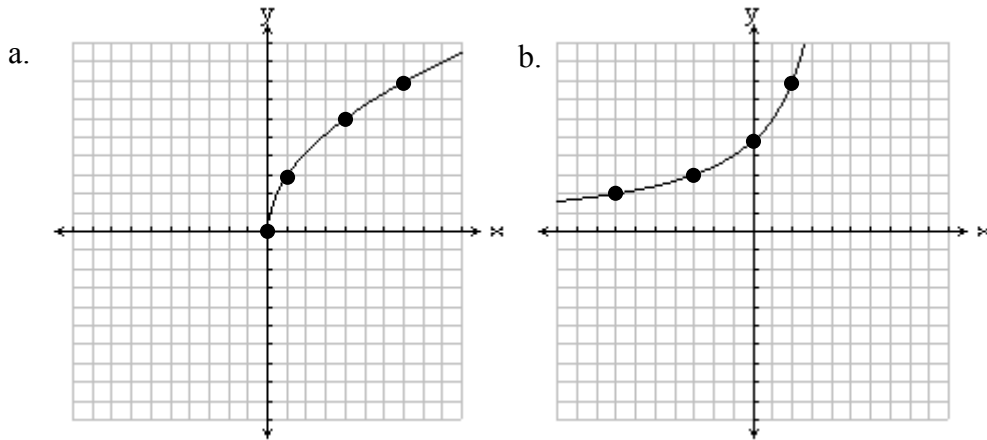
**B**  $f^{-1}(x) = 3x - \frac{1}{5}$

**C**  $f^{-1}(x) = 3x - 5$

**D**  $f^{-1}(x) = 3x - 15$

20. If  $g(x) = 9x - 10$ , determine the inverse function,  $g^{-1}(x)$ .

21. For the graph of each function below, sketch the inverse function.



For questions 22 through 25, use the following matrices:

$$A = \begin{bmatrix} 4 & 2 \\ 5 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 1 & 9 \\ 8 & 5 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 4 & x \\ 10 & 15 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 5 & 6 & 3 & 1 \\ 9 & 7 & 3 & 4 & 0 \\ 4 & 2 & 3 & 1 & 8 \end{bmatrix}$$

22. What are the dimensions of the product matrix  $BD$ ?

- A  $2 \times 5$
- B  $5 \times 2$
- C  $5 \times 3$
- D  $3 \times 5$

23. Write an expression for the value of the determinant of matrix  $C$ .

24. For what value of  $x$  will matrix  $C$  not have an inverse matrix?

25. If  $A \cdot B = \begin{bmatrix} 44 & y & 42 \\ 11 & -10 & 36 \end{bmatrix}$ , what is the value of  $y$ ?

26. If  $f(x) = x^2$ , on what interval of  $x$ -values is  $f(x)$  decreasing?
27. What is the right- and left-end behavior of the function  $f(x) = x^3 - 4x - 8$ ?
- As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_                      As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_
28. What are the right- and left-end behaviors of the function  $f(x) = -x^4 - 4x^3 - 4$ ?
- As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_                      As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_
29. Let  $f(x) = x^5 - 3x^2 - 4$ .
- a. How many zeros does  $f(x)$  have?
- A**    0
- B**    1
- C**    4
- D**    5
- b. Based on its graph, how many real number zeros does  $f(x)$  have?
- A**    0
- B**    1
- C**    4
- D**    5

30. Let  $f(x) = x^4 - x - 4$ .

- How many zeros does  $f(x)$  have?
- Based on its graph, how many real number zeros does  $f(x)$  have?

For each set of data in questions 31 and 32 below,

- Determine the degree of the polynomial that models the data.
- Use the regression feature of your calculator to write a function that models the data.

31.

$x$	1	2	3	4	5
$f(x)$	5	14	37	80	149

32.

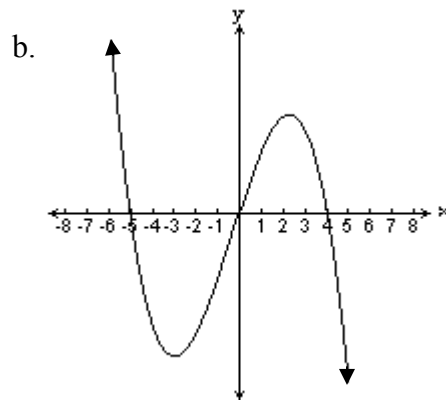
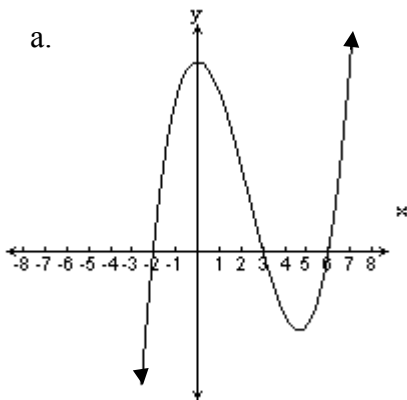
$x$	0	1	2	3	4
$f(x)$	3	6	11	18	27

Solve.

33.  $x^2 - 3x + 11 = 0$

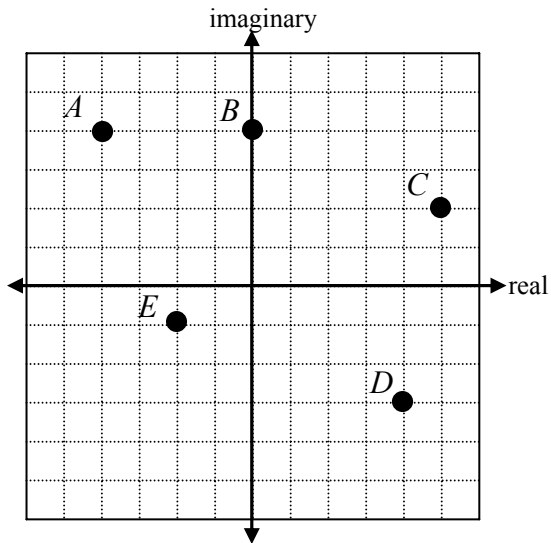
34.  $3x^2 + x + 3 = 0$

35. Write an equation in factored form for the graphs shown below.



36. Given  $f(x) = x^3 + 3x^2 - x - 3$
- Find the zeroes of the function and sketch a graph.
  - Complete: As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_
  - Complete: As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_
37. Let  $f(x) = 4x^5 - 7x^3 + 5x^2 + 15$ . Is each number below a possible rational root (zero) according to the rational root theorem? Write yes or no for each number.
- 5
  - $-\frac{1}{4}$
  - $\frac{3}{2}$
  - 4
  - $\frac{1}{3}$
38. Using the rational root (zero) theorem, list the possible rational zeros of  $f(x) = 5x^4 - 7x^2 + 4$ .
39. Is each expression below a factor of  $x^3 + 6x^2 - x - 30$ ? Write yes or no for each expression.
- $x - 2$
  - $x + 6$
  - $x - 5$
  - $x + 2$
  - $x + 3$
  - $x + 5$
40. Fill in the blank with the number that completes the square.
- $x^2 - 6x +$  \_\_\_\_\_
  - $x^2 + 10x +$  \_\_\_\_\_

41. Write the complex number represented by points A – E on the graph below.



42. Identify each of the following as real, pure imaginary, and/or complex.

- $\sqrt{3}$
- $\sqrt{-9}$
- $5 + 2i$

For questions 42 through 46, perform the indicated operations.

43.  $(3 + 2i) + (5 - 7i)$

44.  $(5 - 6i)(1 - i)$

45.  $(8 - 2i)(2 + i)$

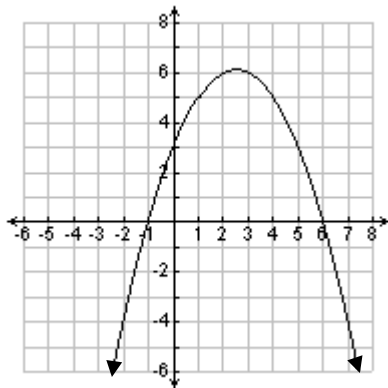
46.  $(2 - 7i)^2$

47.  $\frac{7i}{2 - i}$

48.  $\frac{2 + 7i}{1 + 6i}$

49. The solutions of  $x^3 - x^2 - 50x - 48 = 0$  are  $x = -6, -1,$  and  $8$ .  
Write the polynomial  $x^3 - x^2 - 50x - 48$  in factored form.

50. The solutions of  $x^3 - 6x^2 - 19x + 84 = 0$  are  $x = -4, 3,$  and  $7$ .  
Write the polynomial  $x^3 - 6x^2 - 19x + 84$  in factored form.
51. Barry's Burgers offers three different types of burgers at three different prices. The types are the Hamburger, the Cheeseburger, and the Barry Burger. One time, 3 Hamburgers, 5 Cheeseburgers, and 6 BarryBurgers cost \$25.24. Another time, 2 Hamburgers, 7 Cheeseburgers and 5 BarryBurgers cost \$25.68. The last time, 4 Hamburgers, 4 Cheeseburgers, and 7 BarryBurgers cost \$26.59.
- Write a system of equations that represents the situation. Be sure to define the variables.
  - Represent the system in matrix form.
  - Determine the cost of each type of burger.
52. Factor.
- $x^3 - 125$
  - $x^3 + 64$
53. Find all zeros of each function.
- $f(x) = x^3 - 27$
  - $g(x) = x^3 + 1000$
54. Look at the graph of  $f(x) = -x^2 + 5x + 6$  below.



Use the graph to solve the inequality  $-x^2 + 5x + 6 \leq 0$ .

For questions 55, grid in and bubble your answers in the grids below each question.

55. An arrow is shot from a height of 32 feet with an initial velocity of 56 ft/sec. The equation of the height  $h(t)$  (in feet) of the arrow at time  $t$  (in seconds) is  $h(t) = -16t^2 + 56t + 32$ , where  $0 \leq t \leq 4$ .

- a. When will the arrow hit the ground?
- b. How high will the arrow be at  $t = 1.5$  seconds
- c. What is the maximum height the arrow will reach?

**55a**

	7	7	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

**55b**

	7	7	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

**55c**

	7	7	
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

56. Complete the chart. Round values to three decimal places.

Function	Values of any local maximums	Values of any local minimums	Interval(s) where the function is increasing	Interval(s) where the function is decreasing
$f(x) = \frac{x^3}{3} + 2x^2 + x + 3$				
$g(x) = x^4 - 5x^2 + 4$				

57. Sam throws a ball up from the top of a building. The table below shows the height of the ball above the ground.

Time (sec) $x$	Height (ft) $f(x)$
0	400
1	434
2	436
3	406

- Write a quadratic function that best models the data.
- How high is the ball after 4 seconds?
- When does the ball hit the ground?